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ELECTRICITY
AND MAGNETISM

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PHYSICAL PRINCIPLES
OF
ELECTRICITY AND MAGNETISM

BY

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BLACKIE & SON LIMITED
LONDON AND GLASGOW
1930

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FROM THE PREFACE TO THE FIRST GERMAN EDITION

This book is essentially based on the Electricity section of my course of lectures on Experimental Physics and is intended for students of science and engineering. In the choice and arrangement of the material my aim has been to make the relationships between experimental facts clear and to give the reader an idea of the wide range of modern electrical theory. I have also sought to make it easier for the student to pass from books on Experimental Physics to those dealing with Theoretical Physics.

In all numerical examples I have employed the units in terms of which electrical instruments are always graduated, namely the ampere and the volt.

The experiments described are all simple and require only a small stock of apparatus. As far as possible they have been designed so as to exhibit the essential phenomena without any distracting external features.

PREFACE TO THE ENGLISH EDITION

This translation is based on the text and figures of the second German edition, from which it deviates only in a minor detail, namely the choice of the letters used for the various physical quantities. A number of examples have been added at the desire of the publishers.

Throughout the book I have attempted to make the explanations as simple as possible. I trust that this aim will be appreciated by English-speaking readers also.

Göttingen, February 1930.

R. W. POHL.

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CONTENTS

CHAPTER I

INSTRUMENTS FOR MEASURING CURRENT AND PRESSURE

SECT. PAGE

1. Introduction 1
2. Current and Pressure 2
3. Properties of a Current 3
4. Galvanometers and Ammeters 9
5. Properties of a Pressure 13
6. Electrostatic Voltmeters or Electrometers 14
7. Voltmeters traversed by a Current 15
8. Illustrations of the Size of the Volt and Ampere. The Potentiometer. 17

CHAPTER II

THE ELECTRIC FIELD

1. The Field and its Lines of Force 21
2. Electric Substances or Charges 28
3. Gradual Decay of the Electric Field owing to Conduction 29
4. The Mechanism of the Decay of the Field 30
5. Measurement of Electric Substance 32
6. An Example of the Measurement of Electric Substance or Quantity of Electricity 36
7. Capacity of a Condenser 37
8. Other Types of Condenser. Relative Dielectric Constants 38
9. Application of the Condenser Equation $Q = CE$ 40
10. Mobility of the Atoms of Electricity in Conductors 40
11. The Situation of the Atoms of Electricity on the Surface of the Conductor in the State of Equilibrium 42
12. Electrostatic Induction 46
13. An Application of Induction: the Occurrence of Atoms of Electricity in Pairs 48
14. An Application of Induction: Electric Screening 49
15. An Application of Induction: Induction Machines 50
16. The Situation of the Atoms of Electricity in and upon Insulators . . 56

SECT. PAGE
17. The Dielectric Constant 57
18. Atoms as Electric Fields. Ions 60
19. "Frictional Electricity" 61
20. Measurement of the Electric Field. **R**, the Intensity of the Field, and **D**, the Displacement-density 63
21. The Connection between **R**, the Intensity of the Field, and **D**, the Displacement-density 67
22. The Pressure E as the Line-integral of **R**. 67
23. Distribution of **R** in Fields of Various Forms. Calculation of Capacities 68
24. Forces and Work in the Electric Field 71
25. The Energy of an Electric Field 74
26. Calculation of the Forces in various Electric Fields 76
27. Change of Energy in the Electric Field while Work is being done. Further Examples of Forces in Electric Fields 79
28. Sensitive Electrostatic Voltmeters or Electrometers. 82
29. The Charge on a Single Atom of Electricity 83

CHAPTER III

THE MAGNETIC FIELD

1. Some Typical Magnetic Fields 86
2. **H**, the Intensity of the Magnetic Field, and the Magnetometer. . . 91
3. Iron in the Magnetic Field of a Coil, and Molecular Currents. . . . 93
4. The Essential Difference between the Electric Field and the Magnetic Field. 96

CHAPTER IV

THE CONNECTION BETWEEN ELECTRIC FIELDS AND MAGNETIC FIELDS

1. Preliminary Remarks 100
2. The Phenomena of Electromagnetic Induction 101
3. The Law of Electromagnetic Induction 104
4. The Magnetomotive Force M 106
5. A More Thorough-going Interpretation of the Phenomenon of Electromagnetic Induction. Closed Lines of Electric Force. Maxwell's Second Equation 113
6. Direct Current, Alternating Current, and Displacement Current . . . 115
7. The Magnetic Field of the Displacement Current and its Magnetomotive Force. Maxwell's First Equation. 120
8. Concluding Remarks 122

CHAPTER V

MAGNETIC FLUX, SELF-INDUCTION, MAGNETIC ENERGY, FORCES IN MAGNETIC FIELDS

SECT. PAGE
1. **B**, the Magnetic Induction, and $B = \mathbf{B}A$, the Magnetic Flux. Their Magnification by means of Iron 124
2. The Magnetic Circuit 129
3. The Permeability of Iron and the Hysteresis Loop. 130
4. Self-induction. 134
5. Induction Phenomena in Conductors, some Parts of which are in Motion relative to the other Parts 137
6. The Energy of a Magnetic Field 138
7. Forces in various Magnetic Fields 141

CHAPTER VI

ELECTRICAL GENERATORS AND MOTORS

1. Generators 148
2. The Transformation of Energy in Generators; Lenz's Law and Rotary Fields 155
3. Eddy Currents produced by Alterations of Magnetic Flux. The Permeability of Diamagnetic Substances 158
4. Electromotors 160

CHAPTER VII

THE INERTIA OF THE MAGNETIC FIELD AND ELECTRIC OSCILLATIONS

1. The Inertia of the Magnetic Field as a Result of Self-induction. . . 168
2. Transformers and Induction Coils. 175
3. Electrical Oscillations 180
4. Some Applications of Electric Oscillations. 185

CHAPTER VIII

THE MECHANISM OF CONDUCTION CURRENTS

1. The Mechanism of Conduction in a Model Experiment. 194
2. Dependent Conduction in the Air of the Room with Visible Electricity Carriers. The Meaning of Ohm's Law 198
3. Dependent Conduction in Air. Ions as Electricity Carriers. 202
4. Dependent Ionic Conduction in the Air of the Room. X-rays as an Ionizing Agent Ionic Mobility 204
5. Dependent Ionic Conduction in Air. The Saturation Current. 205
6. Dependent Conduction of Electricity in a High Vacuum 208

SECT. PAGE
7. The Atomic Weight of the Electron deduced from Observations on Cathode Rays 210
8. Applications of the Dependent Conduction of Electrons through a High Vacuum 212
9. Independent Conduction in Gases and Vapours at Low Pressure . . . 215
10. Some Applications of Independent Conduction in Gases at Low Pressure 218
11. Independent Conduction of Electricity in Gases at High Pressure. (Discharge from Points, Brush Discharge, Spark, Arc.) 225
12. Conduction in Liquids. General Considerations 230
13. Electrolytic or Ionic Conduction in Aqueous Solutions 231
14. The Ionic Charge. Faraday's Law of Equivalents. Avogadro's Number N 233
15. Ohm's Law for Electrolytic Conduction. The Velocity of the Ions . . 235
16. How the Heating Effect of the Current Arises 240
17. Practical Applications of the Electrolysis of Aqueous Solutions . . . 242
18. Ionic Conduction in Fused Salts and in Supercooled Liquids (Glasses) 243
19. Conduction in Liquids of High Specific Resistance 245
20. Conduction in Metals 246
21. The Effect of Temperature on Metallic Conduction. Superconduction 250
22. The Connection between Electrical Conductivity and Thermal Conductivity in Metals 254
23. Independent Conduction of Electricity in Solid Non-metals 256
24. Dependent Conduction of Electricity in Insulating Crystals 258
25. Concluding Remarks on the Mechanism of Conduction 259
26. The Connection between Induction Phenomena and the Mechanism of Conduction 260

CHAPTER IX

ELECTRIC FIELDS IN THE BOUNDARY LAYER BETWEEN TWO SUBSTANCES

Preliminary Remarks 262
1. Cataphoresis and Convection Currents 262
2. Electric Fields in the Boundary Layer between Two Metals 266
3. Metals as Conductors of the First Class. Thermo-electric Pressures . . 268
4. Electric Fields in the Boundary Layer between Metals and Electrolytes. Conductors of the Second Class, Cells 271
5. Polarization in Electrolytic Conductors 273
6. Accumulators 274
7. Unpolarizable Electrodes and Cells. Standard Cells 276
8. The Demonstration of Polarization by the Change in the Surface Tension of Liquids 278
9. Electric Fields in the Boundary Layer between Gases and Liquids . . 279
10. Emission of Electrons from a Surface. The Microphone 281

CHAPTER X

RADIOACTIVITY

SECT. PAGE
1. The Rays from Radioactive Substances 285
2. The Observation of Single Electrons and Ions. 289
3. The Direct Evaluation of the Universal Constant ***N***, the Number of Molecules in a Gram-molecule (Avogadro's Number). 293
4. The Decay of Radioactive Atoms. Atoms of Electricity as Essential Constituents of the Elements 294
5. The Artificial Disintegration of Atoms. The Question of the Ultimate Constituents of Matter. The Electrical Nature of Mass. 298

CHAPTER XI

ELECTRIC WAVES

1. Preliminary Remarks 303
2. The Production of High Frequency Alternating Currents by Undamped Oscillations 304
3. Forced Electrical Oscillations. 309
4. The Electric Dipole 311
5. Stationary Electric Waves between Two Parallel Wires. 315
6. The Importance of Stationary Electric Waves in Wires. The Propagation of Electric Fields with the Speed of Light 318
7. Direct Measurement of the Velocity of Progressive Waves in Wires. . 321
8. The Displacement Current of the Dipole. The Radiation of Free Electric Waves 323
9. Half-free Waves. Wireless Telegraphy. 331
10. The Essential Identity of Electric Waves and Light Waves. The Whole Spectrum of Electric Waves 333

EXAMPLES 337
SOLUTIONS AND ANSWERS 343
INDEX 349

ELECTRICITY AND MAGNETISM

CHAPTER I

Instruments for measuring Current and Pressure*

1. Introduction.

The ideas of *length*, *mass*, and *time* form the starting-point of any thorough-going exposition of mechanics, and modern measuring instruments are used from the very beginning. No one ever dreams of explaining in all its detail the history of the choice of the second as a unit of time, or of using a sundial or clepsydra to measure time when he is demonstrating scientific experiments. Without the slightest hesitation the scientist proceeds to use a watch, or an up-to-date stop-watch which enables him to take readings correct to $^1/_{100}$ of a second. He may perhaps say a word or two about the fundamental principles which govern the working of all clocks, but no more is expected of him, for one can use a clock without knowing anything about the detailed construction of clocks or about their historical evolution.

In passing on to the theory of heat two new ideas are usually introduced, namely *temperature* and *quantity of heat*, and modern forms of thermometers are utilized in experiments from the start.

Curiously enough, this approved method is abandoned in the case of electricity. In order to follow out the historical order of its development, one toils wearily through whole chapters on end without using instruments of precision. All the experiments are performed under the auspices of the bar of sealing-wax and the inevitable catskin.

With all due respect for historical order, we consider this method obsolete. We choose to follow the method which has for long been customary in mechanics and heat. *We begin by introducing two new ideas, current and pressure, and at once proceed to use up-to-date measuring instruments. We shall consider only the fundamental principles of the latter, and not the details of their construction.*

---

* [The German word *Spannung* has been translated by *pressure* throughout. Other terms used in English are *potential difference* (P.D.), *voltage*, and sometimes *tension*. The term *electromotive force* (E.M.F.) is frequently used as practically synonymous with the above.]

The passage from mechanics and heat to electricity involves two specifically electrical ideas. The choice of current and pressure is convenient, but perfectly arbitrary. In theory we could equally well start from quantity of electricity and capacity or from some other suitably chosen pair.

2. Current and Pressure.

The terms "electric current" and "pressure" are well known in ordinary everyday speech. By these words the layman understands something like the following:

1. Every piece of electrical apparatus has two "terminals", "poles", or "contacts", e. g. the familiar plug contacts for connecting up an electric lamp. If the fingers tingle when the two poles are touched, there is a "pressure" between the poles.

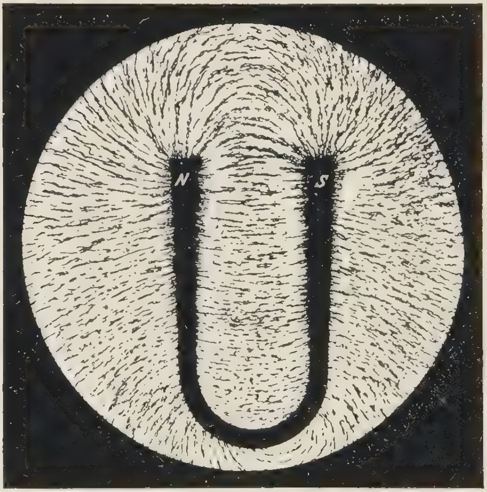


Fig. 1. Lines of magnetic force, shown by iron filings.

2. If the two poles are connected by a piece of string, wood, or glass, nothing happens. These materials are "insulators".
3. If a wire is used instead of the string, etc., there is a display of fireworks. The wire melts or explodes, and we speak of a "short circuit" or of an "arc".
4. If we connect any piece of apparatus, e. g. a table lamp, to the poles by means of two wires, "a current flows through these wires and through the lamp".

This is about the extent of the meaning of current and pressure in ordinary speech. We shall now restate the ideas of current and pressure a little more accurately. On pp. 3—9, 13—14 we shall discuss the characteristic properties of current and pressure.

3. Properties of a Current.

We cannot tell merely by looking at a metal pipe whether water is flowing through it or not. We must find out e. g. whether there is a rushing noise, by putting our ear to the pipe, or the like. Similarly, it is only by certain effects that the flow of an electric current is made noticeable to us, and these we regard as properties of the current.

Before enumerating these properties, we shall add two facts which have long been known:

1. Between the "north pole" and the "south pole" of a permanent magnet there run "*lines of* (*magnetic*) *force*". In order to make them visible, we lay e. g. a horse-shoe magnet on a sheet of paper or glass and strew iron filings on the paper or glass, gently tapping the sheet meanwhile. What we see is shown in fig. 1.

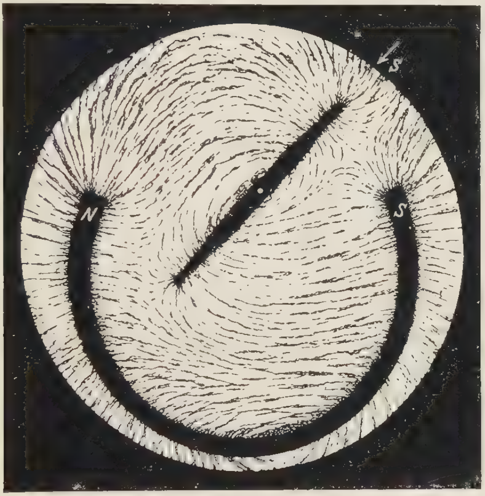


Fig. 2. Lines of force. The horse-shoe magnet NS makes the compass-needle turn in a counterclockwise direction.

2. A magnet exerts mechanical force on another magnet and on soft iron. In both cases the iron filings give us a remarkably clear picture of the lines of force. Fig. 2 shows a horse-shoe magnet "trying to make a compass-needle turn". Fig. 3 shows a horse-shoe magnet "attracting" a piece of soft iron (a key). Here we are purposely using a somewhat primitive form of expression.

Having made these preliminary remarks, we now state the three characteristic properties of an electric current:

(1) *The current produces a magnetic field.* A wire carrying a current is surrounded by ring-shaped lines of force.

In fig. 4 these lines of force are shown by means of iron filings on a glass plate. The wire was perpendicular to the plane of the paper and was subsequently removed from the hole in the middle.

The magnetic field of a current is capable of causing various mechanical motions. We shall give six different examples, a—f.

(a) A bar magnet (compass-needle) *NS* is suspended above a straight conducting wire *KA* and parallel to it (fig. 5a). When the current is switched on the magnet sets itself obliquely to the conductor (fig. 5b).

(b) The process can be reversed. In fig. 6a the bar magnet *NS* is held fast. Beside it there hangs an easily movable ribbon of woven metal, *KA*. When the current passes the conductor sets itself obliquely to the magnet: the ribbon coils itself round the magnet in a spiral (fig. 6b).

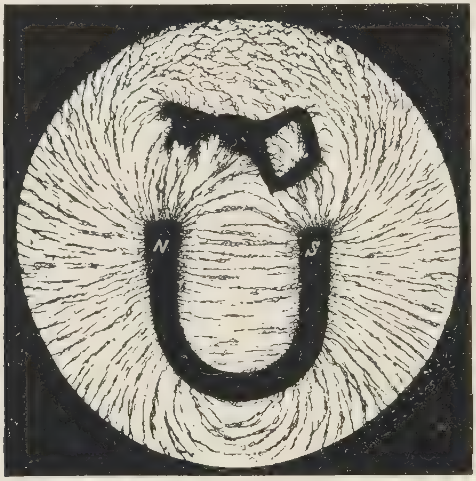


Fig. 3. Lines of force. Attraction of an iron key by a horse-shoe magnet.

(c) We bring a straight conductor *KA* into the magnetic field of a horse-shoe magnet *NS* (fig. 7a). The conductor is suspended like a trapeze. When the circuit is closed we see a motion in one or other of the directions indicated by the arrow (fig. 7b).

(d) We replace the straight conductor by one wound in a coil. When the circuit is closed the conducting coil turns about its axis *KA* (fig. 8a and b).

(e) In all the cases up to now the magnetic field of a conductor acted on a magnetic field of a permanent magnet. We can replace the magnetic field of the latter by that of a second conductor carrying a current. In figs. 9a and b the current flowing in at *K* is made to divide into two branch currents which meet again at *A*. The parts of the conductor between *K* and *A* consist of two bands of woven metal which are slightly stretched. When the current is off they hang almost parallel to each other; when it is on, they fall together till they are almost touching.

Figs. 9c and d show a variant of this experiment which is often put to practical use. The two movable bands are replaced by a fixed coil and a coil free to turn, which are both traversed by the same current (fig. 9c). The movable coil sets itself parallel to the fixed one (fig. 9d).

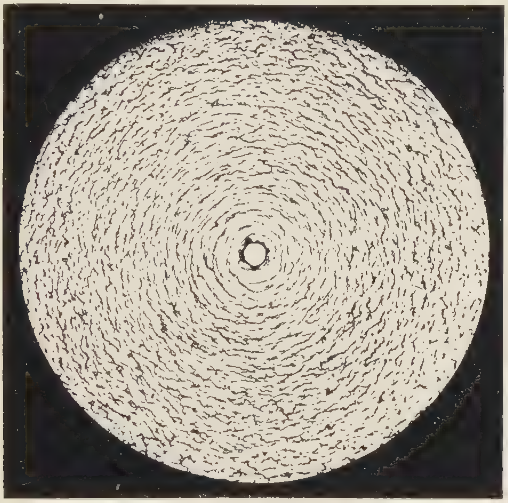


Fig. 4. Circular lines of force of a wire carrying a current.

(f) Finally (in analogy with fig. 3) we take a piece of soft iron Fe, as shown in fig. 10. It is attracted into the magnetic field of a conducting coil (fig. 10b).

So much for our examples of mechanical motion caused by the magnetic field of a current.

(2) *The conductor through which a current is flowing is heated.* It may be heated white-hot, as is the case with incandescent electric lights. Fig. 11 shows a very simple experiment on the expansion of a wire owing to the heating effect of the current.

All this refers to *solid* conductors; we have used metal wires.

A *liquid* conductor likewise exhibits magnetic and heating effects. To prove the existence of the magnetic field, we use a glass coil filled with acidulated water (fig. 12). A small compass-needle, the ends of which are made conspicuous by having small paper flags stuck on to them, is placed inside the coil. Two wires *K* and *A* serve as leads.

In the case of liquid conductors we observe a third effect, in addition to the magnetic and heating effects:

(3) *In liquid conductors the current causes chemical (so-called electrolytic) phenomena.* Examples:

(a) Two platinum wires K and A are introduced into a dish of acidulated water to serve as "electrodes" (fig. 13). When the current passes, bubbles of oxygen come off the electrode A, and bubbles of hydrogen off the electrode K. In accordance with convention, we call the electrode K, from which hydrogen is evolved, the negative pole. The other pole is called the positive pole. *That is, we define the difference between the negative pole and the positive pole electrolytically.*

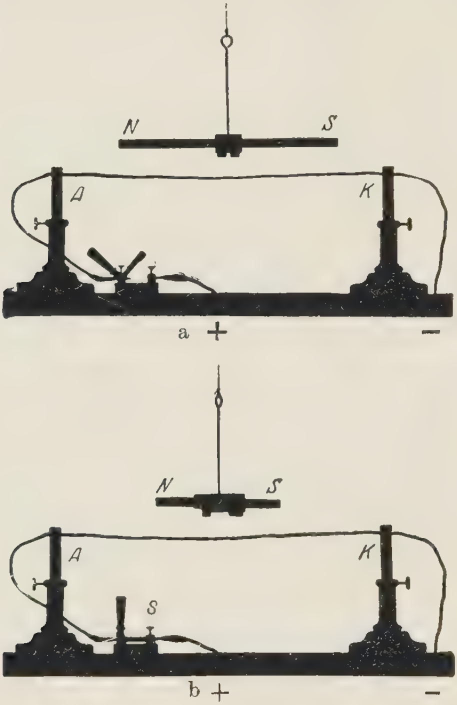
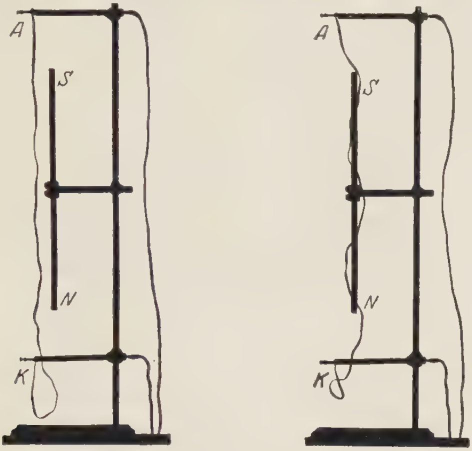


Fig. 5a, b. Firmly fixed conductor KA and movable suspended bar magnet NS; switch on left. (The end of the magnet marked N projects from the plane of the paper in the direction of the observer.)

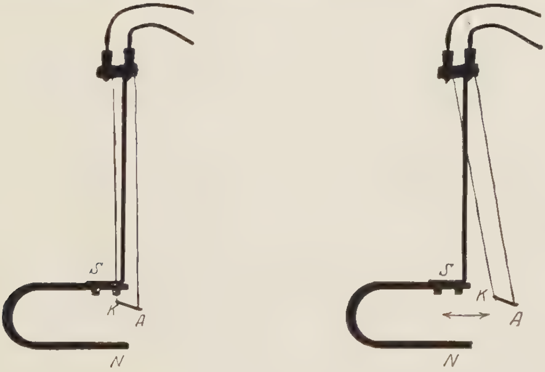
(b) Two electrodes consisting of lead wire project into a dish of aqueous solution of lead acetate. When the current passes an exceedingly graceful "lead tree" of crystal plates is formed at the negative pole as we watch (fig. 14). In this case, therefore, the electrolytic effect consists of the separation of a metal.

Finally, instead of a solid or liquid conductor we take a conducting *gas*. The U-shaped tube in fig. 15 contains the inert gas neon. Two metallic electrodes K and A again serve to convey the current to and from the tube. A small rider at the top of the tube carries a compass-needle NS. We connect A and K with the town mains. At once we see all three effects of the current. The magnetic needle is deflected; the tube becomes warm; and a dazzling orange-red light filling the



a b

Fig. 6a, b. Firmly fixed bar magnet NS and flexible movable conductor KA, of woven metal ribbon.



a b

Fig. 7a, b. Fixed horse-shoe magnet and straight movable conductor KA, suspended like a trapeze by woven metal leads*: also, diagram of a "string galvanometer".

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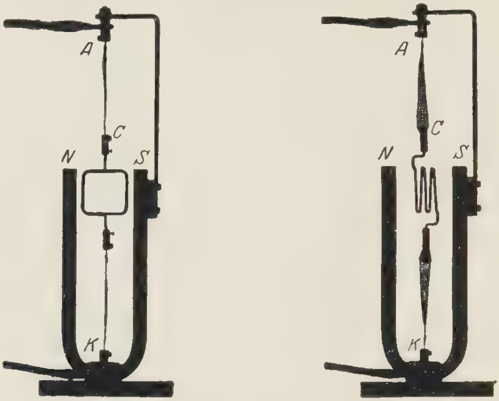
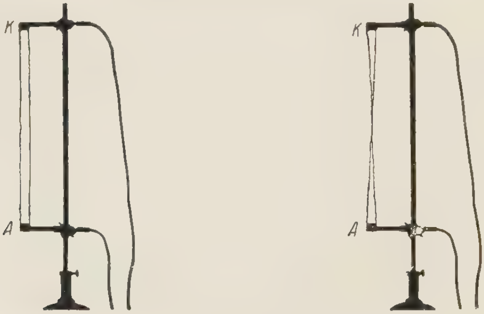
* [The conductors conveying electricity to and from some piece of apparatus under consideration are called *leads*].

tube indicates radical changes in the gas molecules, such as we find in the case of chemical processes in flames.



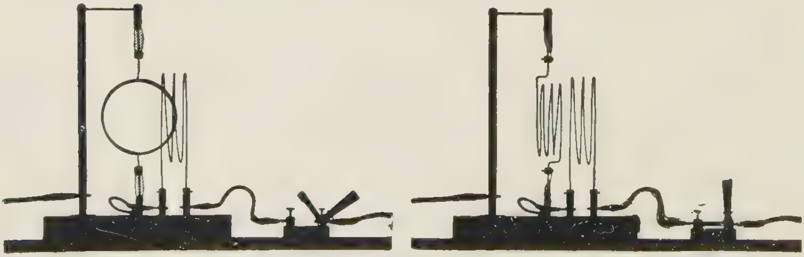
a b

Fig. 8a, b. Fixed horse-shoe magnet *NS* and conducting coil *KA* free to turn, with woven metal leads: also, diagram of a "moving-coil ammeter" or "moving-coil galvanometer".



a b

Figs. 9a, b. Mutual attraction between two conductors (metal bands) carrying a current.

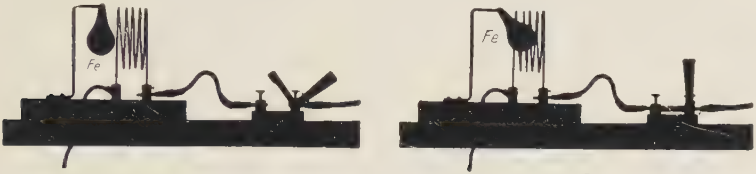


c d

Figs. 9c, d. On the right a fixed coil; on the left a coil free to turn, with leads of woven metal.

In short, up to now we have observed the following properties of a current in a conductor:

1. Magnetic field } in all conductors.
2. Heating effect }
3. "Chemical" effects (in an extended sense) } in liquid and gaseous conductors.



Figs. 10a, b. Fixed coil and soft iron Fe suspended so as to be free to move.

4. Galvanometers and Ammeters.

All three properties of electric currents given in the last section can be used in the construction of measuring instruments:

(a) *Galvanometers on a magnetic principle.* The mechanical forces make an axis carrying a pointer turn; e. g. the *moving-coil ammeter.*

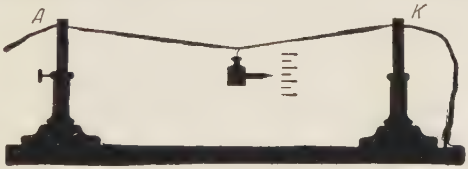


Fig. 11. Expansion of a wire *KA* heated by a current.

This is an easily understood development of the arrangement shown in fig. 8: imagine a pointer of some kind fixed to the moving coil of fig. 8. Fig. 16a shows the coil of such an ammeter with a mechanical pointer. An *optical lever* is used instead of the latter in sensitive instruments. The moving part carries a mirror *R* by means of which a beam of light is reflected on to a scale (fig. 16b).

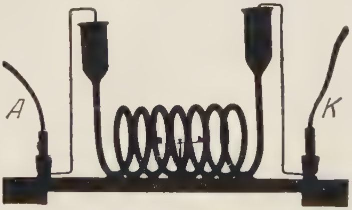


Fig. 12. A compass-needle in the magnetic field of a coil of liquid carrying a current.

In future we shall represent all "moving-coil instruments" by the symbolic diagram of fig. 18.

Technical variations are the *loop galvanometer* (fig. 17a) and the *string galvanometer.* These galvanometers combine high sensitiveness with great rapidity of action. They are indispensable when self-registering instruments are required. In the loop galvanometer (oscillograph) the place of the coil is taken by a stretched loop with a very small light mirror. In the string galvanometer the trapeze-like conductor of fig. 7 is replaced by a stretched wire, the motion of which is observed by means of a microscope (see fig. 17 b).

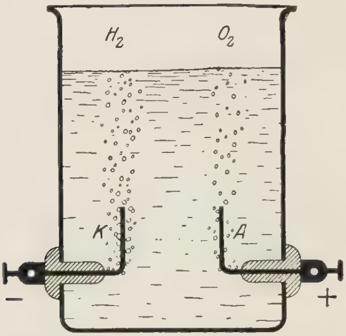


Fig. 13. Formation of hydrogen (H_2) and oxygen (O_2) when a current passes through dilute sulphuric acid.

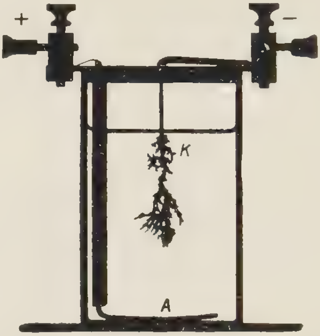


Fig. 14. Formation of lead crystals when a current passes through an aqueous solution of lead acetate.

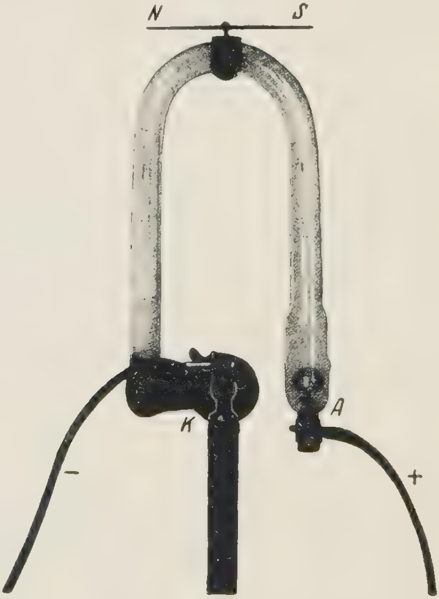
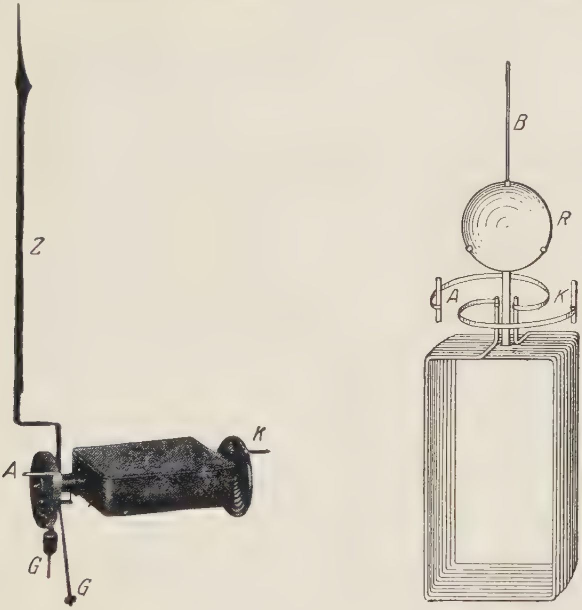


Fig. 15. The inert gas neon as a gaseous conductor in a glass U-tube: *K* and *A* are metallic electrodes, *NS* a compass-needle.

(b) *Galvanometers in which the heating effect is used.* The current to be measured heats a wire *KA*. The wire becomes longer and the expansion is transferred to a pointer arrangement in some way or other, e. g. as in fig. 19. *We shall henceforth indicate these "hot-wire instruments" in our sketches by means of the diagram of fig. 20.*

(c) *Galvanometers on an electrolytic principle.* The amount of metal deposited at the negative pole by the current is ascertained by weighing, and the corresponding time is measured by a clock. The amount liberated per second is a measure of the current. Arrangements for reading off this quantity directly from a pointer and scale are easy to design but are not used at all.



a b

Fig. 16a, b. Two forms of the coil of moving-coil ammeters: (a) with mechanical pointer Z and cup bearings, as in the ammeters of figs. 31, 32, and 25, p. 35; GG are weights to counterbalance the coil: (b) with mirror R and optical lever, and fibre suspension B, as in the "sensitive ammeters" or "galvanometers" in figs. 33, p. 19, 16, p. 30, and 5, p. 128; K and A are spiral leads; in addition, K and A, or B, cause a restoring couple, i. e. bring the coil back to the zero position when there is no current passing.

Galvanometers alone do not enable us to measure a current: we must in addition settle on our *unit of measurement.* There is great confusion in elementary textbooks about the definition of physical units.

All units are arbitrary: e. g. in measuring electric current it is a matter for choice whether the magnetic, the electrolytic, or the heating effect of the current is used to define the unit.

Consequently there is a whole set of different systems of units. We are emphatically in favour of the *international system of electrical units* alone.

In the international system of electrical units the unit of current is defined electrolytically. A current which in one second liberates 1·1180 milligrams of silver has unit strength. This unit current is called "one ampere". The definition of the ampere thus depends on the use of two particularly reliable instruments, the clock and the balance. The ampere unit satisfies the condition of being easily reproducible. The peculiar decimals arise from historical considerations. They enable the international unit to be con-

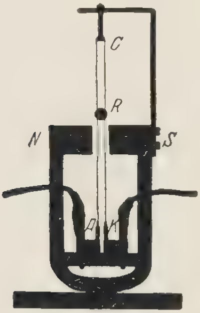


Fig. 17a. Model of a loop galvanometer or oscillograph. *ACK* is a stretched loop of wire carrying a mirror *R*. (N. B. The loop of wire must on no account be in resonance with the alternating current to be recorded).

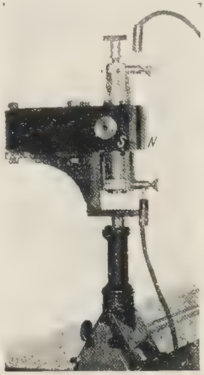


Fig. 17b. A string galvanometer for projection work: the microscope objective forming the image is seen above and to the left of *S*.

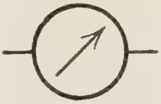


Fig. 18. Symbolic diagram of a moving-coil ammeter. This diagram will also be used later in the case of ammeters which are graduated for use as voltmeters.

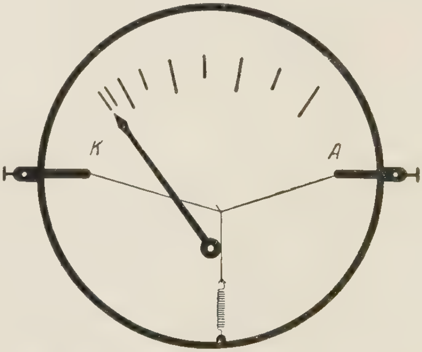


Fig. 19. Diagram of a hot-wire ammeter. The thread between the stretched spring and the hot wire *KA* is to be imagined as being wound round the axis of the pointer.

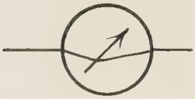


Fig. 20. Symbolic diagram of a hot-wire ammeter. This will also be used later in the case of hot-wire ammeters which are graduated for use as voltmeters.

verted into others previously in use, merely by multiplying by powers of ten.

Besides the international unit of current, two other units still maintain their ground in physical literature. Both belong to the so-called "absolute" or "c. g. s." system. These units are never used for measuring with, but only for calculation. Ammeters are never graduated in terms of them.

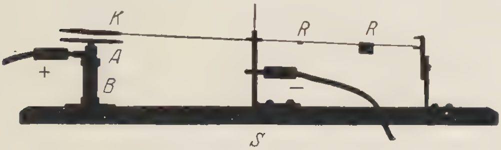


Fig. 21. "Pressure balance". *B* is an amber insulator.

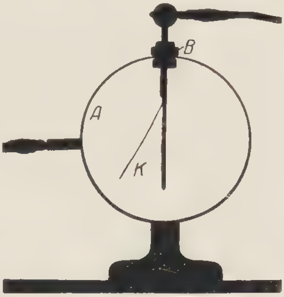


Fig. 22. Electroscope with a gold-leaf pointer.

5. Properties of a Pressure.

1. *A pressure is capable of producing a current in a conductor.*

If two contacts, binding-screws, etc. between which there is a pressure are connected by a conductor, a current flows through the conductor. In view of what has been said on p. 2, this requires no further explanation.

2. *Two bodies between which there is a pressure exert forces on each other.*

This can easily be proved by means of an instrument for measuring force, e. g. a balance. Fig. 21 shows a balance beam of light aluminium tubing, resting on the metal pillar *S*. On the left arm there is a metal disc *K*, on the right small paper riders *R* to act as a counterpoise. Under the metal disc *K* there is another fixed metal disc *A* at a distance of several millimetres. The disc *A* and the pillar *S* are each connected by a wire to a pole of the town mains. The beam of the balance is immediately deflected. The electrical pressure between *A* and *K* therefore gives rise to forces which can easily be shown to exist.

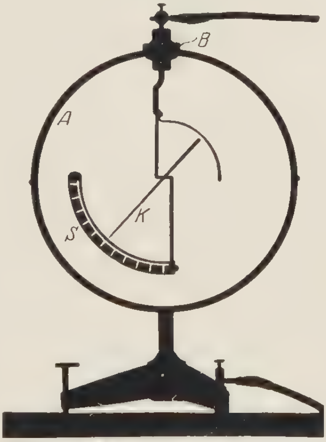


Fig. 23. Electrostatic voltmeter with an aluminium pointer on cup bearings. This can be used from several hundred to about 10000 volts.

The two properties of pressure given in this section may be used in the construction of instruments for measuring pressure or "voltmeters". Accordingly we distinguish between:

(a) Voltmeters traversed by a current.
(b) Electrostatic voltmeters or "electrometers".

We shall consider each group separately in the next two sections.

6. Electrostatic Voltmeters or Electrometers.

These instruments utilize the forces arising from electrical pressure. They are on the same principle as a *letter balance*: the forces due to the pressures cause deflections which can be read directly on a scale. Out of a whole series of different forms we shall mention only three:

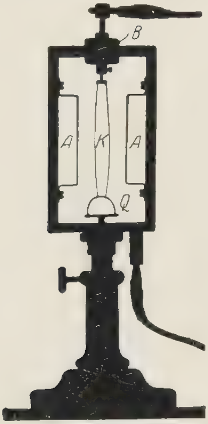


Fig. 24a. Model of a "bifilar electrometer" or "bifilar voltmeter". Range from about 30 to 400 volts.

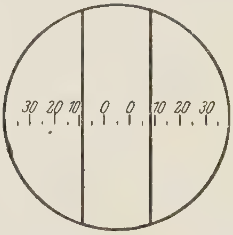


Fig. 24b. Field of view of a bifilar voltmeter.

(a) *The gold-leaf electrometer.* A metal peg, insulated by amber (B) or the like, projects into the metal case A. At the side of the peg there is a small piece of gold-leaf K which acts as a movable pointer. A pressure is produced between A and K, e. g. by connecting them with a source of current. The gold-leaf pointer is attracted by the wall and the size of the deflection is read off on a scale.

(b) *The pointer electrometer* (fig. 23). Everything is the same as in (a), except that the gold-leaf is replaced by an aluminium pointer K on cup bearings.

(c) *The bifilar electrometer* (fig. 24a). In this type also a metal peg insulated by amber is introduced into a metal case A. From the peg there hangs a loop of fine platinum wire K which is stretched by a small semicircular piece of quartz Q at the foot. Electrical pressures between K and A make the threads approach the walls, or more accurately, wire loops fixed to the walls. The distance between the threads therefore increases and the increase is measured by a microscope. Fig. 24b shows the field of view and scale. The bifilar voltmeter is very well suited for projection work. On account of its rapidity of action it is an uncommonly convenient instrument for measuring and demonstration. Its usual range is from about 30 to 400 volts.

Fig. 25 shows an older variant of this instrument. Instead of the two metal threads we have two small gold-leaves acting as pointers. Each pointer is attracted by the wall next to it, so that the two leaves fly apart. In older accounts this is frequently explained as a repulsion of the pointers by each other. This simple type of instrument is quite sufficient for many experiments.

In future we shall always use the symbolic diagram given in fig. 26 to denote an electrostatic voltmeter.

The unit of pressure in the international system of electrical units is the volt. The latter is fixed by defining the pressure of a "standard cadmium cell" to be 1·0187 *volts.* The decimals again arise from historical considerations. Had it not been for tradition and the existence of earlier systems of units the pressure of the standard cell would have been defined as one volt.

The international standard cell does not have zinc and carbon for its electrodes, like the well-known pocket flash-lamp batteries, but mercury and cadmium (p. 277).

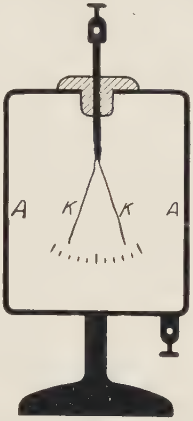


Fig. 25. Electrostatic voltmeter with two gold-leaves *KK*.

7. Voltmeters traversed by a Current.

In principle these instruments are re-graduated ammeters. The possibility of this re-graduation depends on the validity of *Ohm's law* for metallic conductors.

In general we define the resistance r of any conductor* to be the quotient

$$\frac{\text{Pressure }(E)\text{ between the ends of the conductor}}{\text{Current }(i)\text{ in the conductor}}.$$

This quotient E/i, which is defined to be the resistance r, is not in general constant for the conductor in question. The resistance r depends in a complicated way on the current i and on numerous conditions of experiment. *It is only in particular cases that the quotient E/i is constant. In these particular cases Ohm's law* — which is one of our most important experimental results — *is true.* In words:

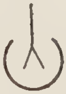


Fig. 26. Symbolic diagram of an "electrostatic voltmeter" or "electrometer": when without scale it is also called an "electroscope".

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* The word "resistance" is used in two senses. To begin with, it denotes E/i, the quotient of pressure by current, measured for an arbitrary conductor. It is also used to denote a piece of apparatus, e. g. a coil of conducting wire, such as is shown in fig. 30. In this book we shall always use R to denote a resistance apparatus.

The pressure E between the ends of a conductor is proportional to the current i in the conductor, or

$$E = ir. \tag{1}$$

A metallic conductor at constant temperature is an example of a particular case where Ohm's law holds. This is shown by means of the arrangement sketched in fig. 27. A source of current sends a current through a metallic conductor KA, in the form of a band or strip, for example. The ammeter G gives the current strength i, the voltmeter V the pressure E between the ends of the conductor KA. We alter the current strength i by using various sources of current, e. g. one time a few cells, another time the town mains. If we divide the values of the pressure E by the corresponding values of the current strength i, we always find that the quotient E/i, which is defined to be the resistance r, is constant.

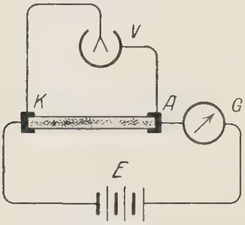

Fig. 27. Demonstration of Ohm's law.

We therefore measure the resistance r by the quotient volts/amperes, just as we measure e. g. velocity in cm./sec. The attempt was made to introduce the word "cel" as an abbreviation for cm./sec., but it has not been adopted. Similarly the *quotient volts/amperes has been denoted by the word "ohm".* This word is in general and international use.

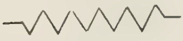
Fig. 28. Diagram of a resistance (cf. the note on p. 15).

If e. g. in the experiment of fig. 27 we find that for the conductor KA the ratio defined as the resistance is 500 volts/amperes, we say for short that the conductor KA has a resistance $r = 500$ ohms.

In future diagrams we shall represent a conductor whose resistance is important by the symbol shown in fig. 28.

The fact that Ohm's law is true for metallic conductors enables ammeters to be re-graduated very simply. All that is required is to apply Ohm's law to the metal wires in the ammeter. We shall explain this in detail by means of an everyday example, namely, the scales used for counting money.

A letter balance is generally graduated in ounces. We can easily re-graduate it in pence: then it will at once give us the value in pence of a heap of coppers piled on the balance-pan.

Legally, one ounce of bronze is equivalent to 3 pennies, or no. of pence = no. of ounces × 3. We have only to multiply the numbers on the ounce scale by 3 in order to change the ounce readings into pence readings.

Similarly, for the wire carrying a current inside an ammeter, we have Ohm's law

$$\frac{\text{no. of volts}}{\text{no. of amperes}} = r, \quad \text{or no. of volts} = \text{no. of amperes} \cdot r,$$

where r denotes the resistance of the wire in the ammeter. Hence we have merely to multiply the numbers of the ampere scale by r in order to get the scale in volts.

That is, voltmeters using a current are in principle, as we have already said, nothing but re-graduated ammeters. For this reason we shall denote them also in our sketches by the diagram of figs. 18 or 20, in contradistinction to fig. 26, which is the diagram of an electrostatic voltmeter.

All the details we have given about voltmeters and ammeters are by no means meant as exhaustive. They are only to be taken as a superficial explanation of the simplest principles of modern practical ammeters and voltmeters and their graduation. This information enables us to work just as well with modern electrical instruments as we can with modern clocks when we are explaining mechanics, without knowing anything about the details of their construction.

Next, it is advisable that we should form a definite idea of the order of magnitude of the volt and ampere from a few examples.

8. Illustrations of the Size of the Volt and Ampere. The Potentiometer.

(a) Pressures of the order of 1 volt exist between the poles of electric cells such as are used for electric bells, etc.

(b) The pressure between the terminals of the town mains amounts to some hundreds of volts. In Glasgow it is 220 volts.

(c) In the case of pressures of thousands of volts, sparks pass. About 3000 volts enable the current to cross an air-gap of 1 mm.

(d) The pressures between the conductors of long-distance transmission installations are generally about 15000 volts (occasionally as much as 200000 volts).

(e) The pressures between clouds and the earth which cause lightning are much higher still. Several hundred million volts is probably too low a figure.

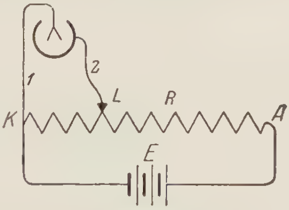

Fig. 29. Diagram of a potentiometer.

For many experiments we require *variable* pressures. There is a simple technical device for producing these pressures as fractions of a high pressure. We use a *potentiometer* (fig. 29). We connect the two poles of the source of current E by a "resistance" R[*]. In practice this is always a badly-conducting wire of some definite alloy, wound spirally on a cylinder. Then between the ends KA of the resistance there is the whole pressure of the source of current. Between one end of the resistance and the middle there is half the pressure, and so on for the other fractions. We therefore connect a wire 1 to one end of the resistance and a second wire 2 to a metal slider *L*. Then by moving the slider between *K* and *A* we can produce any pressure between zero and the highest pressure.

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[*] Cf. the note on p. 15.

Fig. 30 shows two handy forms of variable resistance for use as potentiometers. The knob *G* serves to move the slider.

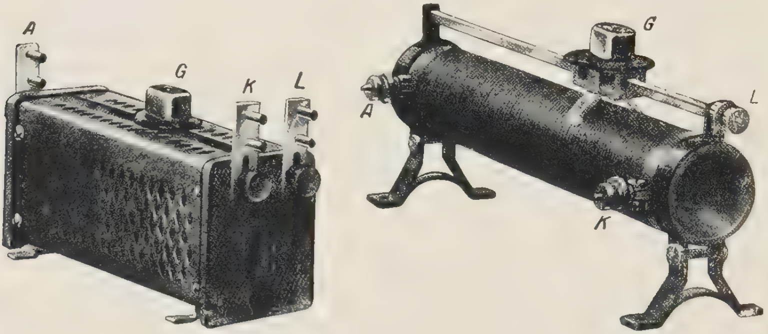


Fig. 30. Two practical forms of resistance with sliding contacts; that on the left has a protective casing.

Now for some examples of *current strengths in amperes.*

(a) Currents of the order of 1 ampere, but often only a few tenths of an ampere, flow through ordinary electric lights in rooms.

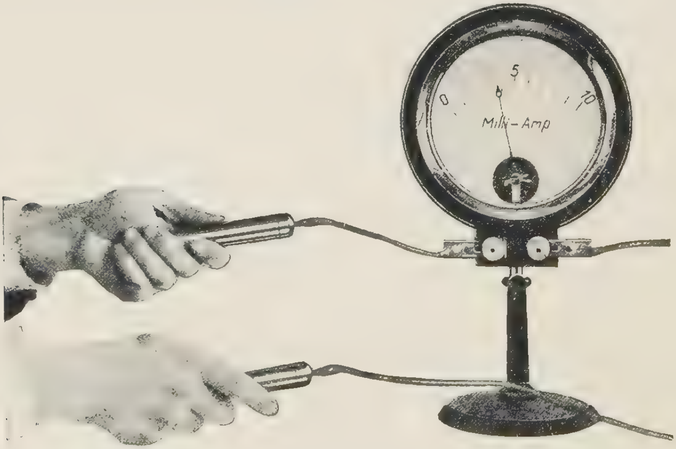


Fig. 31. Bringing of a person under experiment into circuit. Ammeter on the principle of fig. 8. The handles contain invisible safety resistances which prevent any defects in the circuit from endangering the experimenter.

(b) 100 amperes is about the current required by a tramcar.

(c) $^1/_{1000}$ ampere is called a "milliampere". 3—5 milliamperes is the smallest current that our bodies can detect. This is easily shown with the arrangement of fig. 31. The person under experiment is brought into the circuit by means of two metallic handles. The pressure required is gradually and uniformly increased by the process of dividing up the pressure explained above.

(d) The toy known as the "induction machine" gives a current of about 10^{-5} ampere. As is shown in fig. 32, we measure this current directly by means of a practical ammeter. Beginners frequently imagine that an induction machine supplies "static" electricity, but that

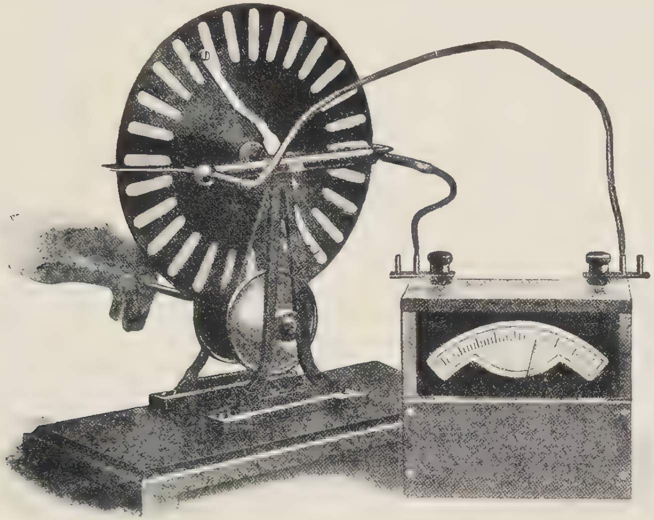


Fig. 32. Measurement of the current produced by a Holtz induction machine by means of a moving-coil ammeter (of the type in fig. 8).

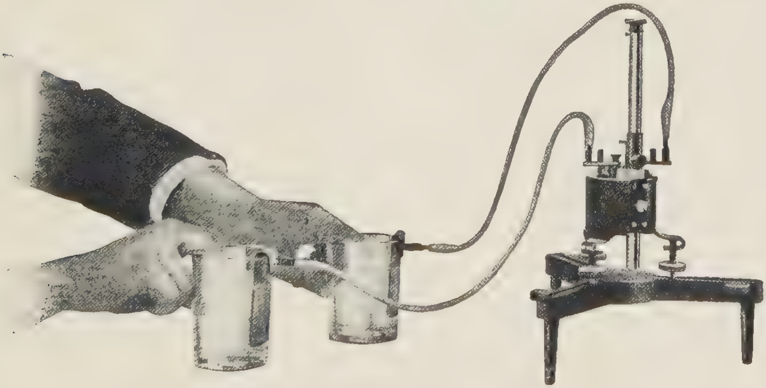


Fig. 33. Observation of feeble currents when the muscles of the fingers are stretched. The moving-coil galvanometer (of the type in fig. 8) has a mirror reflecting a beam of light on to a scale and stands on an iron base. The type of galvanometer shown is distinguished by its particularly short time of vibration. T, the period of a complete (not a half) vibration, is only about 4 seconds.

an ammeter measures only "current" electricity. *There is no difference between static and current electricity!*

(e) 10^{-6} ampere is called a micro-ampere. We can easily produce currents of this order of magnitude with our bodies. As shown in fig. 33, we dip two fingers of each hand into two glass dishes filled with conducting water, and place our thumbs against the outsides of the dishes. Each of the dishes has a zinc plate dipping into it. Conducting wires lead from the two zinc plates to a galvanometer with mirror and scale (usually called a mirror galvanometer). If we keep our hands relaxed, no current is observed. If we stretch the finger muscles of *one* hand, the galvanometer indicates a current of the order of 10^{-6} ampere. If we stretch the fingers of the other hand we observe an equal current but in the opposite direction.

(f) Good mirror galvanometers enable us to measure currents down to about 10^{-11} ampere.

This lower limit is fixed by the Brownian movement of the moving system (moving coil, etc.). In the case of greater sensitivity (lighter coil or finer suspension) the zero-point of the instrument moves just as irregularly as a particle of dust exhibiting the Brownian movement, but much more slowly.

CHAPTER II

The Electric Field

1. The Field and its Lines of Force.

Fig. 1 shows two parallel metal plates *A* and *K*, the supports of which contain amber insulators *B*. We connect the plates with the town mains by two wires and with a bifilar voltmeter by two other wires. We then have the diagram of fig. 2a, which there will be no difficulty in following.

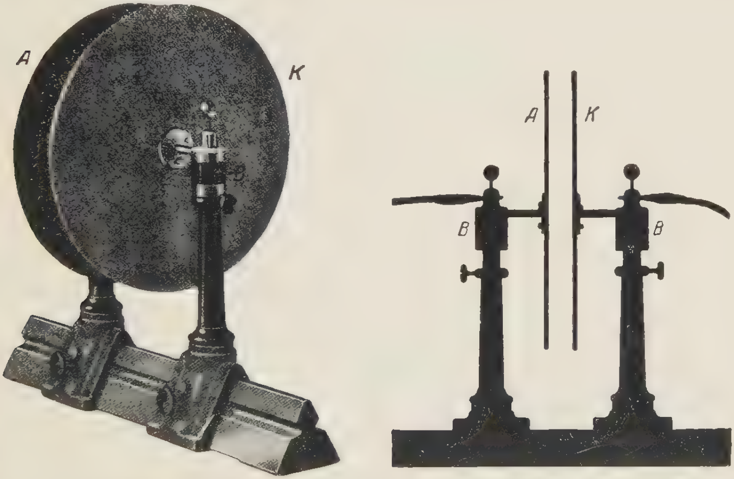


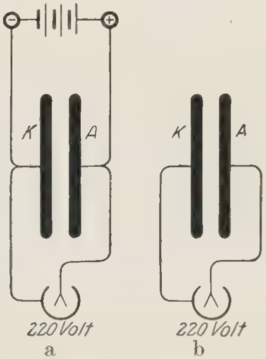
Fig. 1. Photograph and silhouette of a plate condenser with amber insulators *B*. Diameter of plates about 22 cm.

The voltmeter shows that there is a pressure of 220 volts between the plates.

The cause of this pressure we at first take to be the fact that the plates are in connection with the mains; but experiment contradicts this view. *The pressure still remains after the mains have been disconnected* (fig. 2b). This is highly important.

The question arises: in what way do the plates, between which there is a pressure, differ from the same plates in their ordinary condition? Where is the cause of the pressure located? On one plate, or on both? Or perhaps even in the space between?

A simple experiment shows that the space between the plates plays an essential part. Without touching the plates, we put an insulating plate, e. g. of vulcanite, into the space between them (fig. 3). The pressure falls, say to 100 volts. We take the piece of vulcanite out and the former pressure of 220 volts is restored.



a b

Figs. 2a, b. *KA* plate condenser, a in, b after, connection with the town mains.

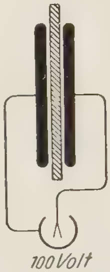


Fig. 3. An insulating plate between the plates of the condenser.

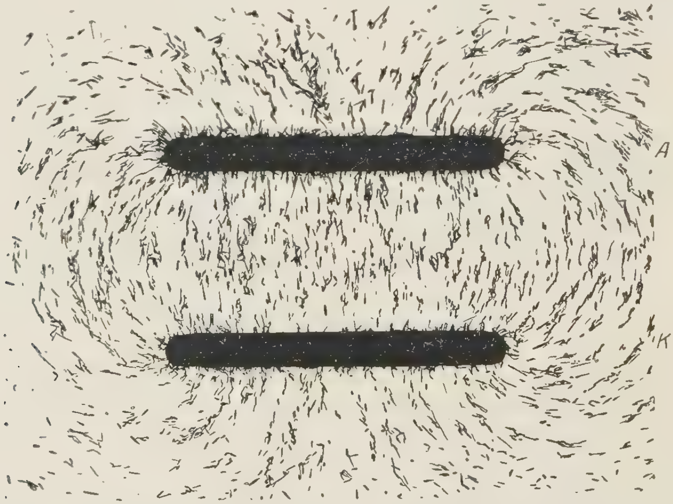


Fig. 4. Lines of (electric) force of a plate condenser, made visible by means of gypsum crystals. Neither this nor any of the following illustrations of lines of force have been retouched.

The result of the experiment is similar if we introduce a metal plate instead of an insulating one. The space between the plates therefore plays an essential part in the production of the pressure.

We ask further: what is there in the space between the plates? The physicist replies: *an electric field.*

At first sight this seems a mere phrase used as a cloak for ignorance. This is not so, however. *"Electric field" is a technical expression with which a sharply-defined conception entirely derived from experiment has been associated in physics.* We shall therefore proceed to investigate the properties of the electric field by experimental means.

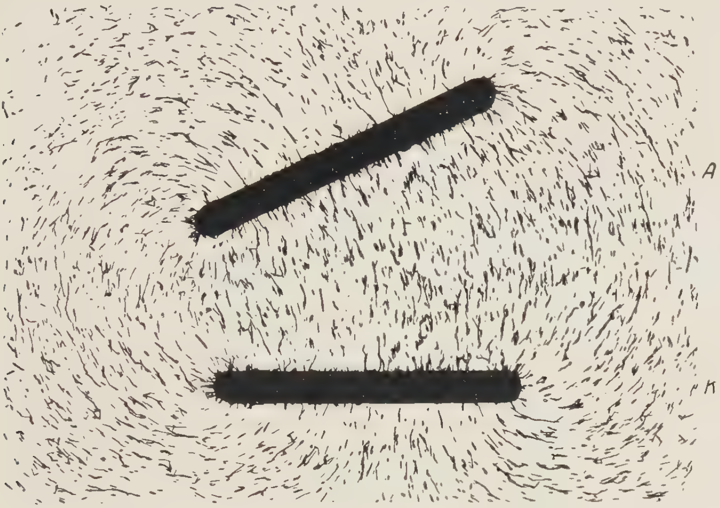


Fig. 5. Lines of force between oblique plates.

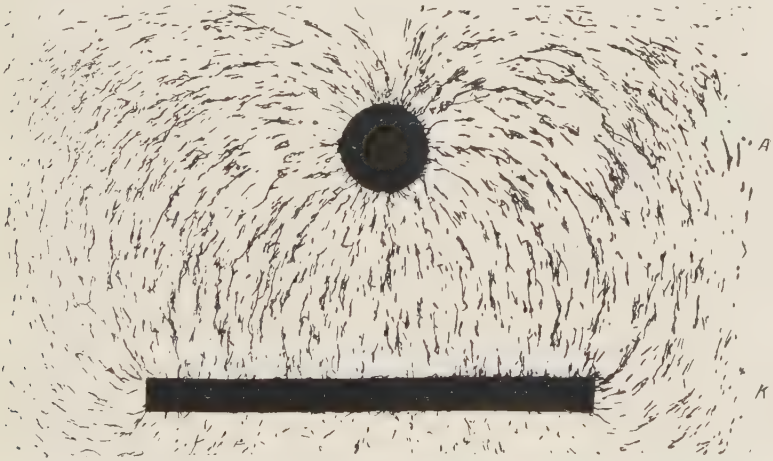


Fig. 6. Lines of force between a plate and a sphere or wire.

We begin by producing a "two-dimensional" electric field. Instead of using the plates of fig. 1, we fix two strips of tinfoil K and A on to a plate of glass. The strips, which correspond to the plates previously used, are connected to a source of current at high pressure, e. g. the induction machine which we have already referred to as a toy. By turning the handle quite slowly we can produce a pressure of about 3000 volts between the strips. We then strew freshly-powdered crystals of gypsum on the glass plate, tapping the latter carefully. The little crystals arrange themselves in a peculiar way along lines, giving a diagram of the "lines of (electric) force" (fig. 4). Superficially the latter resemble the lines of force between magnetic poles, which we made visible by means of iron filings in figs. 1—3, pp. 2—4.

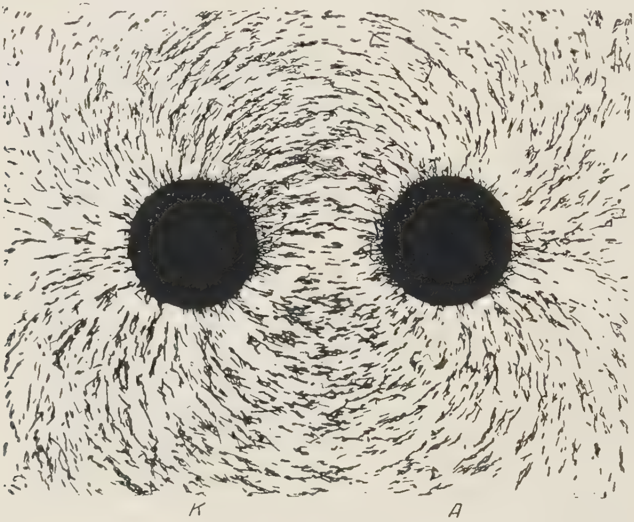


Fig. 7a. Lines of force between two spheres or parallel wires.

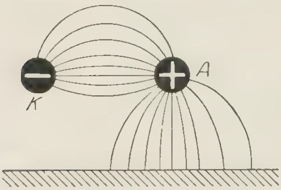


Fig. 7b. Sketch of the lines of force between the town mains and the wall of the room.

Of course the two bodies between which an electric field exists do not need to be *parallel* metal plates: for example, the plates may be inclined to one another. In two dimensions we then have the appearance of fig. 5.

Further, one plate may degenerate into a wire: we then have the lines of force of fig. 6.

Quite generally, we call two conductors, between which an electric field exists or can be produced, a "condenser". We mention four other examples:

(a) *Two neighbouring spheres or wires.* Fig. 7 gives an idea of the paths of the lines of force.

The field between the poles of our electrical leads is somewhat like that of fig. 7a. If, as is usual in municipal electricity installations, one of the mains is permanently connected to earth, we have the field sketched in fig. 7b: in this figure it is the negative pole that we have taken to be "earthed".

(b) *A sphere or cylinder standing on an insulating glass support in a room.* In older textbooks such a body is called a "conductor". The conductor is only one half of the condenser. The other is formed by the earth or by the walls of the room, the furniture and the experimenter. Fig. 8 is a diagram of the lines of force.

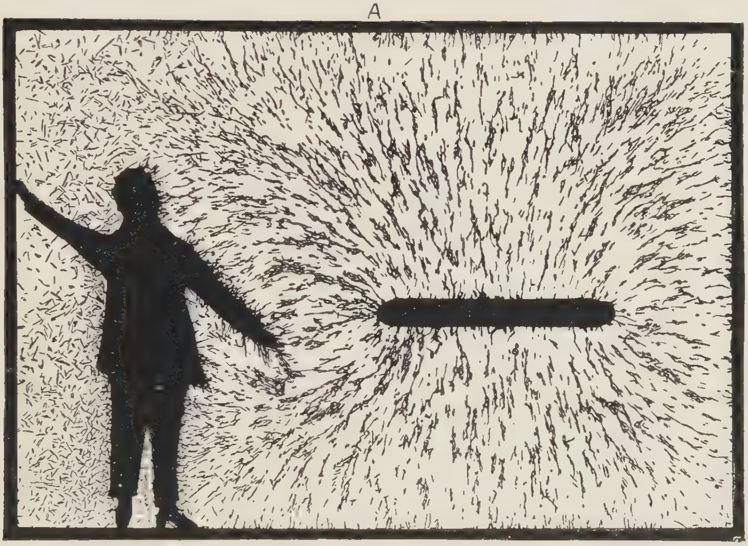


Fig. 8. Lines of force between an old-fashioned "carrier of electricity" ("conductor") and its surroundings. Note how they are changed by the movements of the experimenter.

(c) *A wireless antenna and the hull of a steamer* (fig. 9). We can see the lines of force passing from the antenna to the masts and the hull.

(d) Finally, fig. 10 gives the diagram of the lines of force of the *electrostatic voltmeter.* Such a voltmeter is nothing but a condenser. We can almost imagine we see the pressure pushing the pointer up by the lines of force.

The close connection between lines of force and pressure is strikingly shown by the experiment sketched in fig. 11. The plate condenser is connected to the bifilar voltmeter, i. e. we have the arrangement of fig. 11a. Let the pressure, caused by putting the plates temporarily in contact with the town mains, be 220 volts. We increase the space between the two plates and thereby lengthen the lines of force: the pressure rises (fig. 11b). We make the plates approach each other and shorten the lines of force: the pressure falls ((fig. 11c). We return to the original position: the pressure is again exactly 220 volts. The pressure indicated by a bifilar voltmeter follows even the smallest and most rapid alterations in the length of the lines of force with striking precision.

All the experiments described in this section give practically the same result when the condenser is in a high vacuum as when it is in air. An electric field can exist even in empty space. The air is certainly not essential for the production of an electric field. It is of quite subordinate importance as regards what we observe in the electric field. Except when we are considering sparks and the like, its influence is recognizable only when very exact measurements are made. It may be mentioned in passing that at ordinary atmospheric pressure some numbers turn out different by six parts in ten thousand from those observed in a high vacuum (see p. 67 below).

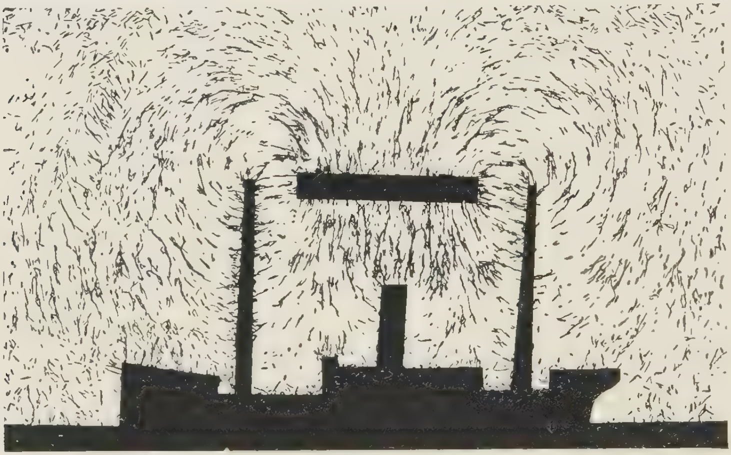


Fig. 9. Lines of force between antenna and hull of a ship.

It follows that the electric field is a special state of space. We become acquainted with it exclusively through experiment. A space in which an electric field exists is not the same as a space with no electric field. In an electric field we observe phenomena different from those in ordinary space: this is the distinction between them. One of these phenomena, the first we met with, is the chain-like arrangement of gypsum crystals which we take as a picture of the lines of force.

To speak of electric lines of force between two bodies instead of an electric field is merely a graphic way of expressing ourselves. *The conception of "electric lines of force" expresses in a pictorial way the directive effect which distinguishes an element of space in the electric field from one where there is no electric field.*

We must draw on experiment for the fundamental concepts of the world of electricity and then state them in words, just as we do for the fundamental concepts of the mechanical world. This must never be forgotten. We cannot study mechanics unless we have made ourselves acquainted with a phenomenon like gravity by many and varied experiments. *We cannot penetrate into the world of electricity unless we make ourselves acquainted by experiment with what we call the electric field.* To begin with we ought to proceed in a perfectly naïve and unprejudiced fashion. It is not a calamity if we finally end by identifying an electric line of force with a visible chain of gypsum crystals in a violently crude way. Later we shall come to know how to distinguish between electric lines of force and the rough intuitive picture of them.

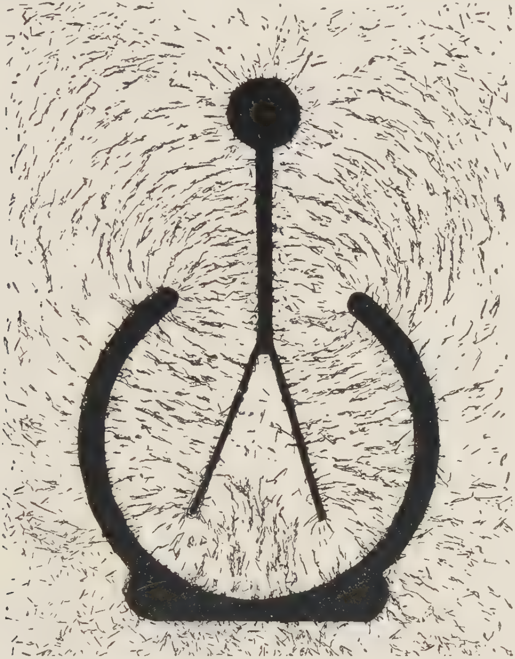
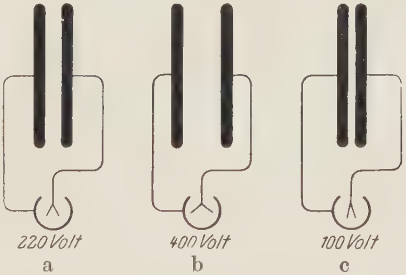


Fig. 10. Lines of force in the electrostatic voltmeter or electrometer considered as a condenser.



220 Volt
a

400 Volt
b

100 Volt
c

Figs. 11a—c. Alteration of the pressure by altering the length of the lines of force.

2. Electric Substances or Charges.

The electric field and the lines of force are situated in the space between the two plates etc. which form the boundary of the condenser. The lines of force end on the surfaces of the plates (figs. 4—10). *The electric substances are found where the lines of force end. As we shall soon see, they consist of single atoms of electricity.*

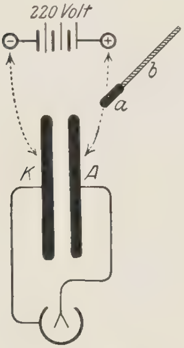


Fig. 12. Transference of electric substances from the poles of the town mains into the plates of a condenser.

First we shall show that the electric substances can be transferred from one place to another. In order to do this, we again connect our plate condenser to the bifilar voltmeter, as in fig. 12. The two circles are the poles of the town mains. We also have a proof-plane, a metal plate *a* on an amber handle *b* (see fig. 13). It is the half of a condenser of the kind mentioned on p. 25 under (b).

We move this proof-plane to and fro several times along the two dotted paths between the poles of the mains and the plates of the condenser. Even after very few moves backwards and forwards we have carried so much positive substance to the right-hand plate of the condenser and so much negative to the left that the voltmeter shows a deflection of over 100 volts.

We then proceed to move the proof-plane along paths which cross each other. The plate which previously received positive electric substance now receives negative, and vice versa. The pressure sinks, as the atoms of electricity which are successively brought together are of opposite sign and annul each other. From this we see clearly that there are two different kinds of electric substance which are distinguished from each other as positive and negative quantities are.

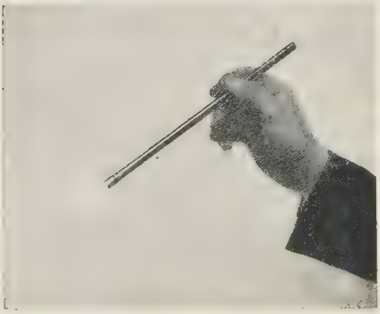


Fig. 13. Small "conductor", "electricity carrier", or "proof-plane".

We can simplify the experiment just described to some extent. We lead the electric substance to one of the plates of the condenser by means of a conducting wire and then bring up only the other with the proof-plane. We then have the arrangement of fig. 14.

The terms "electric charge" and "quantity of electricity" are also used instead of "electric substance". We definitely favour the term "electric substance" because all the substances which we know are atomically subdivided and because the existence of positive and negative atoms of electricity is of fundamental importance for our whole conception of the physical universe. For the sake of clearness, we shall remark even at this early stage that an atom of negative electricity is usually called an *electron* for short. Later developments will show why the atom of negative electricity has been favoured in the naming process.

At the beginning and end of the electric lines of force there are situated atoms of electricity of opposite sign. This is the gist of this section, and it is of great importance.

3. Gradual Decay of the Electric Field owing to Conduction.

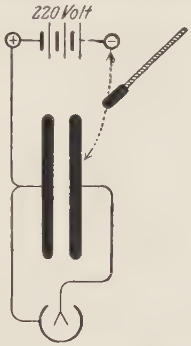


Fig. 14. Positive atoms of electricity are brought to the left plate of the condenser by a lead and negative atoms of electricity to the right plate by an "electricity carrier".

The plates of our condenser (see fig. 1) are supported by amber. We know from everyday life that amber is like porcelain, vulcanite, etc., in that it is an "insulator". Similarly, every man in the street knows that metals conduct. We have also learned that metals not only conduct the current, but oppose it with a resistance which can be measured in ohms. We shall now go on to a more thorough investigation of these properties, of which our knowledge is so far very superficial. We shall begin by showing that it is possible to pass continuously from insulators to conductors.

If we connect the two plates of our condenser by a metal, the electric field at once vanishes. The threads of the voltmeter fall together within a time too short to be measured.

But any body whatever destroys the electric field sooner or later. We show this by successively bridging the gap between the two plates of the condenser with a handkerchief, a piece of wood, a glass rod, a piece of string, etc. It is only a matter of a few seconds, or perhaps occasionally as much as a minute, before the field breaks down.

Even amber, porcelain, and the like do not enable the field to persist: but the breakdown follows more slowly, say after some hours. Amber, porcelain, etc. are only extreme members of the series in which bodies can be arranged in order of their "conductivity". At the other end of this series stand the good conductors or metals.

Plainly no such things as conductors and insulators exist. Every insulator conducts to some extent, i. e. destroys the field, though it may take a long time to do so. No conductor is perfect, i. e. a finite, but possibly very short, interval of time always elapses before the field is destroyed. A perfect insulator, in which the electric field can persist for an unlimited length of time, does not exist among tangible (material) bodies.

4. The Mechanism of the Decay of the Field.

What is going on in a body which is joining the two plates of a condenser and causing the field to decay? What happens while the lines of force and (as we shall crudely say) their ends, the atoms of electricity, are vanishing? We shall first find this out experimentally for a good conductor, namely a metal wire.

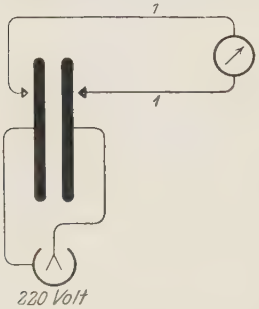


Fig. 15. When a field is destroyed by a conducting wire 1, a galvanometer in circuit shows a ballistic deflection.

In order to do this, we again take the plate condenser connected to the bifilar voltmeter (fig. 15). By connecting the former to the mains for a short time we produce a field of 220 volts. 1 is the conducting wire for destroying the field, with a sensitive galvanometer (mirror type) in circuit. If we make the ends of the wires touch the plates, the threads of the voltmeter immediately fall together. At the same time the spot of light from the galvanometer shows a ballistic deflection or fling. The experiment is shown in fig. 16.

We may suppose that a current of short duration flows and that its magnetic field causes the deflection of the galvanometer coil. In order to go ahead with certainty, however, it is important to be able to demonstrate the two other properties of the current, *the heating effect* and *electrolysis.*

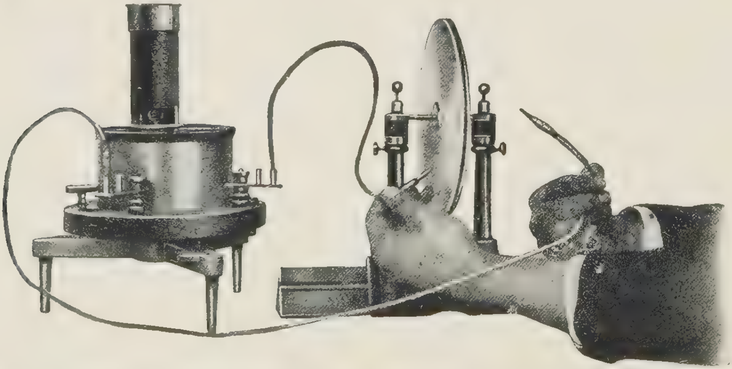


Fig. 16. Practical arrangement of the experiment of fig. 15. On the left is a highly sensitive moving-coil galvanometer (mirror type) on an iron stand. The mirror which throws the spot of light on to the scale is visible through the rectangular window at the foot of the tower. The period of this galvanometer is about 34 seconds. On the right is the plate condenser, with the plates about 1·5 mm. apart. For a numerical example see p. 37.

This can easily be done in both cases. We have only to replace the plate condenser which we have hitherto used by a type commonly used in practice, with large surfaces. These practical condensers are constructed in a simple way made clear by fig. 17. Two long strips of tinfoil *K* and *A* and two strips of insulating paraffined paper *PP* are laid on top of each other, rolled up, and pressed together. The paper insulation is far from being as perfect as that of amber and air (fig. 1). But the purchaser quietly puts up with the relatively poor durability of the electric field in these "paper condensers", for their extraordinarily compact construction has the advantage of being very cheap and taking up very little space.

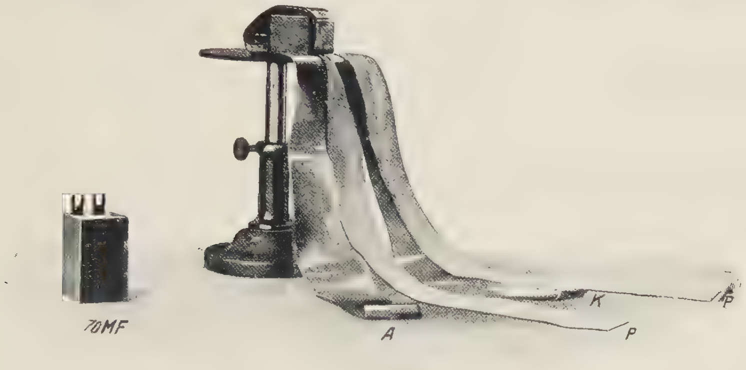


Fig. 17. On the right a partly unwound practical paper condenser of a capacity of 10 microfarads, on the left a similar condenser put together. The two strips of tinfoil have each an area of about 4 square metres. Their distance apart, or the thickness of the strip of paper *P*, amounts to about 0·02 mm.

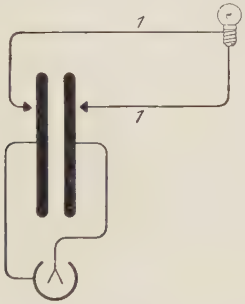


Fig. 18. When the field is destroyed by a conducting wire 1 a lamp in the circuit lights up.

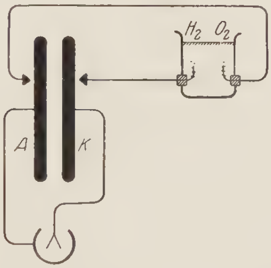


Fig. 19. When the field is destroyed by a conducting wire electrolytic effects are shown in a liquid conductor in the circuit.

We take a very large practical paper condenser, made up of fifty of the small ones shown on the left of fig. 17. We fix a wire to each of its tinfoil plates, and join the wires to the poles of the town mains; we thereby "charge" the condenser, one plate receiving negative electric substance and the other positive. An electric field of 220 volts is produced in the condenser. We then join the wires of the condenser to an incandescent lamp. It lights up brightly, showing the heating effect of the current (fig. 18).

Finally, we replace the lamp by the electrolytic vessel shown in fig. 19. When the wires are joined on, bubbles of gas rise from the electrodes for a moment (fig. 19). Hence the electrolytic effect is also established.

We thus reach this result: *While the electric field and the ends of its lines of force are vanishing, an electric current flows through the conductor for a short time.* We shall illustrate this fact later in fig. 1, p. 100.

5. Measurement of Electric Substance.

The decay of the field owing to conduction brings an important advance within our reach. For complete mastery of a physical phenomenon we require to be able to *measure* it. So far we have had no measure either for the electric field or for the electric substance at the ends of the lines of force. The decay of the field owing to conduction will now enable us in the first place to measure the amount of electric substance at least. The measurement of the electric field will then follow later on pp. 63—66.

Here, as always, we start with the field of the plate condenser, which is particularly easy to observe. The field is again produced by temporary connection with the poles of the town mains. According to our fundamental ideas, both plates then receive a definite quantity of electric substance. On the one plate there are atoms of positive electricity, on the other an equal number of atoms of negative electricity. The equality of the numbers of the two kinds of atoms may be inferred in a rough provisional way from the complete symmetry of the diagram of the lines of force in fig. 4 (p. 22).

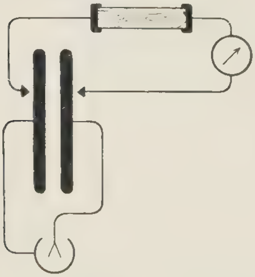
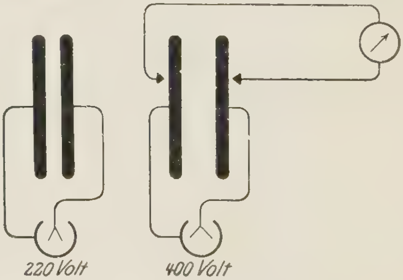


Fig. 20. Slow decay of the field through a badly conducting piece of wood.



a b

Figs. 21 a, b. Decay of the field after previous lengthening of the lines of force.

The quantity of electric substance which has once been brought up should not be altered by what is done with the condenser later on. Otherwise there would be no sense in using the idea of electric substance. It should make no difference whether we subsequently alter the pressure of the condenser by pulling out the lines of force or by introducing an insulating plate into the space between the plates. The amount of electric substance must also turn out the same whether the field persists for a long or a short time or decays slowly or rapidly.

After these preliminary remarks let us return to observation. We repeat the fundamental experiment of last section. We let the field of our plate condenser be destroyed by a metal wire with a mirror galvanometer in circuit (figs. 15 and 16). We again begin by charging the plates to 220 volts. The galvanometer again gives the same deflection as before, namely 100 cm.

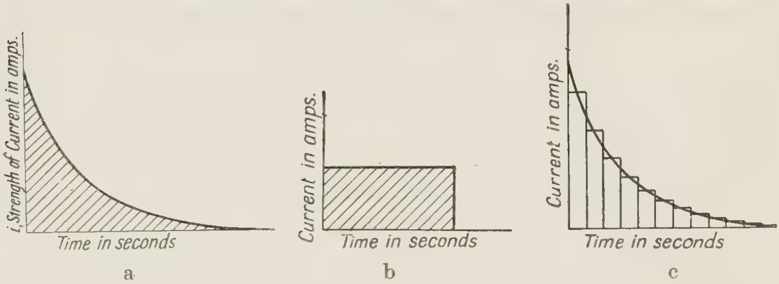
We repeat this experiment, letting the field decay through a bad conductor (otherwise a bad insulator). We include a piece of wood or glass in the metallic circuit to act as a bad conductor (fig. 20). The field is destroyed slowly, i. e. in say 3—4 seconds: we follow this on the voltmeter. Surprising to relate, our galvanometer still gives exactly the same fling of 100 cm. The size of the galvanometer fling therefore does not depend on the resistance of the conductor which is used to destroy the field.

We repeat the experiment for the third time with a further variation. We provide the condenser with a pressure of 220 volts (fig. 21 a), and then increase the distance between the plates by a few millimetres. The lines of force become longer, and the pressure rises to over 400 volts (fig. 21 b). Now we again let the field decay through the conductor. Once more we observe a fling of 100 scale divisions, although the current is now driven through the resistance of the circuit by a much higher pressure.

The fling of the galvanometer is therefore constant under those very conditions in which we must postulate that the electric substances are constant. Hence we may regard the fling of the galvanometer as a measure of the quantities of positive and negative electricity present on the plates of the condenser or situated at the ends of the lines of force. As we have mentioned, the two quantities are equal, since equal numbers of lines of force end on each plate or, in general, on each boundary of the field.

Of course the galvanometer fling is in the first instance only a relative measure and as such is of subordinate usefulness only. Fortunately, however, it has a very simple physical meaning, which can be related to the international electrical volt-ampere system of units in a perfectly clear way. We must therefore explain what a fling or ballistic deflection of the galvanometer means, in contradistinction to a constant deflection.

We state the result in advance. It is as follows: *The extent of the fling of the galvanometer depends only on the time-integral of the current, as long as the time of flow of the current is small compared with the period of vibration of the galvanometer.* By time-integral of the current ($\int i\,dt$) we mean the shaded area in the diagram of fig. 22a. This area takes a rectangular form in the case where the current i is constant during the time of flow t (fig. 22b). In this case the time-integral of the current is simply equal to the product of the current and the time, measured in ampere-seconds.



Figs. 22a—c. Three examples of "time-integrals of current", measured in ampere-seconds.

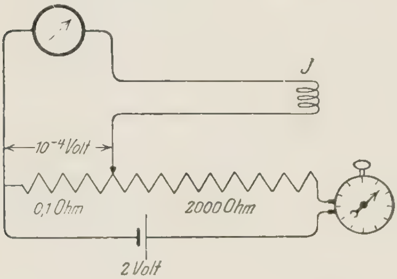


Fig. 23. Graduation of the ballistic deflections of a long-period galvanometer in ampere-seconds.

In order to confirm our assumption, we use as direct a method as possible. *We send known currents i through the galvanometer for short but accurately measured periods of time t.* We do this by means of a switch which forms part of the works of a $^1/_{50}$ second stop-watch. The switch remains closed as long as the watch is going.

A known current i of suitable magnitude is obtained by means of the arrangement shown diagrammatically in fig. 23. By means of a potentiometer (p. 17), a pressure of e. g. $^1/_{100}$ volt is produced in a circuit which contains a resistance of 10^6 ohms and our ballistic galvanometer with a period of 34 seconds. The current i flowing through the galvanometer then amounts to $10^{-2}/10^6 = 10^{-8}$ ampere, by Ohm's law. By means of this apparatus we observe the deflections α for various values of the product $i\,t$. We then repeat the measurements with two larger currents, again marking off a definite but arbitrary time between a few tenths of a second and say two seconds. If we form the quotient $i\,t/\alpha$ for the various measurements, we obtain the same value, say $5 \,.\, 10^{-10}$ ampere-seconds per centimetre of deflection, independently of the current i and the duration of the current t. That is, the extent of the ballistic deflection really depends only on the product of current and time. We have proved our assumption for time-integrals of current of the form drawn in fig. 22b. The proof for the general case follows, however, for any arbitrary time-integral can be built up of simple products $i\ t$, as is shown by fig. 22c.

In short, we have found that *a galvanometer fling of 1 cm. represents a "time-integral of current" of $5 . 10^{-10}$ ampere-seconds. We have graduated the deflections in terms of the international system of electrical units.*

In order to explain the term "ampere-second" we shall give yet another experiment. We again take a practical "paper condenser" C (fig. 25) instead of the plate condenser. We first put a carbon resistance R of about 10^7 ohms into the circuit. We charge the condenser to 220 volts and subsequently let the field decay through the ammeter circuit. We see that the current at the beginning is only a few micro-amperes and that it flows for about a minute and dies away (fig. 26b).

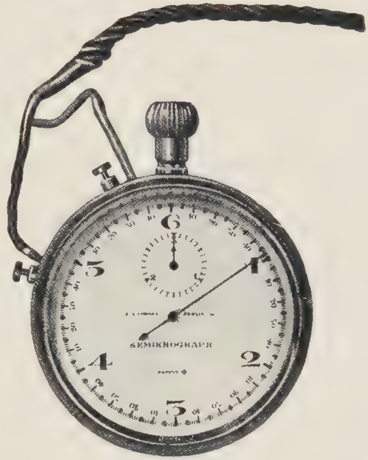


Fig. 24. This stop-watch, which can be read to $^1/_{50}$ second, completes a circuit while it is going. It is a convenient means for producing a known number of ampere-seconds.

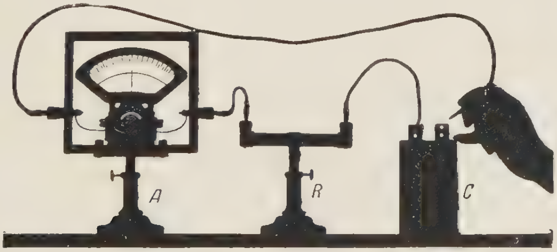


Fig. 25. Decay of the field in a practical paper condenser C with the passage of time.

We then repeat the experiment, after putting in a carbon resistance ten times as small, i. e. of only 10^6 ohms. While the field is decaying the current is now ten times as great at the beginning and dies away in about ten seconds, as is shown in fig. 26a. Moreover, the areas of figs. 26a and 26b are equal, so that the plates of the condenser originally received the same quantity of electric substance in both cases, that is, the same number of ampere-seconds.

We can also observe the ammeter while the condenser is being charged, instead of during the "discharge". The number of ampere-seconds is the same during charge as during discharge.

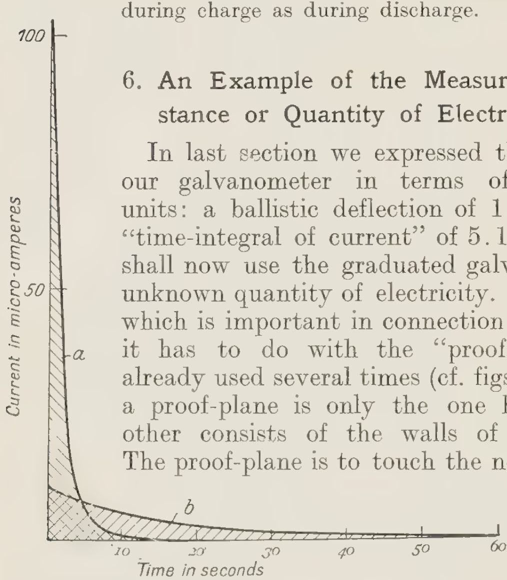


Fig. 26. The same number of ampere-seconds for fields decaying at various rapid speeds.

6. An Example of the Measurement of Electric Substance or Quantity of Electricity.

In last section we expressed the ballistic deflections of our galvanometer in terms of international electrical units: a ballistic deflection of 1 centimetre represents a "time-integral of current" of $5 . 10^{-10}$ ampere-seconds. We shall now use the graduated galvanometer to measure an unknown quantity of electricity. We choose a special case which is important in connection with subsequent sections: it has to do with the "proof-plane" which we have already used several times (cf. figs. 13, 14, pp. 28, 29). Such a proof-plane is only the one half of a condenser: the other consists of the walls of the room, etc. (p. 25). The proof-plane is to touch the negative pole of the mains momentarily; how many ampere-seconds does it then pick up? The apparatus shown in fig. 27 serves to answer this question. We first touch the pole of the mains with the proof-plane and then bring the latter into contact with the free pole of the galvanometer by a quite arbitrary path. No matter what the path is, we always obtain the same deflection, namely 1·5 cm. in our case. Hence $7 \cdot 5.10^{-10}$ ampere-seconds is the amount of electric substance which the proof-

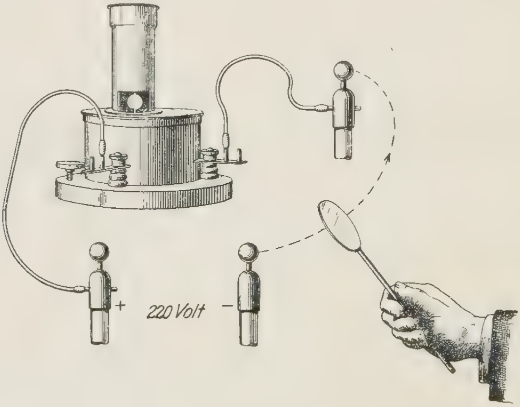


Fig. 27. Measurement of the charge of an "electricity carrier".

plane takes up on touching the pole of the mains. This simple experiment will be of great use to us in the investigations on pp. 66 and 194—197. For one thing, the proof-plane is the original type of "electricity carrier".

7. Capacity of a Condenser.

Different condensers can take up different amounts of electric substance. This we have already seen more than once. Practical condensers, such as we used in the experiments on pp. 30—35, contain greater quantities of positive and negative electric substance on their plates than the plate condenser of fig. 1. The fact that we can measure the electric substance in ampere-seconds makes it possible to give the electrical "capacity" of a condenser as a number.

If we have a balloon into which we force a gas, that is, an ordinary substance, with the pressure E, the amount Q of gas contained in the balloon is given by

$$Q = C \cdot E. \tag{1}$$

That is, $C = Q/E$ is a measure of the capacity of the balloon. The capacity is the number of litres which can be forced into the balloon per atmosphere of pressure.

Similarly we have the equation

$$Q = C \cdot E \tag{1}$$

for any condenser. *In this Q means the amount of positive electric substance on one of the plates of the condenser or the equal amount of negative on the other. Less accurately, but conveniently, we may speak of the "charge of a condenser": but by this we mean the two equal and opposite charges on the two plates.*

According to equation (1), Q, the number of ampere-seconds stored up, should be proportional to E, the pressure between the boundaries of the condenser. This can easily be proved by means of the ballistic deflections of the galvanometer. We use the arrangement sketched in fig. 15, p. 30.

The factor of proportionality $C - Q/E$ has a simple meaning: it is the number of ampere-seconds which the condenser stores up when the pressure is 1 volt.

C, which is called the capacity of the condenser, is therefore measured in $\frac{\text{ampere-seconds}}{\text{volt}}$.

The plate condenser in fig. 16 gave a galvanometer deflection of 100 cm. when the pressure was 220 volts and the plates were about 1·5 mm. apart. According to p. 36, this corresponds to $100 . 5 . 10^{-10} = 5 . 10^{-8}$ ampere-seconds; that is, the capacity C amounts to $5 . 10^{-8}/220 = 2 \cdot 3 . 10^{-10}$ ampere-seconds/volt.

As we have seen, it is usual to employ the abbreviation "ohm" instead of volt/ampere. Similarly, instead of "ampere-second" we generally *write "coulomb", and instead of "ampere-second/volt" we write "farad" almost without exception.* Also, we call 10^{-6} farad a "microfarad".

Our plate condenser with plates about 2 mm. apart accordingly has a capacity of about $2 . 10^{-10}$ farad or $2 . 10^{-4}$ microfarad.

8. Other Types of Condenser. Relative Dielectric Constants.

We frequently use condensers whose capacity can be continuously varied. For this purpose we may use the plate condenser shown in fig. 1, p. 21. It is only a matter of making the distance between the plates variable by means of a sliding adjustment or the like. In practice, however, another arrangement is mostly used. One of the plates can be displaced in its own plane, so that different fractions of the total area of the plates come opposite to each other. This gives us the adjustable condenser which is now well known everywhere.

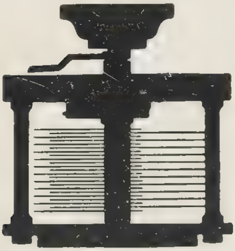


Fig. 28. Silhouette of an adjustable condenser.

Instead of air, condensers often have liquid or solid insulators between their plates to act as the "dielectric"; other gases, usually strongly compressed, are occasionally used instead of air. For example, we have already illustrated the practical paper condenser, in which the two metal plates are separated by paraffined paper, in fig. 17 (p. 31).

There are two reasons for the use of these dielectrics:

(1) *The strength of the insulation is increased.* The condenser will stand a higher pressure without sparks passing between the plates.

(2) *The capacity is increased.*

If the capacity with a dielectric other than air is K' times the capacity with air (i. e. $C_D = K' \cdot C_{\text{air}}$), we say that the dielectric has the dielectric constant* K'. Table I gives its value for a few dielectrics.

Table I.

Substance	Relative Dielectric Constant K' with respect to air = 1	Substance	Relative Dielectric Constant K' with respect to air = 1
Liquid air	1·5	Petroleum	1·9—2·3
Ice	3·1	Paraffin	2·3
Sulphur	4·0	Amber	2·8
Quartz	4·5	Vulcanite	about 2·9
Diamond	5·5	Various glasses.	about 5·5—7
Water	81	Porcelain	4·5

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* [The term *specific inductive capacity* (abbr. s. i. c.) is often used.]

The values of the dielectric constant are relative values, air being taken as arbitrary standard. On p. 67 we shall come across values of dielectric constants in terms of volt-coulomb units.

The raising of the capacity K' times means that the pressure of a charged condenser is reduced in the ratio $1/K'$ by including the dielectric. We saw this qualitatively as early as p. 22. We shall refer to the fact later on p. 58.

To the class of condensers with a solid dielectric belongs the "Leyden jar", which is much used and has been known for a long time. A primitive form of it is shown on the right-hand side of fig. 29; it consists of a glass cylinder with layers of tinfoil pasted inside and outside. Fig. 30 shows a Leyden jar constructed in accordance with practical requirements, with thick electroplated copper coats. It is easy to make jars which have capacities of 10^{-8} farad and which will stand pressures of 5.10^4 volts.

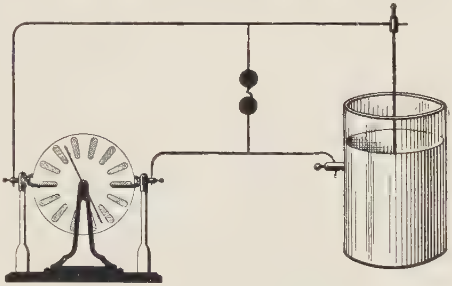

Fig. 29. Charging a Leyden jar.

A small induction machine produces currents of about 10^{-5} ampere (p. 19): that is, its current must be conveyed to such a jar for about 30 seconds in order to charge it to 3.10^4 volts. When this pressure is reached a spark jumps with a loud crack between the balls (about 1 cm. apart) of a spark gap connected across the terminals. The duration of such a spark is about 10^{-6} second, as can easily be shown by means of a rapidly rotating photographic plate. The current in the spark must therefore be $30/10^{-6} = 3.10^7$ times as great as the current of the induction machine, so that it must amount to about 300 amperes. The great strength of the current causes the air to be strongly heated, and this results in the production of the sound wave.

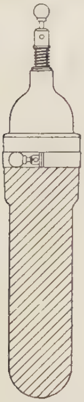

Fig. 30. A modern practical Leyden jar.

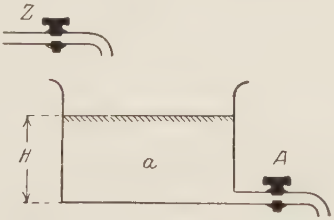

Fig. 31. Water model to explain how time can be measured electrically.

9. Application of the Condenser Equation $Q=CE$.

As an application of the equation $Q = CE$ we shall consider the measurement of a very short interval of time. *We shall determine the time of flight of a pistol bullet along a path of a few centimetres.*

In order to explain the procedure we shall begin by giving a mechanical analogy. In fig. 31, let a be a water-tank, Z the inflow, A the outflow. Let both taps be open initially. Then in the state of equilibrium the water reaches a definite height H; the velocity of outflow, however, depends on the height H. We now close first the inflow Z and then, t seconds later, the outflow A. Meanwhile the level of the water has sunk by a small amount ΔH. This fall in height, ΔH, is proportional to the time t.

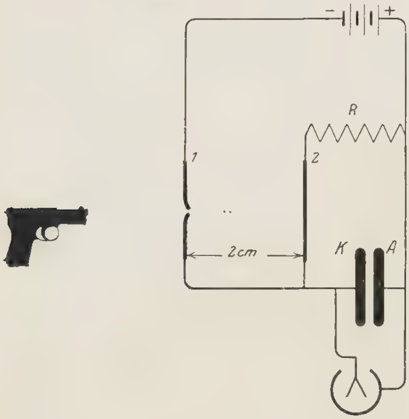

Fig. 32. Measurement of the velocity of a revolver bullet.

The transference of this simple measurement of time to the electrical case is explained in fig. 32. The condenser KA takes the place of the water-holder. Negative electric substance flows in through the conductor 1 and out through the conductor 2. The latter contains a resistance of 10^4 ohms. Instead of the taps we have strips of tinfoil included in both conductors. The pistol bullet first cuts through the tinfoil strip 1 and cuts off the inflow. After a flight of 2 cm. it cuts through the strip 2 and cuts off the outflow also. During the time of flight t the condenser pressure falls by the small amount ΔE, in analogy with the height of the water. Meanwhile the current i flowing through r is practically constant. By equation (1) it carries the quantity of electricity $Q = it = C.\Delta E$ through the resistance.

According to Ohm's law (equation (1), p. 16), $i = E/r$, so that we have

$$t = \frac{C.\Delta E}{E} \cdot r \text{ seconds}$$

for the time of flight.

In an actual experiment we make $C = 2.10^{-7}$ farad (practical paper condenser), $r = 10^4$ ohms, $E = 200$ volts. For a flight of 2 cm. (= 0·02 m.) we observe a decrease of pressure ΔE of 10 volts. This gives the value 10^{-4} seconds for t, the time of flight, which corresponds to a bullet velocity v of $0{\cdot}02/10^{-4} = 200$ metres per second.

10. Mobility of the Atoms of Electricity in Conductors.

Conductors are bodies which let the electric field break down, so that the lines of force and, to use a rough form of expression, their ends, the atoms of electricity, disappear (p. 29). While the field is decaying a current which decreases as time goes on flows through the conductor.

From this fact we conclude that the atoms of electricity in conductors can be moved. *We regard the current* (as we have already done on occasion without saying anything about it!) *as a movement of the atoms of electricity through the conductor.* Atoms of electricity of opposite sign attract each other (cf. fig. 7a, p. 24). Owing to their being free to move in the conductor, they can follow this pull. They can lay themselves so closely together that their lines of force do not appear externally at all. The charges have neutralized each other.* After a short time the stock of separate positive and negative atoms of electricity is used up. The field has broken down and the pressure is gone.

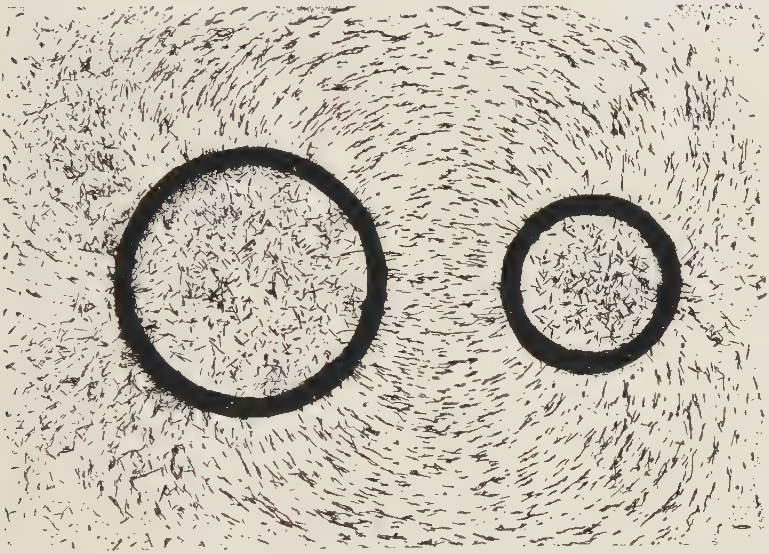

Fig. 33. Lines of force between two hollow spheres. There is no electric field inside the spheres.

At the same time, moreover, this conception shows us what must happen if the pressure between two points of a *conductor* is to be maintained: the losses which occur every second must be replaced. As many atoms of electricity of each sign must be delivered to each point per second as are lost at that point by the movement of the particles and their neutralization of each other in pairs. That is, we have only to connect the two points of the conductor by means of other conductors to a "source of current", i. e. a cell or a machine which supplies atoms of electricity continually. The current can then flow without exhausting the supplies of atoms of electricity at the two points, the supplies which are indispensable for the production of the field between them.

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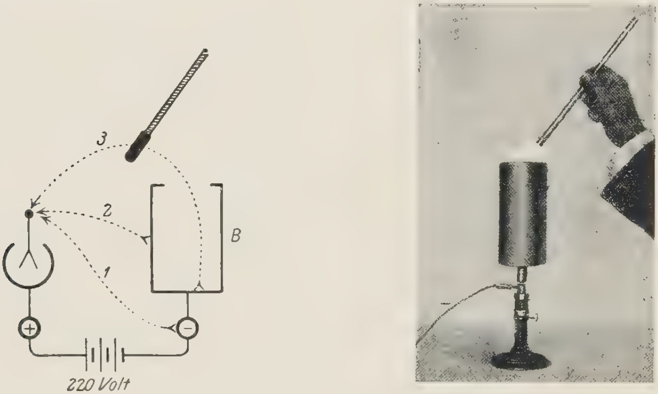
* Here, according to experienced teachers, there arises a serious difficulty for the beginner. The atoms of electricity are said to be substances, i. e. they are to be indestructible. Hence the conductor must increase both in volume and mass at the place where positive and negative atoms of electricity are united. These considerations are quite correct and in no way contrary to experimental results. The whole of the alleged difficulty is removed later in a very simple way by the quantitative investigation of the mechanism of conduction in Chapter VIII (pp. 259—260).



a b

Fig. 34 a, b. Situation of the atoms of electricity on the outer surface of a box B closed in almost everywhere.

Hence we have only to use one of the arrangements shown in fig. 27, p. 16 and fig. 29, p. 17 (which we have already used empirically) in order to maintain the pressure between two points of a conductor.

The reader will do well to note the following simple statement, which gives the gist of this section. *There is no pressure between two points of a conductor unless a current is flowing.*

11. The Situation of the Atoms of Electricity on the Surface of the Conductor in the State of Equilibrium.

In the last section we became acquainted with a fresh property of electric substances, namely their mobility in conductors. This mobility enables the atoms of electricity to move in a current through the interior of a conductor, so long as a pressure exists or is maintained between points of the conductor.

The mobility of the atoms of electricity in conductors immediately leads to another important experimental fact. *If there is no current,* *the atoms of electricity are found on the outer surface of the conductor and there is no electric field in its interior.*

We illustrate this first for two dimensions in fig. 33, for the case of two hollow spheres carrying atoms of electricity of opposite sign. We may imagine, for example, that each sphere is connected to a pole of the town mains or of an induction machine. Inside the hollow spheres there is no field. The crystals of gypsum lie jumbled at random.

The explanation is simple: the spheres do not touch each other. No current flows through their walls. Consequently there are no pressures between different points on the spheres, and hence there are no lines of force running between two points on the same sphere.

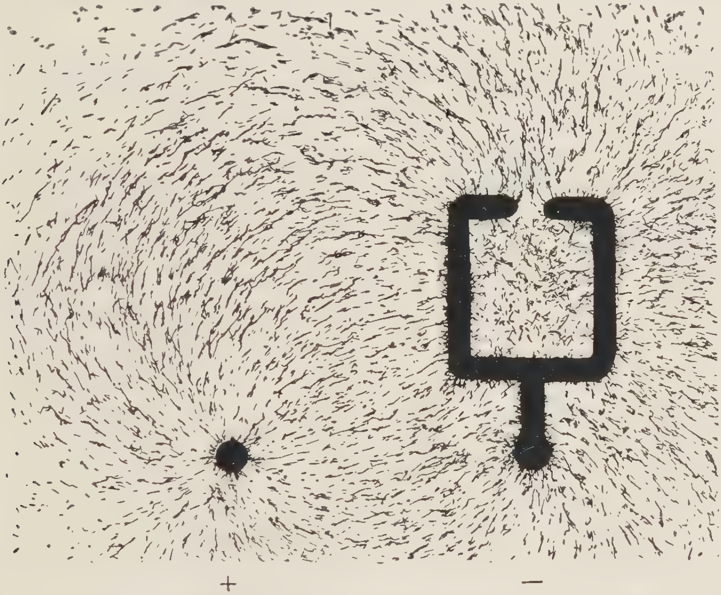


Fig. 35. Diagram of the lines of force between a sphere and a box with a narrow opening.

Further, the diagram shows us how all the lines of force between the two spheres meet the surfaces of the conductors *at right angles.* This was the case in all our earlier diagrams of lines of force: it is necessary in order that the atoms of electricity should remain at rest. Tangential components of the field, that is, lines of force arriving at a surface obliquely, would pull the atoms of electricity along the surface of the conductor, and would thus cause a current to flow until the lines of force were attracting the atoms in a direction at right angles to the surface, when the motion would cease.

All these conclusions are only tentatively drawn from the semi-diagrammatic two-dimensional picture of the lines of force given by gypsum. We now proceed to show, by means of three experiments to which no objection can be made, that the atoms of electricity are really confined to the outer surface of the conductor.

In order to do this, we connect a hollow box B with a hole in the cover to the negative pole of the town mains, as shown in fig. 34. We also connect the case of our bifilar voltmeter to the positive pole of the mains by means of a wire, so that the case receives positive atoms of electricity. The voltmeter is nothing but a condenser (fig. 10, p. 27). We can therefore charge the voltmeter: we have only to move a proof-plane (fig. 13, p. 28) between the negative pole of the mains and the connection to the threads of the voltmeter in such a way that the threads receive negative atoms of electricity (path 1).

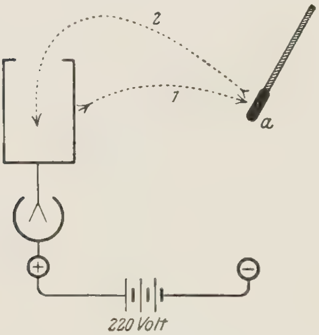


Fig. 36. Removal and return of atoms of electricity by means of the electricity carrier a.

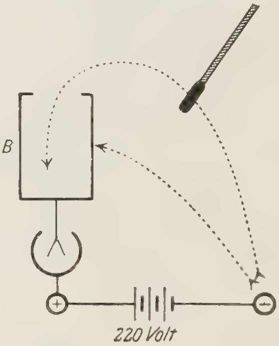


Fig. 37. Production of high pressures between the box B and the case of the voltmeter. (The reader should beware of unintentionally confusing this experiment with the one shown in fig. 11, p. 27.)

The same experiment is immediately successful if we touch the outside of the hollow box instead of the negative pole itself (path 2). *On the contrary, the proof-plane does not transfer a trace of negative electricity if we touch the inner wall of the box and then go to the voltmeter* (path 3). The result of the experiment is perfectly astounding. The box is in electrical connection with the large dynamos which supply the town mains, and yet we cannot skim off from its inner wall even the minute number of coulombs required to charge the voltmeter.

A diagram of the lines of force again makes it easier to understand what happens (fig. 35). We see that the lines of force joining the positive and negative poles of the mains all end on the outside of the box. Scarcely any lines of force reach the inner wall through the small hole in the cover. On the inner wall there are no ends of lines of force, that is, no atoms of electricity.

Another very instructive experiment is as follows. We put a box on the voltmeter (fig. 36). Let the case of the voltmeter be permanently connected to the positive pole and the box temporarily to the negative pole: then there is a field of 220 volts in the voltmeter. We touch the outside of the box with the proof-plane and then remove the latter to a, about a metre away. The voltmeter registers a smaller pressure than before; some of the negative atoms of electricity which were stored up in the box and threads have been brought to a by the proof-plane. If we go to the inner wall of the box by path 2, we bring back all the negative atoms of electricity, and the voltmeter again registers 220 volts. The proof-plane, being a part of the inner wall of a box, cannot retain any atoms of electricity; when we lift it out again it has no charge.

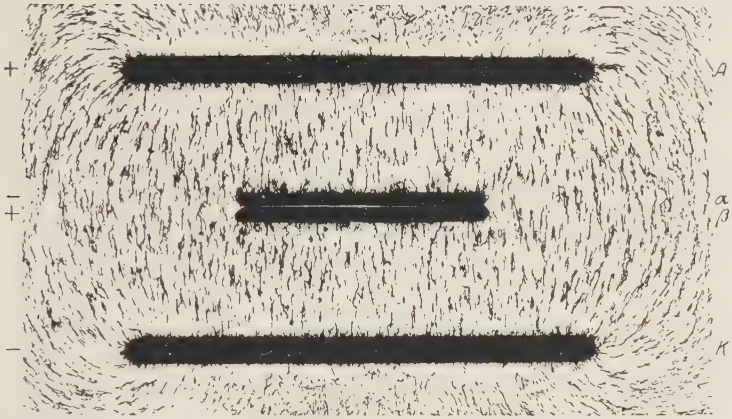


Fig. 38. Diagram of lines of force (gypsum crystals) to explain induction. Two plate-shaped electricity carriers α and β touch each other in the field.

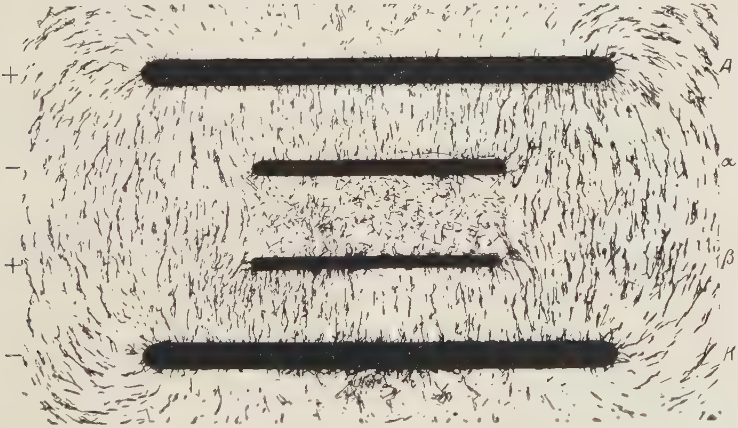


Fig. 39. The two electricity carriers α and β separated in the field.

Finally, we carry out a third experiment with the same apparatus. If we move the proof-plane backwards and forwards between the negative pole and the outside of the box, as shown in fig. 37, we do not raise the voltmeter reading above 220 volts; that is, we cannot produce a pressure higher than the pressure originally available. *If, however, we make the paths of the proof-plane end on the inner wall of the box, we can raise the reading of the voltmeter as much as we like beyond 220 volts. All the atoms of electricity on the proof-plane are given up each time.* This artifice is put to practical use in the construction of the old-fashioned but widely used *induction machine* (pp. 52, 53).

12. Electrostatic Induction.

From the mobility of the atoms of electricity in conductors there results a further fact which is of importance for the understanding of the electric field, namely the phenomenon of electrostatic induction.

We begin with a simple particular case. We are given the field of a plate condenser (fig. 38): let the upper plate contain positive atoms of electricity. Into this field we bring two conducting discs α, β, which touch each other. The lines of force are interrupted by the discs and end on their outer surfaces. At the top of α there are negative atoms of electricity, at the bottom of β positive. *These atoms of electricity which appear when the lines of force are interrupted are said to be induced.* In order to show that they exist separately, we move α and β apart in the field (fig. 39). The space between α and β remains without a field: there is no pressure between α and β. Previously α and β were in electrical contact with each other. We first take out the disc α and touch the inner connection of the voltmeter. There is a deflection of say 100 volts. We then touch the inner connection with the disc β. The deflection sinks back to zero. That is, there must have been induced atoms of electricity on both α and β, and that in equal numbers but of opposite sign.

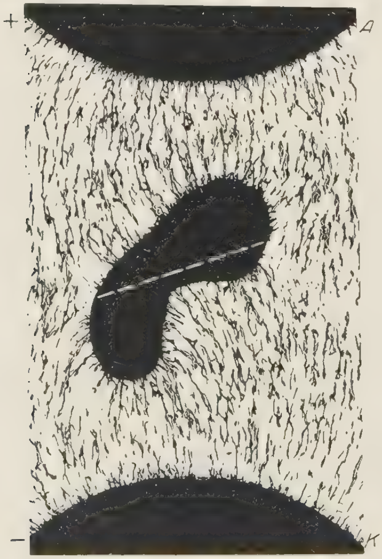


Fig. 40. Example of marked distortion of the field in the induction process.

In the general case the conductor introduced into the field may be of any shape. Then the lines of force are not only interrupted but distorted in addition, as e. g. in figs. 40, 41. Induced atoms of electricity always appear at the places where the lines of force are broken off. They can also be proved to exist separately in every case. We have only to divide the conductor in the field in two at the right place: this is indicated in fig. 40 by the dotted line.

So much for the actual empirical facts. Their interpretation leads us to a very important result. The conductor which we bring into the field must even before this contain within it positive and negative atoms of electricity, and that in equal numbers. But they lie so close to each other in pairs that their lines of force produce no external effect. If we bring the conductor into the electric field, the atoms of electricity are separated from each other. Thanks to their free mobility, they follow the pull of the lines of force until they reach the boundary of the conductor and the lines of force rise normally to its surface. At first, therefore, there is a field inside the conductor also; but the "current" of moving atoms of electricity makes it break down. The time taken for this depends on the conductivity of the conductor only (cf. p. 29).

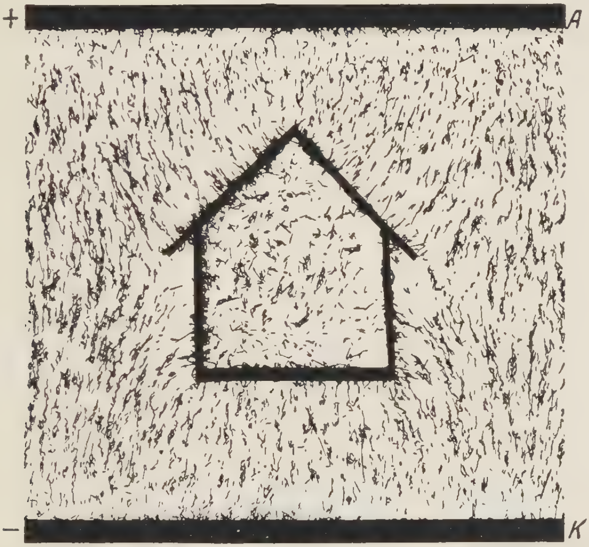


Fig. 41. Interruption of the lines of force by a hollow conductor.

The phenomenon of induction therefore shows us that even neutral conductors, which have not been electrically charged, contain atoms of electricity. Hence what we have hitherto called the electric charge of a body is only the *excess* of atoms of electricity of one sign. That is, the statement: "*A body contains negative atoms of electricity*" now means: "*In the body there are more negative atoms of electricity than in the normal state, in which the body contains equal numbers of positive and negative atoms*".

Induction provides us with the first proof that atoms of electricity are among the permanent ingredients of matter, at least of bodies which conduct electricity. This, however, is by no means the only way in which induction is important. It is capable of being applied in practice in many different ways: we shall consider some examples in the three following sections.

13. An Application of Induction: The Occurrence of Atoms of Electricity in Pairs.

We indicated as early as p. 32 that just as many positive atoms of electricity are found on one boundary surface of an electric field as there are negative atoms on the other. We inferred this to begin with from our gypsum diagrams of lines of force. With the help of induction we can convince ourselves quantitatively that at the two ends of an electric field there are always situated equal numbers of positive and negative atoms of electricity. The atoms of electricity really occur in nature only in pairs; according to our conception they are connected by the invisible band of the lines of force.

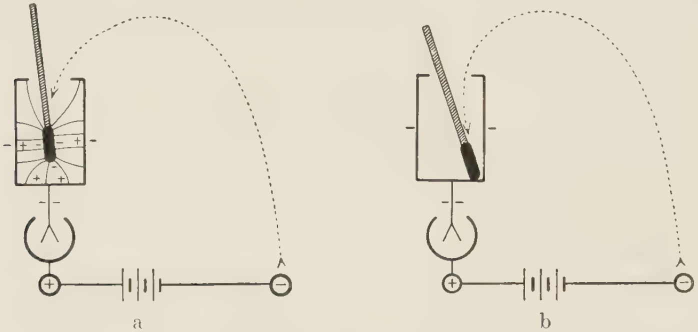


Fig. 42a, b. Equality of the positive and negative charges on the two boundaries of an electric field.

In order to do this, we touch one of the poles of the town mains, say the negative one, with the proof-plane which we have frequently used already. We then bring all the atoms of electricity which the proof-plane takes up from the mains into the box on the bifilar voltmeter (fig. 42a). To do this we must touch the inner wall of the box (p. 45). The condenser formed by the box and the voltmeter thereby receives the pressure E, 50 volts, say. We then repeat the experiment, this time, however, without touching the inner wall of the box with the proof-plane (fig. 42b). The inner wall now picks up all those lines of force which start from the negative atoms of electricity on the proof-plane. This results in the separation by induction of the atoms of electricity in the box. The positive atoms of electricity are situated on the inside of the box and the negative atoms on the outside. The voltmeter is again deflected and gives exactly the same pressure as above, 50 volts in our example. Consequently there are exactly as many atoms of electricity situated at the positive end of the lines of force starting from the proof-plane as there are negative atoms on the proof-plane itself.

14. An Application of Induction: Electric Screening.

We often have to screen off a space from an electric field. The induction phenomenon illustrated in fig. 41 shows that this is theoretically possible. We have only to surround the space to be screened with a conducting cover closed on every side. The field then induces charges on the outer wall of the cover, to be sure, but the inside remains without any field.

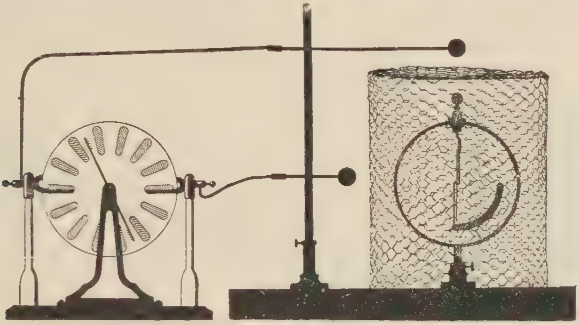


Fig. 43. Screening off an electric field by means of a cage; voltmeter as in fig. 23, p. 13.

The cover does not need to be absolutely closed: a case of wire netting of not too wide a mesh suffices, as this in practice keeps away all the lines of force from the inside. This is illustrated by the apparatus shown in fig. 43.

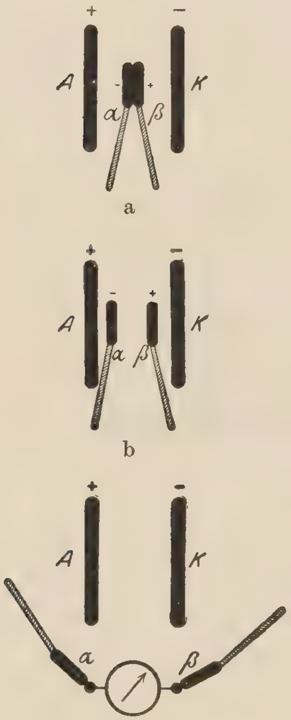
Before the case of wire-netting is put on, the voltmeter gives a large deflection, whereas with the case on the voltmeter indicates no pressure whatever. The lines of force cannot reach the space inside the cage.

We increase the pressure of the machine until crackling sparks jump between the spheres and the cage. *The inside of the case remains free of sparks.* For the occurrence of a spark presupposes the *previous* existence of a field (pp. 226, 227).

During the passage of the spark there is, of course, a weak electric field in the case, as a current flows through the walls of the case and pressures consequently exist between different parts of the case. In general, however, the field is not sufficient to affect the voltmeter visibly by induction.

For demonstration purposes Faraday put small animals in the wire case instead of a voltmeter. Hence the case is often called a Faraday cage.

The electric screen plays a considerable part in the laboratory and in practice. In the latter it is used to give protection against lightning; we surround e. g. powder magazines with wide-meshed wire-netting. But the insulated water-pipe of a fire hydrant must not be introduced as a further safety precaution. Then, of course, the lightning jumps from the wire cage through the building to the water-pipe and the mischief is done, as sad experience has abundantly shown. A hollow space with an insulated conductor introduced into it is not a screen, but a condenser. This must be borne in mind later, particularly in the case of the rapidly alternating fields of electrical oscillations.

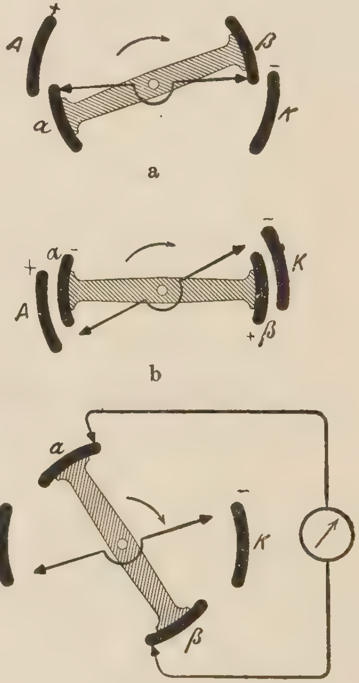


a

b

c

Fig. 44a—c.



a

b

c

Fig. 45a—c.

How an induction machine works.

15. An Application of Induction: Induction Machines.

Induction machines are sources of current of small strength (seldom more than 10^{-5} ampere) but at high pressure (often over 10^5 volts). The principle on which they act consists of periodic repetition of the simple induction experiment. For this we use the field of a plate condenser AK and two movable condenser plates α and β. By means of these we carry out the experiment in three stages (cf. fig. 44):

1. We bring α and β in electrical connection into the field between A and K.
2. We separate α and β in the field.
3. We take α and β out separately and lead the charges produced on them by induction to the place where they are to be used. In our figure the latter is a sensitive galvanometer, which shows a ballistic deflection.

Then the process begins again, and so on for as many times as we like. If the movements succeed each other rapidly enough the slow-moving galvanometer shows a permanent deflection. Periodic movements in rapid succession are easily obtained in practice by rotation. Fig. 45 shows the three stages of our experiment carried out by means of rotatory motion. The plates α and β are situated on an insulating spoke. The black triangles represent sliding contacts.

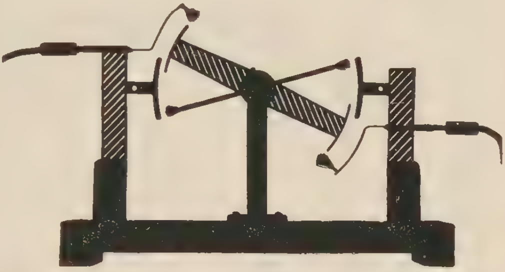


Fig. 46. A simple induction machine. The insulators appearing in the silhouette have been shaded subsequently.

Fig. 46 shows a small demonstration model which works in exact accordance with the scheme of fig. 45. The original field between A and K is produced by connecting A and K temporarily with the town mains. The current then amounts to about 10^{-8} amperes at 10 revolutions per second. It immediately disappears if A and K are connected by a conductor, e. g. the fingers, for a moment, so as to destroy the field.

Induction machines made for practical use generally have an additional arrangement by which the machine itself raises the inducing charges on the condenser plates AK to a high value and maintains them in spite of possible losses owing to insulation defects. Two different methods of doing this have been adopted.

In the first method the rotating plates α and β transfer part of their charges to the condenser plates AK by conduction. This transference is accomplished with the help of the device known as the "Faraday box".

Fig. 47 explains this so-called "multiplier process" in four stages:

1. We bring α and β in electrical connection into the field between A and K.
2. We separate α and β in the field.
3. We transfer a part of the charges obtained by induction to the place where they are to be used, in this case to a pointer voltmeter (fig. 23, p. 13). Part of the charges remains on α and β, for the transference has of course taken place without the Faraday box (cf. fig. 36, p. 44).
4. We transfer the remainders of the charges to the condenser plates A, K by means of the Faraday boxes F_1 and F_2 (note the signs).

Then the process begins again. This experiment is easily carried out. For Faraday boxes it is sufficient to fix two pockets of sheet-metal, F_1 and F_2, on to the sides of the condenser plates A and K. If a field of 220 volts is originally produced between A and K, several thousand volts are recorded on the voltmeter after only a few repetitions.

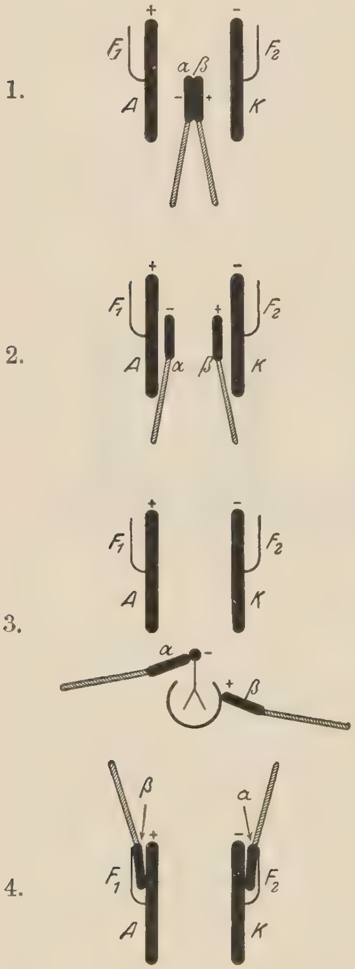


Fig. 47.

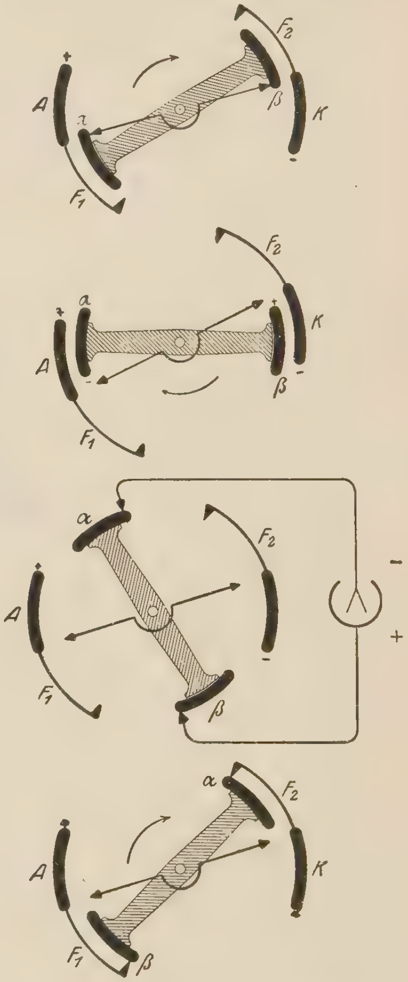


Fig. 48.

Figs. 47, 48. How an induction machine on the multiplier principle works.

Fig. 48 shows the single stages in rotatory movement corresponding to those in fig. 47. The Faraday box, however, is still incompletely developed. It is sufficient that the sheet-metal extensions F_1 and F_2 of A and K should enclose the revolving plates α and β.

Fig. 49 shows a model constructed in exact accordance with this diagram. After a few revolutions it gives thousands of volts. With this model, it is usually quite unnecessary in practice to produce a field between A and K to begin with. There nearly always exists a small accidental pressure between A and K, which is rapidly increased by the multiplier process.

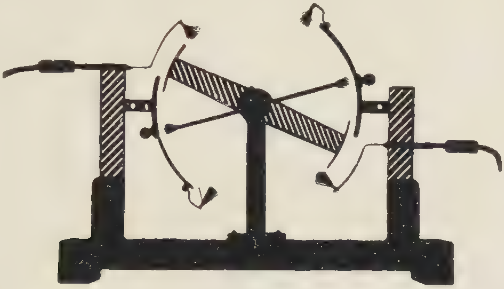


Fig. 49. A simple induction machine on the multiplier principle.

In the usual manufactured forms of these induction machines it is difficult even for experts to recognize the simple principle on which they work. We see such a machine on the left of fig. 50 a. Instead of being on coaxial cylinders, the metal plates are symmetrically arranged on glass plates. The plates A and K and parts of the sliding contacts are made of paper, following old custom. The plates α and β are usually arranged in several pairs, or may even be absent altogether. In the latter case the atoms of electricity cling to the surfaces of the insulating glass plates (cf. fig. 54, p. 56). These well-known machines are useless for purposes of instruction on account of their being so difficult to understand.

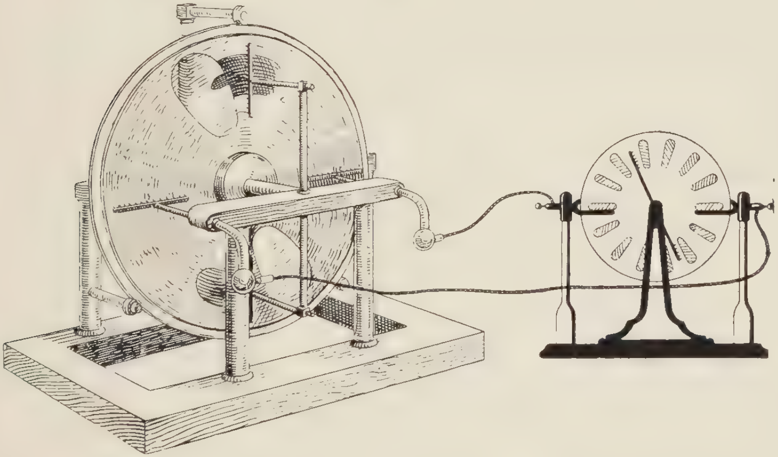


Fig. 50 a. On the left an old-fashioned induction machine acting as a motor, on the right the Holtz machine (semi-diagrammatically) acting as a generator.

The second method utilizes *induction* to increase the inducing charges: the pairs of plates AK and $\alpha\beta$ are used in larger numbers and continually interchange their parts.

Fig. 51 a shows two rings of condensers which can rotate in opposite directions. 1—4 are sliding contacts, metallically connected in pairs.

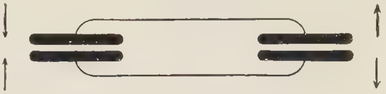
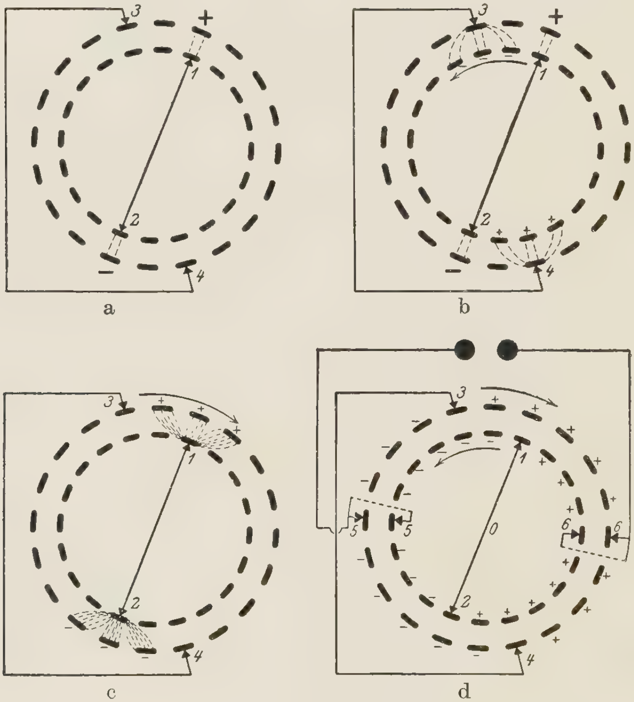


Fig. 50 b. Symbolic diagram of the apparatus of Fig. 50 a.

Let the plate marked with the heavy + sign have an arbitrary positive charge. Between + and 1, and also between — and 2, there is a weak electric field. This is indicated by two lines of force. Let the outer ring remain fixed. We turn the inner one counterclockwise through three divisions. At the end of the turn we have the arrangement shown in fig. 51 b. Three plates of the inner ring have each received charges by induction. Their lines of force (this is the main point) all pass to 3 or 4.



a b c d

Fig. 51. How the Holtz induction machine works.

The inner ring is now held fast and the outer turned clockwise through three divisions. The induction process takes place at 3 and 4.

This time 6 lines of force are effective and the induced charges are three times as large as the initial charge in fig. 51a. At the end of this turn we have the arrangement shown in fig. 51c. All the 3 × 6 lines of force end on 1 or 2.

The inner ring is then turned again, and now there are 18 inducing lines of force coming from 1 or 2, and so on.

In practice, of course, we make both plates rotate continuously in opposite directions instead of using the above intermittent rotation backwards and forwards (fig. 51d). In the region within the obtuse angle 104 both plates are positively charged, in the angle 302 both are negatively charged. Sliding brushes 5 and 6 are fitted to serve as poles from which current may be taken.

The right-hand side of fig. 50a shows in a semi-diagrammatic way a very convenient practical form with condenser plates arranged along the radii. This is the particularly useful Holtz induction machine*. The condenser plates are unnecessary in this case also. The charges can adhere directly to the surfaces of the insulating discs (cf. the next section).

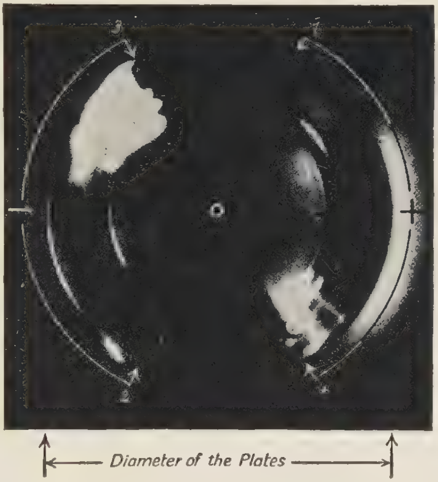


Fig. 52. Distribution of the charges on a Holtz induction machine at work.

The appearance of the machine when running in the dark with the poles 5 and 6 removed is very instructive. The opposite charges on the regions within the angles 104 and 302 are marked out by bright scintillations. There are long reddish brushes on the positive side and a violet glow on the negative (fig. 52).

In practice the induction machine is only used as a *source of current* or *generator*. In theory it can also be reversed and used as a *motor*. In fig. 50a the poles of the two induction machines are connected in pairs by wires (the "mains"). The Holtz machine on the right-hand side is turned by hand to act as a generator. The belting is removed from the old-fashioned machine on the left in order to diminish the friction on the bearings. This machine then runs well as a motor. The experiment seems somewhat trifling, but it explains the nature of the transference of electrical energy quite well. At bottom the process reduces to the simple symbolic diagram of fig. 50b. The reader should turn back to this figure in connection with the discussion on pp. 76—79.

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* [Frequently known as the *Wimshurst machine.*]

16. The Situation of the Atoms of Electricity in and upon Insulators.

The decay of the field through tangible bodies forms the starting-point for all the developments of pp. 32—50. Bodies which let the field collapse very rapidly, in the ideal limiting case instantaneously, we call conductors. We conclude from this decay of the field that the atoms of electricity can move through conductors. This "mobility" then leads us to a general point of view including all the phenomena dealt with on pp. 32—50.

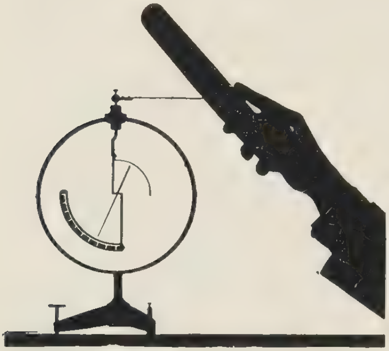


Fig. 53. Scraping off the atoms of electricity from the surface of a bar of sealing-wax.

Bodies which let the electric field collapse slowly, in the ideal limiting case not at all, we call insulators. We conclude from the absence of decay of the field that atoms of electricity cannot move through insulators. *In insulators the atoms of electricity remain fixed in their places.* We shall make this fundamental state of affairs clear by two very simple experiments.

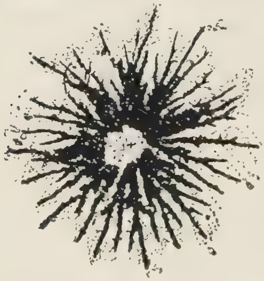


Fig. 54. Electric patch (Lichtenberg's figure).

We transfer atoms of electricity from the negative pole of the town mains to the bifilar voltmeter by means of a metal proof-plane, as shown in fig. 34 (p. 42). It is sufficient to touch the connection to the threads at a single point. The quantity of atoms of electricity supplied is not increased by stroking the connection to the threads with the proof-plane. There is no purpose in bringing all the parts of the proof-plane successively into contact with the connection.

It is quite otherwise with insulators. We take a bar of sealing-wax. It can be electrically charged in the well-known way by means of a catskin. If we touch a condenser plate connected to a voltmeter with the sealing-wax, the pointer shows only a small deflection (fig. 53). It is only when the whole bar is *stroked along* the plate that the pressure continues to rise. We have to scrape off the atoms of electricity, so to speak, from their separate places on the surface of the sealing-wax.

The second experiment is perhaps even more telling. We can spray *patches of electric substance* directly on to insulating surfaces so as to be visible.

We lay an insulating plate, e. g. of glass, between a sheet of metal and a pointed piece of wire. The metal sheet is connected with one pole of a source of current at high pressure, e. g. an induction machine. A small spark is made to pass from the other pole of the source of current to the pointed wire. At first the eye sees nothing: the atoms of electricity on the glass plate are invisible. But they send out lines of force into space. We dust a fine powder, say flowers of sulphur, on to the plate. The endpoints of the lines of force are marked by particles of powder adhering to them, just as in the case of an electric wire on the white walls of a room (see p. 24). Fig. 54 shows one of these "Lichtenberg's figures".

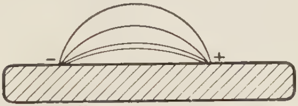


Fig. 55. Lines of force between two points of an insulator, very diagrammatically.

The lack of mobility of the atoms of electricity in insulators has a further important result: in contradistinction to conductors, lines of electric force can run between two points of an insulator without a current flowing. We only require to bring atoms of electricity of opposite signs to two different places on the insulator. In external space we then have e. g. the paths sketched in fig. 55 for the lines of force. We must remember, however, that in reality there exists no ideal insulator which lets the lines of force persist indefinitely. Even the best practical insulator can conduct to a small extent. The atoms of electricity can move through it, even if only very slowly, whenever the lines of force meet the surface obliquely. In the end, though it may be after a long time, the field collapses: the atoms of electricity move right up to each other. In other words, a weak current flows through the un-ideal insulator.

Finally, the lack of mobility of the atoms of electricity in the insulator enables them to exist in the interior of the insulator instead of on the surface only, even in the state of equilibrium. This is shown by any conductor surrounded by an insulating layer. Here again, however, it must be noted that in reality there exists no such thing as an ideal insulator. Hence in every insulator the atoms of electricity move to the surface in course of time, following the pull of the lines of force.

17. The Dielectric Constant.

The atoms of electricity can only move very slowly through an insulator (pp. 29, 56). In the ideal limiting case there is no possibility of movement even in the insulator: the atoms of electricity cannot move in any way between the atoms and molecules of matter. Yet even in the ideal insulator we must not deny the atoms of electricity every kind of mobility. We shall see in this section that the atoms of electricity can be displaced in a certain way, at least *within* the molecules of an insulator.

Every insulator has a distinctive constant depending on the material of which it is made, the so-called dielectric constant (p. 38). If we replace the air between the plates of a charged plate condenser by a plate of paraffin wax (fig. 56), the pressure sinks by about half. If we take a plate of glass instead of paraffin wax the pressure sinks to about a fifth of its original value. Hence we say that paraffin has a relative dielectric constant $K' = 2$, glass a relative dielectric constant $K' = 5$.

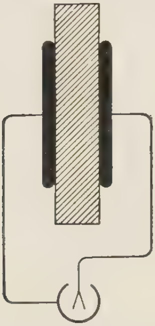


Fig. 56. Dielectric in the field of a plate condenser.

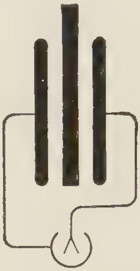


Fig. 57. Conductor in the field of a plate condenser.

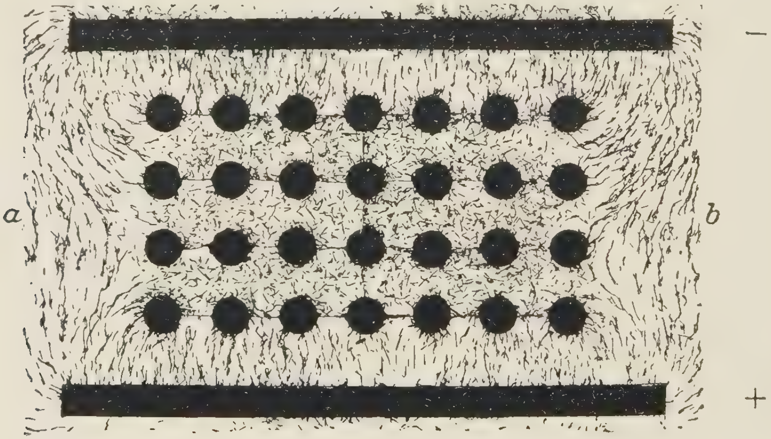


Fig. 58. Model experiment to explain induction in a conductor.

Putting in the insulator therefore has the same effect as replacing part of the interval between the plates by a metal plate (fig. 57). The metal plate shortens the lines of force, because within the conductor the field breaks down.

We shall once more explain the process which we already know as *induction.* Fig. 58 is a diagrammatic picture of the conductor many times magnified. The individual molecules are represented by spheres. The electric field appears only between the plates of the condenser and the surfaces of the conductor. There is no field within the conductor: the crystals of gypsum lie jumbled at random. The positive and negative atoms of electricity have moved to the surfaces, following the pull of the lines of force owing to their free mobility in the conductor. In this two-dimensional model we obtain free mobility of the atoms of electricity by means of a simple device: we connect all the tinfoil-sphere molecules by copper wires, which, though as thin as hairs, are visible in fig. 58.

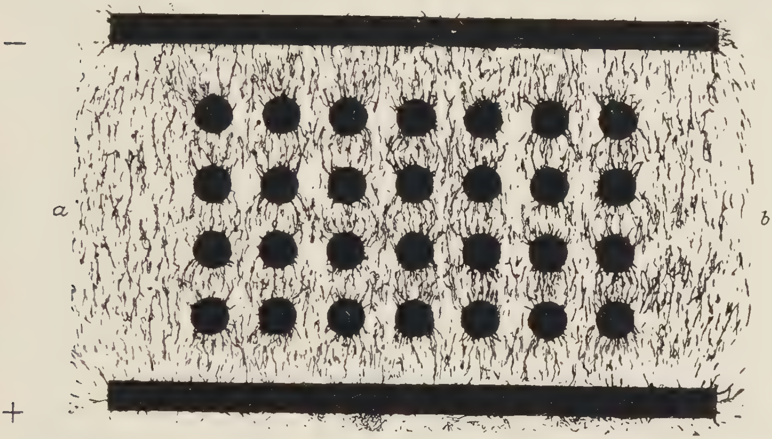


Fig. 59. Model experiment to explain the existence of dielectric constants.

Now the existence of dielectric constants shows us that an insulator also shortens the lines of force. There is no question of the atoms of electricity moving as far as the boundary surfaces as in the conductor of fig. 58, as the atoms of electricity cannot move through the insulator.

In spite of this, a shortening of the lines of force may take place. This is shown by the model experiment of fig. 59. *The whole insulator is penetrated by the field.* The atoms of electricity have been displaced *within the separate molecules only.* Each molecule carries positive and negative induced charges on opposite sides. We call the induction effect on individual molecules of an insulator the "*polarization of the dielectric*".

To avoid misunderstanding, we expressly emphasize that even "monatomic" substances, whose molecules consist of only one atom, possess a dielectric constant. In their case the displacement of the atoms of electricity takes place within the individual atoms.

The polarization of the dielectric causes induced charges to appear on the two boundary surfaces of the insulator also. This reminds us superficially of the induction observed on the surfaces of conductors. *There is, nevertheless, a difference between the induction effect on conductors and the polarization of a dielectric which is of importance in practice*: if we divide the conductor in the field along the line *ab* and take out the two halves separately, the upper carries positive atoms of electricity only in the case of fig. 58 and the lower negative only. The state of affairs is quite different, however, in the case of the insulator of fig. 59. Here both halves of the dielectric turn out electrically neutral in external space. Both halves contain equal numbers of positive and negative atoms of electricity. *We therefore cannot utilize the induction effect in insulators to separate the charges.*

It is perhaps unnecessary to mention that our picture in fig. 59 involves arbitrary simplifications. These do not affect the essential points, however. In fig. 59 it is assumed that the molecules or atoms behave like small conducting spheres. In these the atoms of electricity would be able to move as far as the surfaces of the spheres. Instead of this, we shall in reality have to deal only with definite displacements of the atoms of electricity from their normal positions within the molecules or atoms.

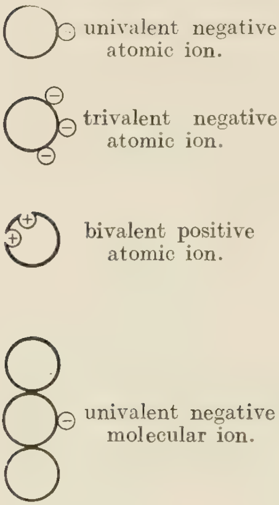

univalent negative atomic ion.
trivalent negative atomic ion.
bivalent positive atomic ion.
univalent negative molecular ion.

Fig. 60. Roughly diagrammatic pictures of "ions".

The gist of this section is the recognition that atoms of electricity are present in the interior of the molecules and atoms of matter and that they can be displaced by external electric fields.

18. Atoms as Electric Fields. Ions.

From the mechanical explanation of dielectric constants we have derived the fundamental fact that atoms of electricity are present in the interior of the molecules and atoms of matter and that they can be displaced within the molecules and atoms (pp. 57—60). Anticipating later developments, we shall remark here that atoms of electricity play an absolutely essential part in the building up of the molecules and atoms of matter. Even to-day it must be taken as certain that the atoms of electricity are the only kind of bricks out of which atoms of matter can be built. This is completely confirmed by experiments. All matter therefore ultimately consists only of electric fields and their ends, the atoms of electricity. The electric field is an absolutely fundamental idea in modern science (pp. 299—301).

The beginner often sees a difference in principle between electric substances and ordinary substances. He sees lines of force going out from the former. We now learn that not even ordinary substances are devoid of lines of force: but these baffle observation in the interior of the atoms or molecules as long as the latter contain just as many positive as negative atoms of electricity. Such atoms or molecules are called electrically neutral.

The opposite to a neutral atom or molecule is an "ion". Let us clear up this technical expression right away. *By "ion" we denote an atom or molecule which has one or more negative atoms of electricity, or "electrons", too many or too few.* Fig. 60 is meant to illustrate this in a rough diagrammatic way. Atoms are simply represented by spheres. molecules consisting of several atoms by chains of spheres. Superfluous electrons are indicated by small attached circles, missing electrons by small gaps. We speak of a *univalent* or *n-valent* positive or negative ion. When necessary we also distinguish between atomic and molecular ions. All this is shown by fig. 60 in a clumsy but simple way.

19. "Frictional Electricity."

It was shown experimentally on pp. 57—60 that atoms of electricity are present in the interior of all the atoms and molecules of matter. This enables us to understand "frictional electricity". The latter represents the historical beginnings of the theory of electricity; for the word electricity is derived from ἤλεκτρον, rubbed amber.

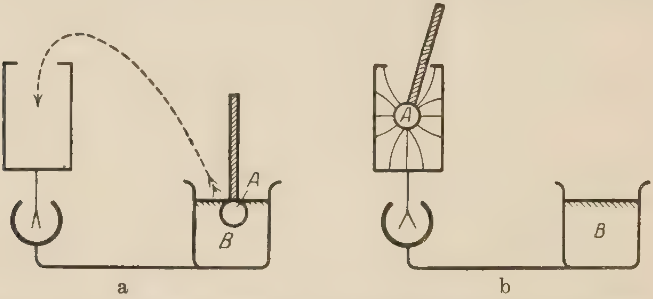
"Frictional electricity" has nothing to do with friction. It is due to the appearance of an electric field in the boundary layer of two different substances in intimate contact.

If we press two different substances A and B on to each other, the molecules of A withdraw electrons from the molecules of B in some way or other. A number of the molecules of A lying on the surface become negative ions. An equal number of positive ions remain on the surface of B. We are still completely ignorant of the details of the mechanism of this process. It is certain, however, that in the shifting or binding of electrons the same forces come into play as unite ordinary atoms to form chemical molecules and cause the cohesion of fluids and solids. Here it is at bottom a question of the fundamental problem of chemistry, of the actions on each other of the various kinds of electric fields of atoms and molecules. The solution of the problem in detail presupposes an exact knowledge of the structure of the atom.

The following fact is sufficient for our present purposes. When different substances are in intimate contact an electric field is formed in the boundary layer. Lines of force appear between the bodies A and B. These lines of force are exceedingly short, of the order of magnitude of the distances between the molecules, i. e. 10^{-8} cm.

If we separate the bodies A and B, the lines of force are stretched. The pressure rises to many thousands of volts. Crackling sparks jump between *A* and *B*.

It is the intimate contact, and not the friction, that is essential for the formation of the electric field in the boundary layer. This can easily be proved. We use the fact familiar to us from mechanics that there is no friction at the boundary between a solid and a liquid. In fig. 61a we take a sphere of paraffin wax on an amber handle for the body *A* and water in a beaker for the body *B*. We dip the sphere into the water and then bring it into the Faraday box of the bifilar voltmeter (fig. 61b). The instrument shows a big deflection. The sphere of paraffin wax is negatively charged. This is easily shown if we previously convey atoms of electricity of known sign to the voltmeter by connecting it for a short time with the poles of the town mains.



Figs. 61a, b. To show the existence of the electric field in the boundary layer between paraffin wax (*A*) and water (*B*).

An equally great positive charge remains behind on the water. This is shown by putting the beaker into the Faraday box. If we dip the sphere in, nothing happens. If we take it out, there is a deflection. If we dip it in again, the deflection again disappears. This proves that the paraffin wax sphere and the water receive equal and opposite charges when in contact.

In general the substance with the higher dielectric constant becomes positively charged: this is Coehn's law of charging.

It is easy to bring a solid and a liquid into intimate contact. *The contact of two solid bodies is disturbed by the mechanical irregularities which are always present. By pressing and rubbing the bodies together larger parts of their surfaces can be brought into contact.*

Here, and here only, lies the significance of friction for the production of the electric field in the boundary layer.

For solids we usually take two insulators: sealing-wax and catskin, glass and silk, or vulcanite and hair. One of the bodies, however, may equally well be a conductor. We take the proof-plane which we have frequently used and "scratch" a few atoms of electricity off it with a dry finger-nail. Or we "wipe" a few electrons off aluminium with our coat-tails. The simplicity of these experiments makes them very effective.

If both the solids in contact are conductors, there is likewise an electric field in the boundary layer. We cannot, however, prove its existence in the way used above, namely, by stretching the lines of force. The reason is clear: if we separate two arbitrary fixed bodies A and B, the separation does not take place at all points at the same time. In fig. 62 let x be the point at which A and B are last in contact. The lines of force between all the other parts of the condenser formed from A and B have already been stretched and the pressure has thereby been considerably raised. Yet the atoms of electricity cannot move over the bridge x as a current, if one at least of the bodies is an insulator: for in insulators the atoms of electricity cannot move.

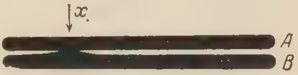


Fig. 62. Contact of imperfectly plane boundary surfaces.

It is different when both bodies are conductors. The atoms of electricity are then capable of moving in both bodies. The condenser AB discharges itself by means of a current across the bridge x, as soon as the pressure is raised, by extension of the lines of force, above the value which originally occurred when the molecules were in contact. We may remark in passing that these pressures due to the molecules being in contact are of the order of a few tenths of a volt in metals.

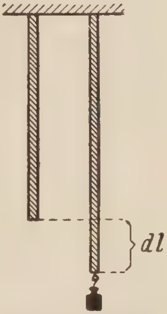


Fig. 63. Tension and extension.

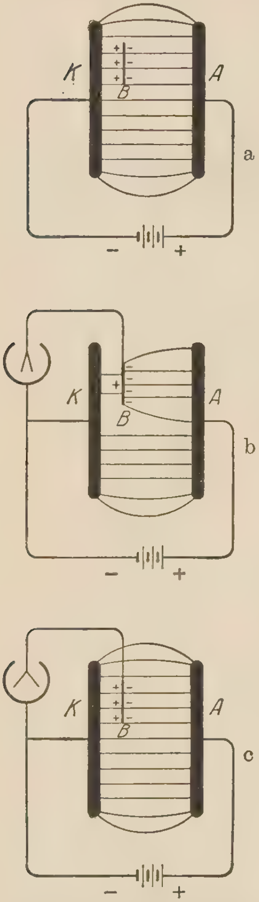
20. Measurement of the Electric Field. R, the Intensity of the Field, and D, the Displacement-density.

We now associate a definite meaning, depending on experiment only, with the words "field" and "line of force". We know, of course, that the idea of the electric field is absolutely fundamental. Hitherto, however, we have only described the electric field qualitatively. Henceforth it must also be measured in numbers.

In the course of historical developments two measures for the electric field have been set up: **R**, *the intensity (or strength) of the field, and* **D**, *the displacement-density.*

The fact that there are two measures may appear surprising, but it is not unusual in other branches of physics. For example, we likewise use two measures to denote the elastic state of a solid body, namely either the tension, measured in kilograms weight per square centimetre, or the extension dl, measured in centimetres (cf. fig. 63). According to Hooke's law, which is satisfied more or less accurately by all bodies, tension and extension are proportional to each other.

By **R**, *the intensity of the field, we mean the pressure along* 1 *cm. of a line of force.* This requires further explanation. Up till now we have spoken only of the pressure between the ends of the lines of force, which we measured in volts. We may, however, equally well speak of a pressure along part of a line of force.



Figs. 64a—c. Measurement of the distribution of pressure in the field of a plate condenser with the help of a proof-wire *B*.

In order to show this, we consider the field of a plate condenser, which we have already used frequently. Its lines of force run in straight lines at equal distances apart within the condenser. Let the pressure between the ends of a line of force be 400 volts. According to our hypothesis we shall expect to find a pressure of 100 volts between the endpoints of the first quarter of the line of force, for reasons of symmetry.

In order to measure this pressure with the bifilar voltmeter, we bring a "proof-wire" to *B*, i. e. we place a thin conducting wire or plate perpendicular to the lines of force. As fig. 64a shows, this does not alter the paths of the lines of force. The lines of force are merely interrupted for a short distance equal to the thickness of the proof-wire. *Induced charges* appear on both its boundary surfaces, as indicated in the figure.

We now expect to find a pressure of 100 volts between the plate *K* and the proof-wire *B*, if we join *K* and *B* to the bifilar voltmeter with wires (fig. 64b). Experiment, however, yields a much smaller pressure, say 30 volts. The reason for our lack of success is at once clear. We have broken the very elementary rule that a measuring instrument must not noticeably alter the quantity to be measured. For example, when measuring the temperature of a very small body we must not use a thermometer which cools the body considerably by contact.

In connecting the voltmeter to the proof-wire we have caused great changes in the whole diagram of the lines of force. Instead of the induction diagram sketched in fig. 64a we now have that indicated in fig. 64b. There are no longer equal numbers of positive and negative atoms of electricity on the proof-wire *B*, but a large excess of negative. Far fewer lines of force run between the plate *K* and the proof-wire *B* than before, and we accordingly obtain a smaller pressure.

It is not difficult to get rid of this disturbance of the electric field caused by including the voltmeter and to reproduce the original lines of force diagram of fig. 64a. *The disturbing excess of negative atoms of electricity is situated on the surfaces of the proof-wire.* If we remove the outer layer of the proof-wire we get rid of the disturbing negative atoms of electricity situated on it. Imagine, say, that two conducting metal sheets are put on to the two boundary surfaces of a proof-plate and are subsequently lifted away in the field by two insulating handles.

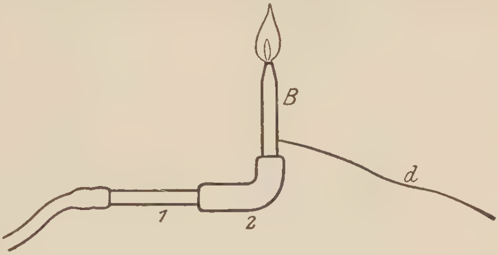


Fig. 65. A flame collector.

In practice it is simpler to remove the surface of the proof-wire in parts. This is done e. g. in the "water-dropper". A drop of water wells out from the dropper. Part of the negative excess of charge is transferred to it. The drop then breaks away, falls, and takes away the excess of charge situated on it, thus acting as an "electricity carrier". The process repeats itself and the excess of charge is speedily removed (fig. 64c).

This water-dropper works satisfactorily, but the use of water is troublesome. It is more convenient and cleaner to work with a "flame collector". We replace the water-dropper by a small gas flame. In the flame there are countless carriers of electricity, so minute as to be invisible even under the microscope, which take the place of the drops of water visible to the naked eye. As we shall see on p. 202, a flame contains countless ions, that is, molecules and atoms which serve as carriers of electricity.

A "radioactive collector" is even more convenient than a flame collector. The proof-wire is covered with a thin coating of radioactive substance, which supplies ions just as a flame does.

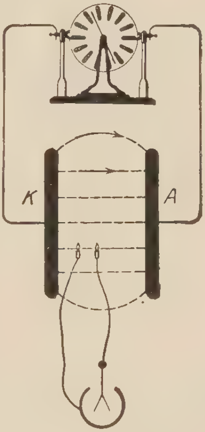


Fig. 66. Measurement of the intensity of the electric field by means of two flame collectors.

Fig. 65 shows a simple flame collector. 1 is the gas pipe, which partly consists of insulating cellon 2. *B* is the actual collector in the form of a burner, *d* the lead to the voltmeter.

Using this flame collector, we immediately obtain the result we anticipated. In fig. 64c we find that the pressure for the first quarter of the line of force is 100 volts, provided that there is a pressure of

400 volts between the plates of the condenser. We obtain similar fractions if we move the flame collector nearer to or further from A.

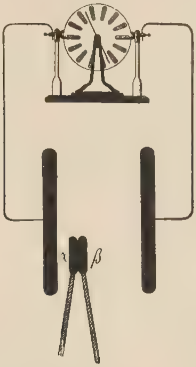


Fig. 67.

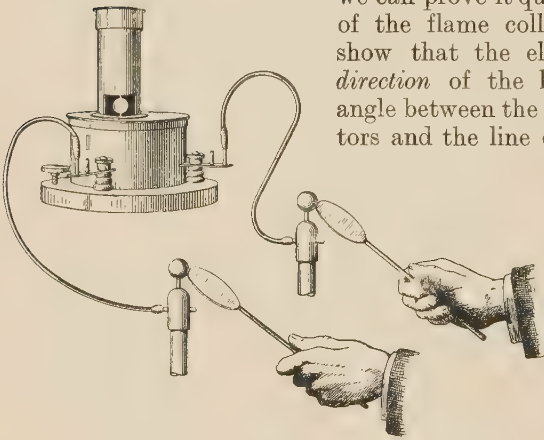


Fig. 68.

Figs. 67, 68. Measurement of **D**, the displacement-density.

Further, using two flame collectors we can now also measure the pressure between two arbitrary points of a line of force (fig. 66).

We define **R**, *the intensity (or strength) of the electric field, as the pressure along a centimetre of a line of force.* That is, we measure the electric field by the number of volts per centimetre along a line of force, or in volts/cm.

R, *the intensity of the electric field, is a directed quantity or vector, symbolized by an arrow.* The very appearance of the lines of force shows this qualitatively: we can prove it quantitatively by means of the flame collector. The collectors show that the electric field is in the *direction* of the lines of force. If the angle between the line joining the collectors and the line of force increases, the pressure falls; when this angle is a right angle, the pressure becomes zero. By the direction of the field we shall throughout this book mean the direction from the negative charge to the positive. Intensities in this direction will be reckoned positive, those in the opposite direction negative.

So much for **R**, the intensity of the field. We now turn to the second measure of the electric field, namely the displacement-density **D**.

If in an electric field we separate two plates α and β placed perpendicularly to the lines of force (fig. 67), there is an induced charge Q_D on both.

There is no difficulty in measuring this charge in coulombs by means of the ballistic deflection of a galvanometer. Fig. 68 shows how this experiment is carried out with the mirror galvanometer which we previously graduated in coulombs (p. 35).

The number of coulombs induced in the field is called the displacement Q_D. It turns out to be proportional to the area A of the plate placed perpendicularly to the lines of force. *We define* **D**, *the displacement-density, as the charge induced on a square centimetre of a surface perpendicular to the lines of force.* That is,

$$\mathbf{D} = \frac{Q_D}{A}. \tag{2}$$

It is measured in coulombs/cm².

21. The Connection between R, the Intensity of the Field, and D, the Displacement-density.

R, *the intensity, and* **D**, *the displacement-density, are proportional to each other.* This is proved by carrying out the experiments shown in figs. 66 and 67 simultaneously.

Measurements of this kind give the equation

$$\mathbf{D} = K\mathbf{R}. \tag{3}$$

The factor of proportionality K is called the absolute dielectric constant. For empty space its value is $8{\cdot}84 \cdot 10^{-14}$ coulombs/volt-centimetre. The value is practically the same for air: it is only 0·06% higher.

Hence we obtain the absolute dielectric constants by multiplying the relative values of the dielectric constant, K', quoted on p. 38, by K, the absolute value for air.

Equation (3) does not only hold for the case sketched in fig. 67, in which the lines of force are interrupted for a short distance somewhere in the middle. It is equally valid when the plate touches one boundary of the field (fig. 69). In the arrangement of fig. 67, Q_D denotes the total charge of the atoms of electricity induced on the surface interrupting the lines of force. In the limiting case of fig. 69, on the contrary, the coulombs Q are a measure of the atoms of electricity from which the lines of force of the condenser start. *Thus in the limiting case of fig.* 69 **D**, *the displacement-density, or number of coulombs per square centimetre, is simply equal to the "surface density" of the atoms of electricity situated on the boundary surface of the condenser.*

22. The Pressure E as the Line-integral of R.

On p. 66 we defined **R**, the intensity of the electric field, as the pressure along one centimetre of a line of force. We also know that E, the total pressure, is the same along every line of force of a condenser. For example, the pressure E indicated by the voltmeter V in fig. 70 is exactly the same for the lines of force ab and cd.

There can, of course, be no pressure between the starting-points ac or bd. No current flows in the conducting condenser plates, and without a current it is impossible for pressures to exist between different points of the same plate.

It follows from this that the pressure E must be the "line-integral" of the intensity of the field $\mathbf{R}$. Here the technical expression "line-integral" has the following meaning. An arbitrarily-chosen line of force ef is replaced by the broken line $s_1\ s_2 \ldots s_m$. The single straight line-elements $s_1, s_2, \ldots, s_m$ are for the time being taken to be so short that the corresponding intensities $\mathbf{R}_1, \mathbf{R}_2, \ldots, \mathbf{R}_m$ can be regarded as constant along them. We then at once obtain the partial pressures

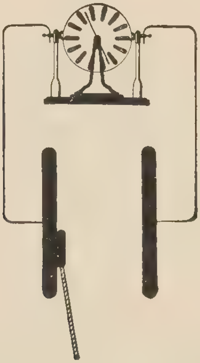


Fig. 69. Displacement at the boundary of the field.

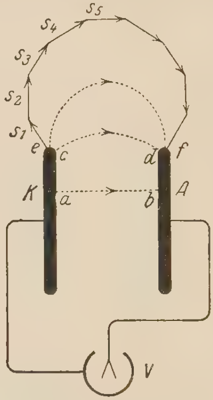


Fig. 70. Connection between $\mathbf{R}$, the intensity of the field, and E, the pressure.

$$E_1 = \mathbf{R}_1\, s_1,\ E_2 = \mathbf{R}_2\, s_2, \ldots, E_m = \mathbf{R}_m\, s_m,$$

from the single line-elements. Adding up $E_1, E_2, \ldots, E_m$ then gives the total pressure E as a "line-integral"; that is,

$$E = \mathbf{R}_1\, s_1 + \mathbf{R}_2\, s_2 + \cdots + \mathbf{R}_m\, s_m = \sum \mathbf{R}_m\, s_m, \tag{4}$$

or, in the limiting case, with another mathematical symbol,

$$E = \int \mathbf{R}\, ds\,. \tag{4a}$$

This equation (4) is the most general expression for the connection between the intensity of the field and the pressure. The path s can be built up quite arbitrarily. It need not, as in this simplified method of deduction, coincide with a line of force. If it does not, we have to multiply the line-element s_m by the component in its direction of the intensity vector $\mathbf{R}$.

23. Distribution of R in Fields of Various Forms. Calculation of Capacities.

The two equations (3) and (4) in theory enable us to calculate the distribution in space of the intensity $\mathbf{R}$, and the capacity C, for any condenser. We shall confine ourselves to two examples:

(a) Flat Plate Condenser with Uniform Field.

Let two plates of area A be at such a small distance d from each other that the "stray" lines of force at the edges can be neglected. From symmetry the intensity $\mathbf{R}$ is constant and the same along every line of force. Instead of the line-integral in equation (4) there simply appears the product $\mathbf{R}d$, so that

$$\mathbf{R} = \frac{E}{d}, \tag{5}$$

a relation which can very frequently be used. By means of it we readily obtain the capacity C. We have

$$C = Q/E, \quad \text{by (1) (p. 37)},$$
$$Q = \mathbf{D}A, \quad \text{by (2)},$$
$$\mathbf{R} = \mathbf{D}/K, \quad \text{by (3)},$$

and hence

$$C = K \cdot \frac{A}{d}. \tag{6}$$

Numerical example (fig. 14): $A = 400$ sq.cm., $d = 0{\cdot}15$ cm., $K = 0{\cdot}884 . 10^{-13}$ (p. 67). That is, $C = 2{\cdot}4 . 10^{-10}$ coulombs/volt or farads, in agreement with our previous measurements (p. 38).

Equation (6) holds when the dielectric of the plate condenser is a vacuum, and practically so for air. For other dielectrics C must be multiplied by the relative dielectric constant of the appropriate dielectric, e. g. by

$$K' = 2 \text{ to } 3$$

for vulcanite, according to the specimen.

Equation (6) is frequently used for approximate calculations. It also enables us to determine the absolute dielectric constants K accurately. For this we must determine C directly in coulombs/volt, as on p. 37, and measure A and d carefully.

(b) Spherical Condenser with Centrally Symmetric Lines of Force.

Fig. 71 shows the lines of force: this time, in contradistinction to e. g. fig. 8, their ends on the remote walls of the room etc. are not shown.

Let the charge of this spherical condenser be Q and its radius a. Then the density on the surface, or the displacement-density $\mathbf{D}$, is $Q/4\pi a^2$. $\mathbf{R}_a$, the intensity of the field at the surface of the sphere, is therefore equal to

$$\frac{\mathbf{D}}{K} = \frac{1}{K} \frac{Q}{4\pi a^2} \tag{7a}$$

by equation (3). As x, the distance from the centre of the sphere, increases, $\mathbf{D}$, the displacement-density, decreases as x^2 does. Hence at a distance x the intensity of the field is

$$\mathbf{R}_x = \frac{1}{4\pi K} \cdot \frac{Q}{x^2}. \tag{7}$$

This is merely a consequence of the fact that the lines of force are spread out in a spherically symmetric way. *Hence equation* (7) *is valid for spherically symmetrical lines of force in general*; and even for arbitrarily charged bodies, provided the lines of force end on remote surfaces on every side and the point of observation is sufficiently far from the surface of the body.

Further, to calculate the capacity of the sphere using (4), we must find the line-integral $E = \sum \mathbf{R}_m s_m$ along a radial line of force from

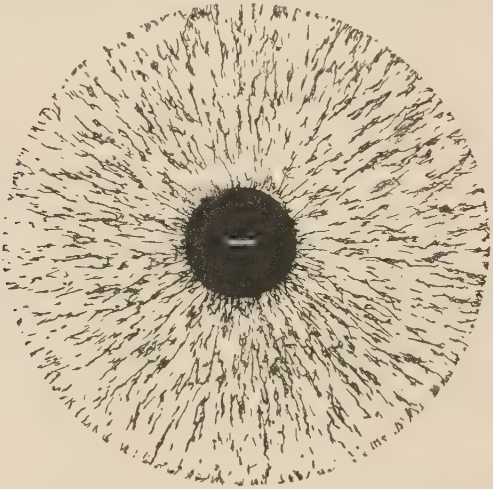


Fig. 71. Centrally symmetric lines of force starting from a sphere.

the sphere to the "infinitely" distant walls of the room etc. By integration we obtain

$$E = \mathbf{R}_a\, a\,. \tag{8}$$

Combining (1) (p. 37) and (7a), we have

$$C = 4\,\pi\, K\, a; \tag{9}$$

that is, the capacity of the sphere is proportional to its radius. This is only true so long as the second boundary of the condenser (cf. p. 25), namely the walls of the room or other distant objects, has no noticeable effect on the field in the neighbourhood of the sphere, i. e. is at an "infinite" distance.

Our earth has a radius of $6{\cdot}37\,.\,10^8$ cm. Hence together with the fixed stars it forms a condenser with a capacity of 708 microfarads, by (9). The earth is permanently surrounded by an electric field. In the neighbourhood of the ground the intensity of the field* is usually about 1 volt/cm. The earth is negatively charged. It is not known exactly where the corresponding positive charges are. They are in part situated in the atmosphere; the rest are probably outside in the system of fixed stars.

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* In meteorological books the term *potential fall* is generally used instead of intensity.

These two examples of the plate and spherical condenser sufficiently illustrate the method of calculation. In geometrically complicated fields the method of direct measurement by proof-wires is generally preferable to calculation. We shall content ourselves with giving a hint which is useful in complicated cases. *Any sharp edge or point may as a first approximation be regarded as a spherical surface of small radius of curvature r.* According to equation (8), **R**, the intensity on the surface, and r, the radius of curvature, are inversely proportional to each other for a sphere. Hence in the neighbourhood of edges and points in the boundaries of a condenser we get very high intensities even for low pressures. When the intensity of the field is high, the air loses its insulating power and becomes conducting. A violet glow shows the far-reaching nature of the changes which are meanwhile going on in the molecules of the air. More details are given in Chapter VIII.

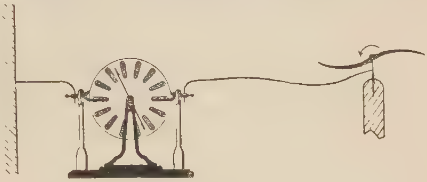

Fig. 72. In the regions of high intensity in front of points the air is no longer an insulator.

The atoms of electricity streaming off points cause reactions corresponding to the recoil of a shell on a gun. As a result of this, the "electric whirl" sketched in fig. 72 revolves in the direction of the arrow, as soon as the pressure between the whirl and the walls of the room exceeds a certain value. This is analogous to the rotating form of garden hose.

24. Forces and Work in the Electric Field.

Both the measures of the electric field, the intensity of **R** and the displacement-density **D**, are specifically electrical in their nature. They depend on the two fundamental electrical quantities of the international system of units, namely on the volt and ampere units.

The most obvious properties of the electric field are of a mechanical kind: the mutual attraction of charged bodies, say of two condenser plates, the arrangement of the gypsum crystals in lines of force diagrams, the rotation of an induction machine used as a motor (fig. 50). In all cases we observe *forces* which move masses and do *work*: or, to use another form of expression, we observe a "transformation of electrical energy into mechanical energy". *The question arises how the magnitudes of the forces, the work, and the energy depend on the electrical quantities.*

In connection with this whole question, which will be dealt with in this and the following three sections, there is a *fundamental experiment* which is perfectly easy to understand. It is illustrated in fig. 73, which represents the silhouette of the apparatus as set up ready for use and not diagrammatically.

We bring a small charged body α at the end of an insulating balance beam into the field of a condenser. It must not appreciably disturb the distribution of the charge on the plates K and A, either by its shape or by its charge. This is taken for granted in what follows. In our apparatus the body α takes the form of a small plate of thin sheet aluminium. That is, we again have the prototype of the "electricity carrier" or proof-plane. The charge is supplied to the plate by means of the movable wire lead 1.

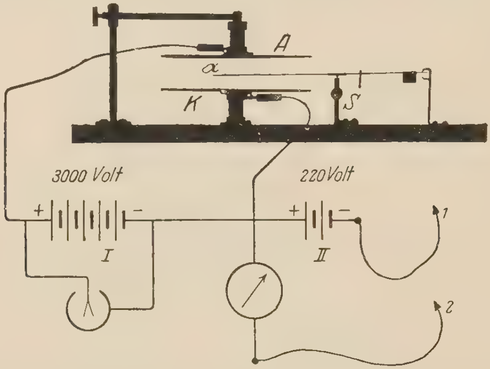

Fig. 73. Fundamental experiment on the dependence of the forces on the charge and the intensity of the field. The quartz balance beam carries on the right-hand side two riders of sheet aluminium and is free to move between two stops. S is a mass which brings the centre of gravity of the beam below the knife-edge. The condenser plates A and K are supported on amber pillars. Owing to their excellent insulation, the battery marked I may be replaced by a thoroughly primitive arrangement (a rubbed bar of sealing-wax)!

We then measure

(a) by means of the balance, the force exerted on the charge on the body by the field, in arbitrary units, e. g. in kilograms weight; sample result, $2{\cdot}9 . 10^{-6}$ kilograms weight. The proof-plane becomes 2·9 mg. "lighter".

(b) $\mathbf{R}$, the intensity of the field, from E, the pressure, and d, the distance between the plates; $\mathbf{R} = E/d$. Result, $\mathbf{R} = 300$ volts/cm.

(c) Q, the charge on the electricity carrier, from the ballistic deflection of an ammeter. This is done by touching the former momentarily with the wire 2 (cf. fig. 27, p. 36). Result, $\mathbf{Q} = 9{\cdot}5 . 10^{-10}$ coulombs.

From such measurements we obtain the following equation for the force exerted on the charge Q by the electric intensity $\mathbf{R}$:

$$\underbrace{\text{Force } F}_{\text{measured in kilograms weight}} = 10{\cdot}2 \cdot \underbrace{\text{charge } Q}_{\text{coulombs}} \cdot \underbrace{\text{electric intensity } \mathbf{R}}_{\text{volts/cm.}}. \qquad (10)$$

That is, a field of intensity 1 volt/cm. exerts a force of 10·2 kilograms weight on a charge of one coulomb, or

$$1 \text{ coulomb} \cdot 1 \text{ volt/cm.} = 10{\cdot}2 \text{ kilograms weight.} \qquad (10a)$$

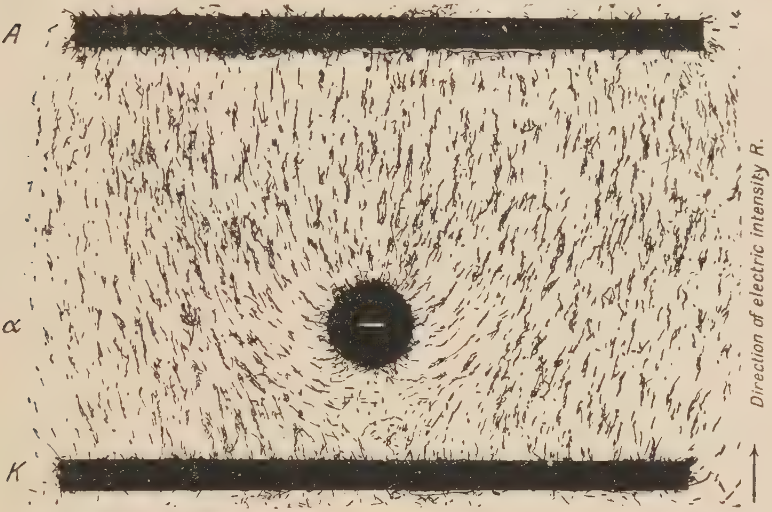

Fig. 74. A small charged body in the electric field of a plate condenser.

The experimentally-determined constant 10·2 depends on our arbitrary choice of the unit of force. If we had measured F in "dynes" instead of in "kilograms weight", equation (10) would have a different constant, namely 10^7 instead of 10·2. That is,

$$1 \text{ coulomb} \cdot 1 \text{ volt/cm.} = 10^7 \text{ dynes.} \qquad (10b)$$

If we insist on making the constant equal to unity, so that

$$F = Q\,\mathbf{R}\,, \qquad (10c)$$

we must reckon F in multiples of 10·2 kilograms weight, a unit which in spite of its convenience is unfortunately nameless.

In view of the fundamental importance of the experiment which we have just carried out, we give the corresponding lines of force diagram (fig. 74). This was again obtained by means of gypsum crystals and a tinfoil model on glass. In a trifling superficial point it departs from the arrangement represented in silhouette in fig. 73: the movable "electricity carrier" α is a small sphere instead of the thin plate of aluminium.

Our knowledge of the force from equation (10) immediately enables us to calculate the work which the field does in moving the charge Q: the work is the force multiplied by the length of path, the force component in the direction of the path being taken. In an element of path s_m in the field the work $\underbrace{Q \cdot \mathbf{R}_m}_{\text{Force}} \cdot \underbrace{s_m}_{\text{Path}}$ is expended. The sum of these separate works gives the total work for a path s as

$$W = Q\mathbf{R}_1 s_1 + Q\mathbf{R}_2 s_2 + \cdots + Q\mathbf{R}_m s_m = \sum Q\mathbf{R}_m s_m ,$$

or, since Q is constant,

$$W = Q \sum \mathbf{R}_m s_m , \tag{11}$$

or, by equation (4), p. 68,

$$W = Q \cdot E \text{ volt-coulombs} . \tag{12}$$

In words: *If an electric field moves the charge Q along an arbitrary path, the field does work amounting to $E \cdot Q$ volt-coulombs, where E denotes the pressure between the beginning and the end of the path.* In the electrical volt-ampere system of units, then, *work* is measured in volt-coulombs. The term "volt-coulomb" is often replaced by "watt-second"*. The connection between this unit of work and those we are familiar with in mechanics follows immediately from equations (10a) and (10b). From (10a) we obtain

$$1 \text{ volt-coulomb} = 10{\cdot}2 \text{ kilogram-centimetres} , \tag{10d}$$

or from (10b)

$$1 \text{ volt-coulomb} = 10^7 \text{ dyne-centimetres or ergs.} \tag{10e}$$

Equation (12) is perfectly general: it is of great importance, however, that the pressure between the endpoints of the path should be correctly determined.

Great attention must be paid to this point, especially later on in the case of the neighbourhood of closed ring-shaped lines of force (phenomena of electromagnetic induction).

In the electric fields we have hitherto met with, pressures exist only between two points separated in space. If the starting-point and the endpoint of the path coincide, there is no pressure: the work along a closed path is equal to zero.

This does not hold at all for the neighbourhood of closed ring-shaped lines of force considered later!

25. The Energy of an Electric Field.

Electric fields are capable of doing work. For example, the attraction between two condenser plates K and A can be made to raise a load (fig. 75). The electric field between the plates must therefore possess capacity for doing work, or energy. The amount of this energy can easily be calculated for a plate condenser. During the attraction the quantity of electricity Q situated on the displaced plate A is caused to move by the electric field. The pressure between the beginning and the end of the path is originally E volts, but it falls continually as the lines of force are shortened (fig. 11, p. 27). Just before the plates touch each other it is practically zero. On the average we may take it to be $E/2$ (to avoid a simple integration). Then by equation (12) we obtain

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* [1 watt-second = 1 joule.]

$$W = \frac{1}{2} Q \cdot E \tag{13}$$

for the total work done in destroying the field completely.

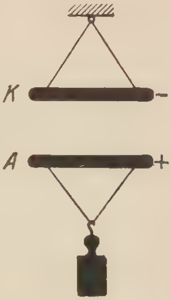

Fig. 75. To illustrate the calculation of the energy of an electric field.

This work represents the total capacity for doing work which an electrical condenser possesses: or, in other words: *A condenser with the pressure E volts and the charge Q coulombs* (cf. p. 37) *contains the electrical energy*

$$W_e = \frac{1}{2} Q \cdot E \text{ volt-coulombs}.$$

That is, the volt-coulomb or watt-second unit is not only a measure of work in the electrical system of units, but of energy in general.

In other branches of physics energy is frequently given in calories (cal.). We often have to transform volt-coulombs or watt-seconds into calories. We have

$$1 \text{ volt-coulomb or } 1 \text{ watt-second} = 0{\cdot}239 \text{ cal.} \tag{10f}$$

This follows immediately from equation (10d), since according to the theory of heat 1 kilogram-centimetre is "equivalent" to 0·0234 cal.

In practice a much larger electrical unit of energy, the kilowatt-hour. is often used instead of the watt-second. It is easy to see at once that

$$1 \text{ kilowatt-hour} = 3{\cdot}6 . 10^6 \text{ watt-seconds} = 8{\cdot}6 . 10^5 \text{ cal.}$$

Owing to the way in which it was deduced, equation (13) holds for any condenser. It is often transformed by using equation (1) (p. 37):

$$W_e = \frac{1}{2} C \cdot E^2. \tag{13a}$$

For the particular case of the plate condenser with a uniform field, equation (13a) can be put in yet another form. We have

$$C = K \frac{A}{d} \quad \text{by (6) (p. 69)},$$

$$E = \mathbf{R} d \quad \text{by (5)}.$$

Further, Ad, the product of the area of the plates and their distance apart, is V, the volume of the condenser. Hence we obtain

$$W_e = \frac{K}{2}\mathbf{R}^2 V \tag{14}$$

as *the energy of an electric field of volume* V (c.c.) *and intensity* **R**. Although equation (14) has been deduced for a particular case, it holds quite generally for any region of space V with a field of constant intensity **R**.

The energy which can be produced in the form of an electric field is only very trifling in amount. According to equation (14), a litre (1000 c.c.), even in the case of the high intensity of 10^5 volts/cm., contains only the minute quantity of energy

$$W_e = \frac{1}{2}\cdot 0{\cdot}88 . 10^{-13} . 10^{10} . 10^3 = 0{\cdot}44 \text{ watt-seconds}$$

or about 0·1 cal.

By embedding the electric field in a fluid or solid dielectric the amount of energy for the same field intensity **R** can be multiplied by a small factor, say by the factor 5, if 5 is the relative dielectric constant (i. e. relative to air). That is, we can then find room for a field energy of about 2·5 watt-seconds in a litre, instead of half a watt-second.

The reader should note that the particular form of the electric field does not matter, only its volume. For **R**, the intensity of the field, cannot be raised above the order of 10^5 volts/cm. in any condenser. It is equally great for the paper condenser in fig. 25, p. 35 and the Leyden jar in fig. 29, p. 39. In the paper condenser **R** is 220 volts/0·002 cm. = 10^5 volts/cm.; in the Leyden jar **R** is 50000 volts/0·5 cm. = 10^5 volts/cm.

It is quite otherwise in the case of the electric fields within molecules. Field intensities of 10^8 or 10^9 volts/cm. and even higher are certainly the rule. By equation (14), an intensity of 10^9 volts/cm. corresponds to an energy of $44 . 10^6$ watt-seconds or 12 kilowatt-hours or 10^7 cal. per litre. The energy of molecular electric fields is therefore amply sufficient to explain the energy observed as heats of reaction in chemistry. For example, in the formation of a litre of water from a mixture of oxygen gas and hydrogen gas, only about $3{\cdot}8 . 10^6$ cal. are given out.

26. Calculation of the Forces in Various Electric Fields.

The smallness of the energy normally contained in electric fields causes the forces observed in electric fields to be small. We shall deal with a few examples quantitatively.

(1) *The attraction between the two parallel plates of a plane condenser.* Like any other condenser, a plate condenser contains electrical energy

$$W_e = \frac{1}{2} C E^2. \tag{13a}$$

If the attracting plates have been made to approach each other till they touch, i. e. till the field vanishes, all the electrical energy must have been changed into mechanical work. This work has been done by the force F moving through the path d (the original distance between the plates), so that

$$Fd = \frac{1}{2} CE^2 .$$

Now, by (6) (p. 69),

$C = K \frac{A}{d}$ (A = area of a plate, K = absolute dielectric constant)

and hence

$$F = \frac{K}{2} \cdot \frac{E^2}{d^2} \cdot A. \tag{15}$$

The numerical value of the force appears in multiples of the nameless force unit, equal to 10·2 kilograms weight, of the volt-ampere system.

As a numerical example, we calculate the force in the case of the experiment with the so-called "pressure-balance" previously shown in fig. 21 (p. 13).

$$E = 220 \text{ volts}, \quad d = 0{\cdot}3 \text{ cm}, \quad A = 20 \text{ sq. cm.}$$

Hence

$$F = 0{\cdot}5 . 0{\cdot}88 . 10^{-13} \left(\frac{220}{0{\cdot}3}\right)^2 . 20 . 10{\cdot}2 \text{ kilograms weight,}$$

$$= 4{\cdot}8 . 10^{-6} \text{ kilograms weight,}$$

$$= 4{\cdot}8 \text{ milligrams weight,}$$

so that the little balance does not require to be very carefully constructed in order to have the necessary sensitivity.

The attraction between two condenser plates is used in practice in the construction of the "condenser telephone". This is a plate condenser, one plate of which is formed by a thin membrane (fig. 76). 1 and 2 are the two wires of the telephone leads. The pressure between them is controlled in some way or other at the sending station by the human voice.

The pressure between the lines and hence also between the plates of the condenser varies with the rhythm of the human voice.

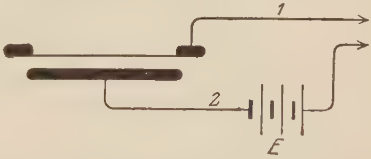

Fig. 76. Diagram of the condenser telephone.

Theoretically speaking, the pressure at the sending station can be made to vary by speaking into a "condenser microphone" constructed in a similar way to the telephone: cf. the circuit diagram of fig. 50b, where the corresponding condensers each stand for an induction machine working either as a generator or as a motor.

In order to make the transmitted speech easily audible, various devices are used to magnify the forces acting on the membrane. One of these devices consists of including a constant equal pressure in circuit, as is indicated at E in fig. 76. The effect of this additional pressure can immediately be understood by considering equation (15).

By equation (15), the forces with which two condenser plates attract each other are inversely proportional to the square of the distance between the plates. Hence condensers with a minute distance between the plates have been constructed for practical purposes. In order to do this, a condenser is made up of a conductor and a half-conductor, one plate being laid on the other and the adjacent surfaces being smooth. Fig. 77 shows a metal plate *M* in contact with a lithographic stone *St*. Both have areas of about 20 sq.cm. The stone has a weight of 200 gm. When a source of current at a pressure of 200 volts is applied, the stone "sticks". It is held up by the metal plate when the latter is lifted by a handle. Of course this condenser is not an insulator. In our example a current of several micro-amperes flows. As we saw on p. 18, our bodies do not perceive currents less than 3—5 milliamperes. Hence we can safely use our body to make the stone "stick", instead of one of the wire leads shown in fig. 77.

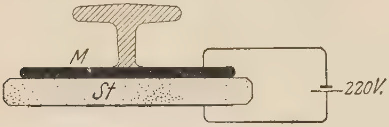

Fig. 77. Attraction between two condenser plates consisting of a good conductor *M* and a bad conductor *St*.

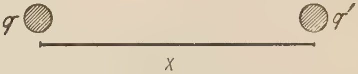

Fig. 78. Two charged spheres at a distance *x* apart.

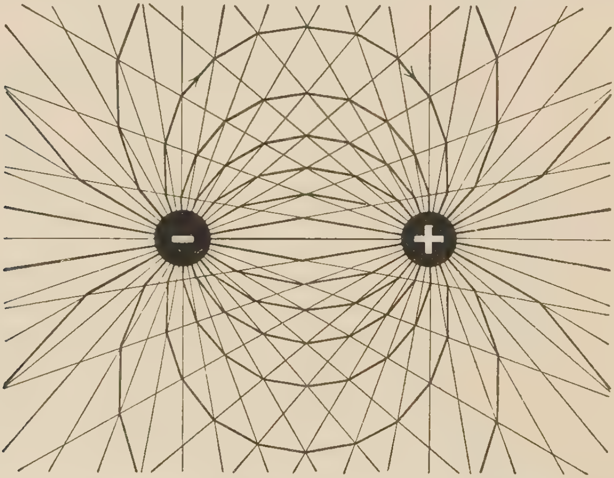

Fig. 79. Construction of the diagram of the lines of force between spheres with equal and opposite charges.

(2) *The attraction between two spheres with charges q and q' at a distance x apart* (fig. 78).

According to equation (7), the left-hand charge q produces a field intensity

$$\mathbf{R}_x = \frac{q}{4\,\pi\,K\,x^2}$$

at a distance x. Suppose that the right-hand sphere is so far removed that it does not disturb the distribution of charge on the left-hand one to an extent worth mentioning. The left-hand charge then exerts a force $F = \mathbf{R}_x . q'$ on the right-hand one, q', by equation (10c); hence

$$F = \frac{1}{4\,\pi\,K} \cdot \frac{q\,q'}{x^2}\,. \tag{16}$$

The force is proportional to the product of the charges and to the inverse square of the distance. This is *Coulomb's law.* When it was experimentally discovered in 1785, the forces F were measured by a torsion balance. One sphere remained fixed, the other being situated on the arm of the balance. The heavy lines in fig. 79 show the corresponding lines of force; the diagram is obtained from the two centrally symmetric fields of the separate charges (cf. fig. 71), shown by the fine lines, by combining the intensity vectors. Coulomb's law forms the foundation of the so-called absolute system of electrostatic units.

27. Change of Energy in the Electric Field while Work is being done. Further Examples of Forces in Electric Fields.

If an electric field does work, if we *obtain* work from it, the energy of the electric field *decreases.* This is what we described on pp. 74—76.

However strange it may sound at first, the reverse may also be true: If we *obtain* work, the energy of the electric field *increases* at the same time. Which of the two statements is correct merely depends on the experimental conditions.

When work is being done by the condenser the plates approach each other. The capacity of the condenser is thereby raised.

Now in the case considered on pp. 74—76 the *quantities of electricity* on the condenser plates were *constant* while the work was being done. Consequently the pressure sank when the capacity rose or the lines of force were shortened. That is, $\frac{1}{2}QE$, the energy of the electric field, decreased while the work was being done.

A field can equally well do work, however, while a source of current keeps the *pressure* between the condenser plates *constant.* Then the quantities of electricity Q increase when the capacity increases, owing to supplies from the source of current. That is, $\frac{1}{2}QE$ *increases* while the work is being done. The supply of work and the simultaneous increase in the energy of the field occur at the expense of the source of current.

It is best to suppose the process split up into two steps. In the first the source of current is switched off and the plates approach each other owing to their mutual attraction. Electrical energy is transformed into mechanical work and the capacity of the condenser is simultaneously increased. In the second step the source of current is switched on and the condenser is again charged to the pressure E.

The reader will do well to get a clear idea of these matters once and for all. He will thereby save himself unnecessary difficulties later on in connection with the energy of the magnetic field.

We are, moreover, in the happy position of being able to maintain electric fields even without sources of current: very good practical insulators exist. Consequently we can reduce every case in which we wish to deduce the forces and motions in the electric field from energy considerations to the same type of problem. If there is a source of current present, we imagine it switched off. We then have the simple case where the quantities of electricity are constant. *For constant quantities of electricity the work obtained is equal to the decrease in the electrical energy.* The forces in the field can only cause such motions as decrease the electrical energy of the field.

In future we shall always tacitly proceed in accordance with this imagined experiment.

Having cleared up this fundamental point, we shall add some further examples of forces in electric fields to those we gave in the last section.

In all the cases we have dealt with hitherto, the forces in the electric field are acting on *electrically charged bodies*, either on a charged body situated in the field of a condenser, of on the charged bodies (e. g. two plates) which actually form the condenser.

In contradistinction to these cases, a whole series of particularly well-known phenomena involve forces of the electric field acting on *uncharged* bodies, either insulating or conducting, e. g.

(1) *The arrangement of gypsum crystals to form lines of force diagrams.*

(2) *The attachment of particles of dust to single electric wires* in living-rooms. The wires become like hairy caterpillars. The places where the ends of the lines of force meet the white walls of the room (cf. fig. 6, p. 23) are marked by a broad black strip of dust.

(3) *The child's toy in which paper figures dance in the electric field* between a rubbed glass disc and the surface of a table.

In all three phenomena the particles as a whole are originally, i. e. before they ever touch the boundaries of the field, electrically neutral. The field can, of course, separate the atoms of electricity in them by induction (cf. p. 59), but can never give them a charge, i. e. an excess of atoms of electricity of one sign.

In spite of this, it is not difficult to establish a connection between these phenomena and the forces dealt with above, the attraction between two condenser plates, say. Once more, we have only to consider the change of energy of the field: the pressure E between the plates of a condenser decreases when an insulator or conductor comes into the region of its lines of force. That is, the energy of an electric field decreases when it attracts uncharged bodies into its region.

If, for example, *an elongated body* is arranged so that it can *rotate*, it causes the maximum decrease of pressure and of energy when it places itself in the direction of the lines of force. A simple experiment confirms the fact that the motion takes this course. We bring a light bar of the size of one's little finger into the field of a plate condenser. The bar may be made e. g. of elder pith or very thin sheet aluminium. It is free to turn about a somewhat eccentric horizontal axis (a glass rod) (fig. 80).

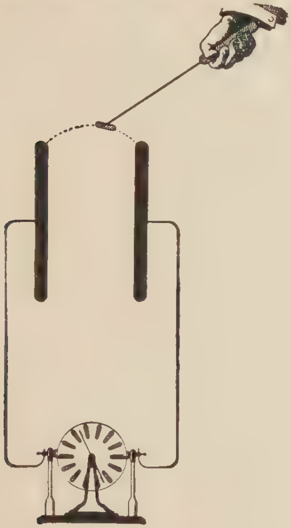

Fig. 80. Forces on uncharged bodies in the electric field.

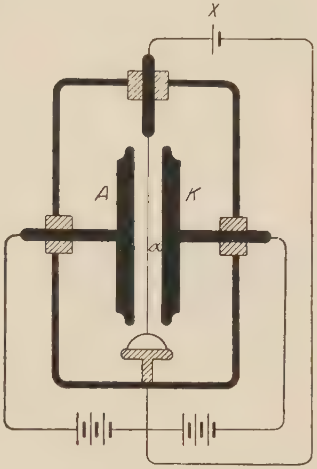

Fig. 81. Unifilar or string voltmeter: X is the unknown pressure which is to be measured.

In the equilibrium position the bar hangs straight down like a pendulum. If we produce a field with a pressure of several thousand volts by means of an induction machine, the bar places itself in the direction of the lines of force, in opposition to gravity and the friction on its bearings. We can follow out the straight course of the field in the middle as well as the curved lines of force beyond the edge of the plates (cf. fig. 4, p. 22).

In this experiment the method of suspension only permits rotatory motion of the insulating needle of elder pith. In the production of lines of force diagrams with crystalline powder on a glass plate, the freedom of motion of the crystals is limited by *friction.*

The small movable bodies may, however, be perfectly free to move. They will then cause the greatest decrease of energy in non-uniform fields at the places where they lower the pressure of the condenser to the greatest extent. But by equation (14) this happens in the region of maximum field intensity **R**. The latter is always at the ends of the lines of force. The attracted particles can therefore remain hanging from, or sticking to, the boundaries of the field. The layers of dust on and under electric lights in rooms arise in this way. If we try to imitate this phenomenon by hanging scraps of paper or the like on to a thread, the experiment does not as a rule succeed. For, as we have often emphasized, no insulator is perfect. Even scraps of paper have some conducting power, although it may be minute. When they touch the boundaries of the field some atoms of electricity pass over to them even within the lapse of seconds and they become charged, e. g. positively on the right-hand side. Hence they are now attracted by the left-hand or negative plate. We see them flying to the left in a graceful curve along the lines of force. The process then repeats itself, just as in the toy mentioned above. It is very interesting to track out the stray field of our plate condenser with a bunch of cotton-wool in this way.

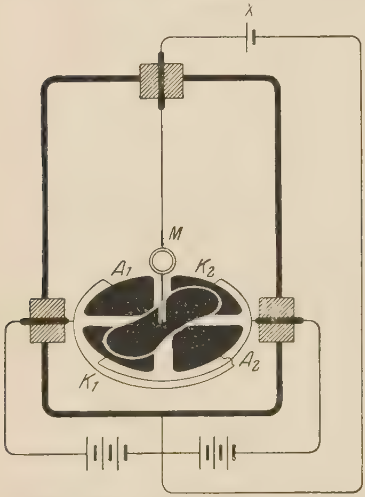

Fig. 82. The quadrant electrometer; X is the unknown pressure which is to be measured.

28. Sensitive Electrostatic Voltmeters or Electrometers.

The bifilar voltmeter, which otherwise has so many advantages, is sometimes insufficiently sensitive for accurate measurements. Even in our work we shall meet with such cases later on. We then resort to electrostatic voltmeters with an "auxiliary field". The most widely used types are the *unifilar electrometer* and the *quadrant electrometer*. *Both these instruments are merely technical variants of the apparatus used for our fundamental experiment* (fig. 73, p. 72).

Fig. 81 shows the unifilar voltmeter. The two condenser plates AK are replaced by two knife-edges, the disc-shaped electricity carrier α by a fine hair-like platinum wire α, stretched by a semicircular piece of quartz. The motion of the fibre is read by a microscope. $^1/_{100}$ volt can easily be measured and the time of coming to rest is a matter of tenths of a second.

Fig. 82 shows the quadrant electrometer. The central symmetry of its arrangement allows rotatory motions only to occur. Instead of each of the condenser plates A and K of fig. 81 two metal plates A_1, A_2; K_1, K_2 in the shape of a quadrant are used; instead of the disc-shaped electricity carrier, a metal "needle" cut in a peculiar shape and suspended so as to turn freely. The readings are usually obtained from a mirror M and beam of light. The sensitivity amounts to about 10^{-4} volt per scale division. On the other hand, the time of coming to rest is very long, corresponding to the time of vibration of the rotating system, which amounts to many seconds. This disadvantage is avoided only in a modern form with a minute needle on a fibre stretched on both sides of the needle, and read by a microscope.

29. The Charge on a Single Atom of Electricity.

On pp. 76—83 we investigated some practical applications of the forces which occur in the electric field. From the standpoint of physical science another application of these forces is of far greater significance, namely a method of measuring electric charges on small bodies floating in the air or in gases. This method forms a remarkably clear proof that the structure of electric substance is atomic.

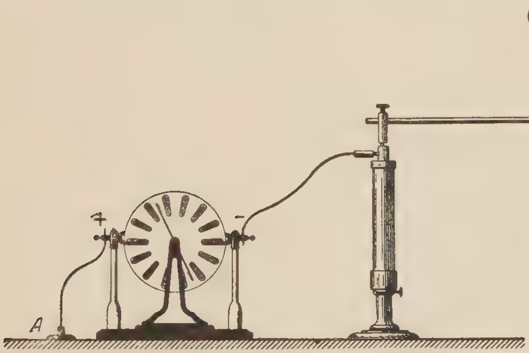

Fig. 83 a. Charging a soap-bubble.

We bring a small charged body acting as an "electricity carrier" into the vertically-arranged field of a plate condenser. That is, we have another variant of our fundamental experiment represented in fig. 73 (p. 72). The reader should not fail to have the corresponding lines of force diagram of fig. 74 at hand for reference. For example, let the body be charged negatively and the upper plate of the condenser plate positively. Then the body is pulled downwards by its weight G and upwards by the force $F = Q \cdot \mathbf{R}$ (equation 10c), or $F = Q \cdot E/d$. If G is greater than F, the body sinks, if F is greater than G, it rises. F can be given any value we like by altering E. If $F = G$, the body remains floating.

All this is very easily shown by using a soap-bubble as electricity carrier. The arrangement sketched in fig. 83 a, which requires no explanation, enables us to produce and charge the soap-bubble. We blow the soap-bubble sideways and downwards from the pipe. An electric field has previously been produced between two metal plates + and — by means of an induction machine. The two plates are held by insulating handles. It is easy to capture the soap-bubble with the electric field between these plates + and — (fig. 83 b). By altering the distance between the plates we alter **R**, the intensity of the electric field*. In this way we can successively make the soap-bubble rise, fall, or remain floating where it is, just as we please. The experiment is a remarkably striking one, especially when the shadows are projected on to a screen.

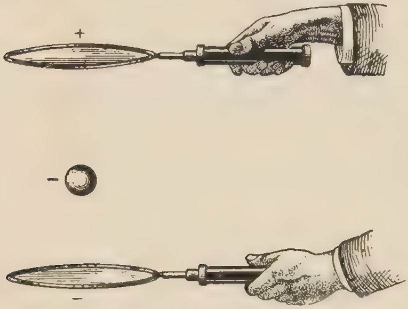

Fig. 83 b. A charged soap-bubble floating in the electric field.

To simplify the experiment, the upper positive plate in fig. 83 b can be left out altogether. The ceiling then takes its place. In both forms the experiment only comes to an end when the arm of the experimenter is tired!

If the charged body remains floating, we have

$$G = Q\cdot E/d \quad \text{or} \quad Q = G\cdot d/E\,. \qquad (18)$$

Hence we know the charge Q in coulombs when the weight of the particle is known. In accordance with p. 73, the weight must be in terms of the nameless unit 10·2 kilograms weight. If it is in kilograms, we have to multiply the right-hand side of equation (18) by the factor 10·2, by equation (10).

There is no difficulty whatever in repeating this experiment on a greatly reduced scale. Small drops of fluid take the place of the soap-bubble of about 5 cm. diameter. Drops of oil or mercury several microns in diameter are mostly used. They are charged by being touched with a solid body ("frictional electricity"). The drops may for example be made to pass along through the nozzle of an atomizer or sprayer by means of a stream of air. The plates of the condenser are set about 1 cm. apart. The motion of the charged droplets in the electric field is observed microscopically. The weight of the particles is obtained by measuring their diameter by the microscope. The volume is calculated from the diameter and the weight G found by multiplying by the specific gravity.

Such experiments on small electricity carriers (which, however, are still easily visible) yield an absolutely fundamental result, the importance of which cannot be sufficiently emphasized. All the charges observed are integral multiples of the charge $e = 1{\cdot}6 . 10^{-19}$ coulombs*. $1{\cdot}6 . 10^{-19}$ coulombs is the smallest charge, whether of positive or of negative electricity, which has ever been observed. $e = 1{\cdot}6 . 10^{-19}$ coulombs is the charge of the single positive or negative atom of electricity, or in other words, the elementary quantity of electricity. The atomicity of electric substance is firmly established beyond all doubt by this simple experiment.

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* This only happens because the electric field between + and — in fig. 83 b is not uniform. If the condenser were plane, that is, if d, the distance between the plates were small compared with their diameter, **R**, the intensity of the field, would be a constant independent of d, the distance between the plates. The plates in fig. 83 b are not connected to a source of current, so that the pressure between them is not kept constant.

The carrying out of the experiment presents no difficulty. It should find a place in every practical course for beginners. It is most striking in its effect when each one observes for himself through the microscope. When the microscopic field of view is thrown on the screen disturbances are apt to arise from air currents in the condenser due to heating by the intense light required for projection.

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[*] Rounded off from $1{\cdot}59 . 10^{-19}$.

CHAPTER III

The Magnetic Field

1. Some Typical Magnetic Fields.

In our introductory survey in Chapter I we mentioned three properties of a current in a conductor: (1) the magnetic field surrounding the conductor, (2) the heating of the conductor, and (3) chemical changes in the conductor.

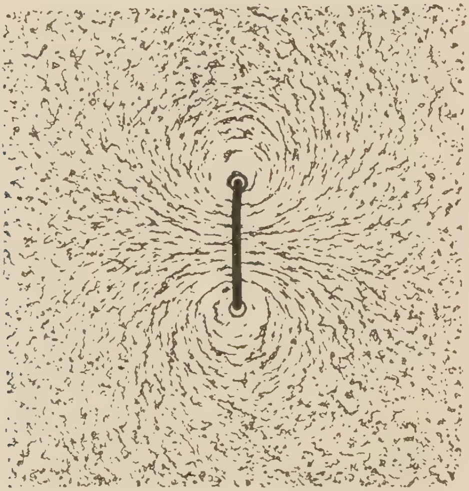

Fig. 1. Lines of force of a circular ring carrying a current, made visible by iron filings.

These three properties are by no means of equal value to us. Chemical changes do not occur in the conductors which are most important in practice, namely the metals. Even the heating of the conductor may be absent under certain conditions (superconduction, p. 253). The magnetic field, however, remains under all circumstances. The magnetic field is the inseparable companion of the electric current.

The magnetic field can exist in empty space just like the electric field. The presence of molecules of air at atmospheric pressure is of quite subordinate importance in this connection. The magnetic field is also a special condition of space, with which we become acquainted by experiment only. A space in which there is a magnetic field is different from a space without one. In a magnetic field we observe phenomena different from those in ordinary space; this is the distinguishing fact, just as in the case of the electric field. We have already come across two of these phenomena:

(1) The chain-like arrangement of iron filings, which we regard as a pictorial diagram of the magnetic lines of force.

(2) The forces exerted on compass-needles and conductors carrying a current.

We now proceed to investigate the magnetic field further. We begin by studying some typical forms of magnetic fields.

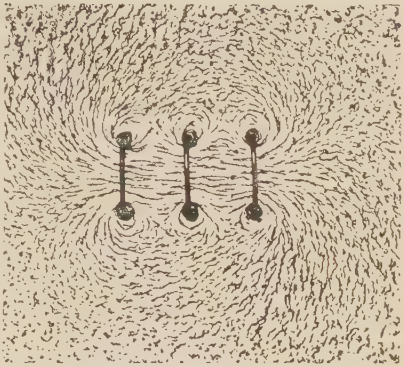

Fig. 2. Lines of force of three parallel rings carrying equal currents.

The lines of force of a long straight conductor carrying a current are concentric circles (fig. 4, p. 5).

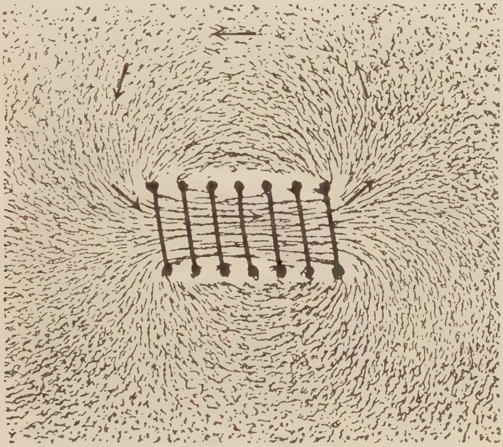

Fig. 3. Lines of force of a short coil carrying a current. The arrows indicate compass-needles; the arrow-heads are the north poles. The positive pole of the source of current is supposed to be connected to the top left-hand end of the coil and the negative pole to the bottom right-hand end.

For a ring-shaped conductor we obtain lines of force like those shown in fig. 1. The "circles" appear to be displaced eccentrically towards the outside and to be somewhat deformed. We put a number of circular turns together: the lines of force diagrams which, according to fig. 1, correspond to the single turns now overlap, and the diagram of fig. 2 results. In this each turn is supposed to be connected with a special source of current. Of course it is much more convenient to send the same current through all the turns. This is very simply done by winding a wire into a spiral (cf. figs. 3 and 4).

A small compass-needle which can be moved about in the magnetic field of the coil* always places itself in the direction of the lines of force. This is shown in fig. 3. The arrowhead always denotes the north pole of the compass-needle. We define the positive direction of the lines of force to be that indicated by the arrows (cf. e. g. fig. 3 above and fig. 1, p. 100).

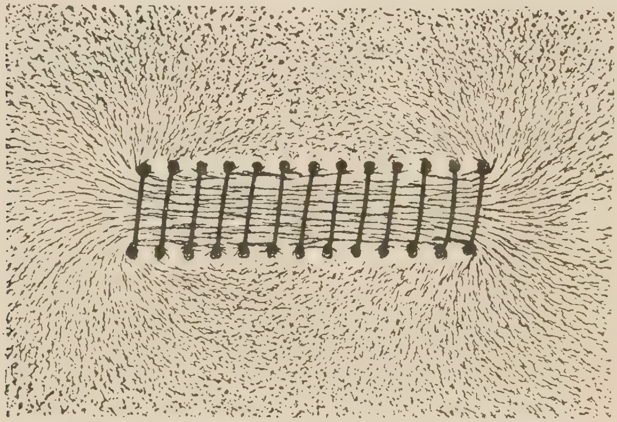

Fig. 4. Lines of force of a long coil carrying a current. Within the coil there is a uniform magnetic field.

The same field is obtained if we replace the coil by a bundle of longer but thinner coils. It is only necessary that the bundle should have the same cross-section, and that all the coils should carry currents of the same strength (number of amperes). Fig. 5a shows a lines of force diagram obtained experimentally in this way.

The experimental result is easily intelligible if we draw a bundle of coils in section. In fig. 6 the cross-section of the coils has been taken as square, for the sake of simplicity. In this figure we see that everywhere inside the bundle the currents in adjacent coils flow in opposite directions. Their effects cancel out and there remains only the effect of the parts of the turns on the surface of the bundle of coils (the heavy lines in fig. 6). That is. only the path of the surrounding coil remains. The approximation is the closer, the smaller the diameter of the single coils of the bundle*.

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* [A conducting coil is frequently called a *solenoid*.]

At the ends of the coil lines of force pass out into external space. They not only emerge from the two openings of the coil, but also leave it sideways between the turns near the ends of the coil. *These regions where the lines of force leave the coil are called the poles of the coil.* This name has arisen in analogy with the poles of a magnetic needle. If a coil carrying a current is suspended so as to be free to move in a horizontal plane, it sets itself north and south just like a compass-needle. If iron filings are strewn over it, a thick growth of them, like a beard, becomes attached to its poles, just as in the case of the poles of a magnetic needle. As with the latter, the middle part of the coil remains without iron filings. The lines of force come only from the ends, which we called the poles.

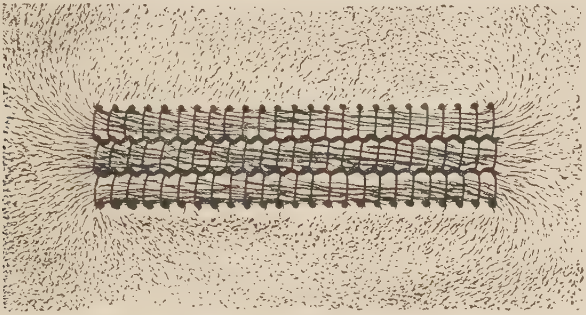

Fig. 5a. Magnetic field of a bundle of coils. The single coils were quite separate in this experiment. The apparent zig-zag connection between them is caused by the iron filings heaping themselves together between neighbouring wires.

If we increase the length of the coil carrying the current, other things remaining the same, the regions of the field which we have called the poles diminish in importance compared with the field inside the coil. Compare, for example, figs. 3 and 4.

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* In the case of a single coil the lines of force do *not* change their direction when they enter the inside of the coil: note the paths of the lines of force in fig. 3, following the arrows. The same is true for the bundle of coils in fig. 5a. The plane of the diagram of fig. 5a is taken *through the middle* of a layer of single coils.

Instead of this the plane may be taken as being between two neighbouring layers of coils. This is done in fig. 5b. The bundle of coils is not included in the photograph; its outline is merely indicated by the dotted lines. The lines of force now change their directions as soon as they enter the bundle of coils, at N and S. It looks as if magnetic lines of force can also have a beginning and an end. In this way we may reach the erroneous conclusion that at N and S there are magnetic substances, charges, or quantities, which somehow or other behave similarly to the positive and negative electric substances. We shall return to this later (p. 96).

It is even possible to make coils which have no poles at all. For this the coils must be wound to form closed rings. An example is shown in fig. 7, in which the cross-section of the turns of the coil is everywhere the same. This, however, is not necessary: by suitable choice of the distance between adjacent turns we can produce coils of variable section which have no poles.

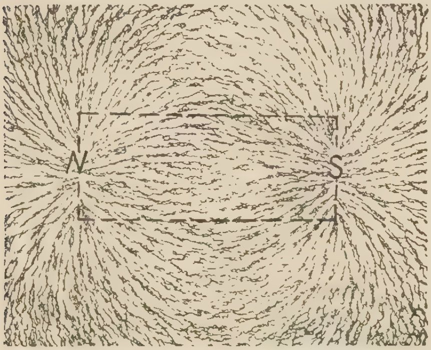


Fig. 5b. Lines of force *between* the coils of a bundle.

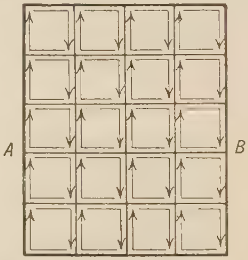


Fig. 6. Diagram of a bundle of rectangular coils: AB is the plane of fig. 5b.

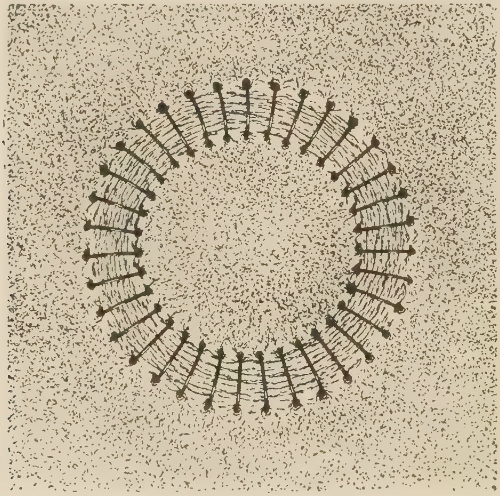


Fig. 7. Lines of force in the field of a ring coil.

In long coils the lines of force in the interior are practically straight lines, apart from the short regions near the poles. Elsewhere the lines of force are everywhere at the same distance apart and we have a *uniform* field.

The uniform magnetic field of a long coil plays the same part in connection with the magnetic field as the uniform electric field of a sufficiently plane plate condenser does in the theory of the electric field. We shall make frequent use of it.

The magnetic fields of permanent magnets differ in no way from those produced by coils carrying a current. We can imitate the magnetic field of any permanent magnet with a coil of the same size and shape, merely by paying attention to the proper arrangement of the winding; that is, we can replace any permanent magnet by a conducting coil of the same size and shape.

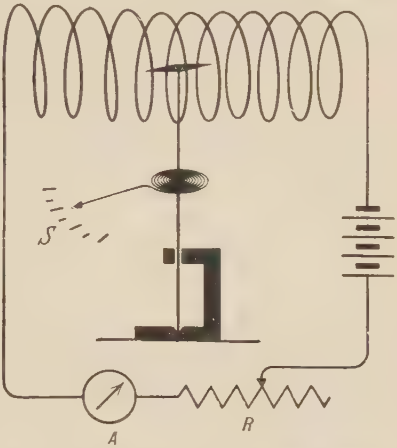


Fig. 8. Magnetoscope or magnetometer; the coil is like that in fig. 5, p. 104.

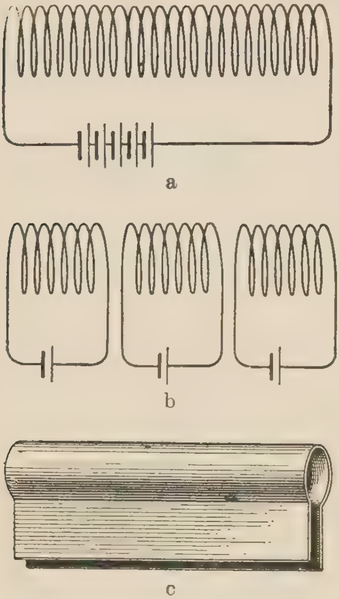
2. H, the Intensity of the Magnetic Field, and the Magnetometer.

Hitherto we have only considered the magnetic field and its lines of force qualitatively. We must now proceed to deal with the magnetic field quantitatively, using the international electric units, the volt and the ampere.

To solve this problem we need two things: (1) a uniform magnetic field, such as we can produce with a long coil (see p. 90 and fig. 4); (2) an arbitrary indicator of the magnetic field (a magnetoscope). The indicator need do no more than enable us to recognize two magnetic fields separated in space or time as being equal. It is not for *measuring* the fields with, but merely for establishing the *equality* of two fields.

As an indicator of this kind we use the turning moment, or torque, exerted on any compass-needle by the field. A compass-needle is fixed to a torsion balance as shown in fig. 8. The equilibrium position of the compass-needle is determined by the spring; that is, for the sake of simplicity we get rid of the weak magnetic field of the earth, which would normally make the compass-needle point towards the north. We bring the compass-needle into the uniform field of a coil, placing the needle perpendicular to the axis of the coil, that is, perpendicular to the direction of the lines of force shown in fig. 4. We close the circuit and send a current of strength i through the coil by means of the sliding resistance R. The needle is deflected. We then stretch the spring by turning the pointer until the needle reaches its original position of equilibrium. The angle of torsion α, read on the scale S, is a measure of the turning moment exerted on the needle by the field.

We now replace the first coil by others with different cross-sections A, different lengths l, and different numbers of turns n. Some of the coils are made up of one layer, others of many layers.



Figs. 9a—c. To illustrate the meaning of the quantity "ampere-turns per centimetre".

In this way we obtain a very simple result by a purely experimental method, namely, that magnetic fields are equal whenever the quantity in/l, or the number of amperes multiplied by the number of turns per centimetre of the length of the coil, is the same for them all. The cross-section of the coil does not come in, and it is all the same whether the coil has one layer or many.

Hence the product in/l, *or, for short, the "ampere-turns per centimetre" is an unambiguous measure of the intensity of the magnetic field in a long coil. We call the product the intensity (or strength) of the magnetic field,* **H**, *and we write*

$$\mathbf{H} = \frac{in}{l}. \qquad (1)$$

The unit for **H** is one ampere-turn/centimetre.

We shall give another example to explain the idea of ampere-turns per centimetre. We shall produce a magnetic field of intensity $\mathbf{H} = 10$ ampere-turns/cm. in a space of about the size of a finger. This can be done in many different ways, e. g.

(a) A coil 20 cm. long and about 2 cm. in diameter is made up of 100 turns, and a current of 2 amperes is sent through it (fig. 9a).

(b) Four coils of equal thickness but each only 5 cm. long are arranged in a row. Each one has 1000 turns, possibly in more than one layer. Each coil is supplied from a separate source with a current of 50 milliamperes (fig. 9b).

(c) A strip of sheet metal 20 cm. wide is bent into a tube which is closed except for a narrow longitudinal gap, the rest of the metal forming a broad lead for a current of 200 amperes (fig. 9c).

Such examples can be multiplied to as great an extent as we please.

The compass-needle of the torsion balance has hitherto served only as an indicator of the magnetic field. It was only meant to prove the equality of two fields from the equality of the turning moments measured by it.

We can now complete the little instrument and make it into a *magnetometer*. We vary i, the current in one of our coils, and hence **H**, the intensity of the field, by equation (1). We find that the turning moment is proportional to **H**, the intensity of the field. For example, in our demonstration model 2 ampere-turns per centimetre correspond to twisting the spring through one scale division. We have thus graduated our magnetometer, and, moreover, in international electric units.

Once the magnetometer has been graduated with the help of a long coil, it can thereafter be used to measure any magnetic field, no matter what its origin.

Suppose, for example, that we have to measure the magnetic field of a bar magnet NS at the point P. We first detach the needle of the magnetometer from the torsion balance arrangement and leave it free to move: it places itself in the direction of the lines of force at P. We then fix the needle firmly to the torsion axis. A torsion of 3·7 scale divisions brings it perpendicular to the direction of the lines of force (the latter has just been ascertained). Then the intensity of the field at the point P is given by $\mathbf{H} = 2 . 3{\cdot}7 = 7{\cdot}4$ ampere-turns/cm.

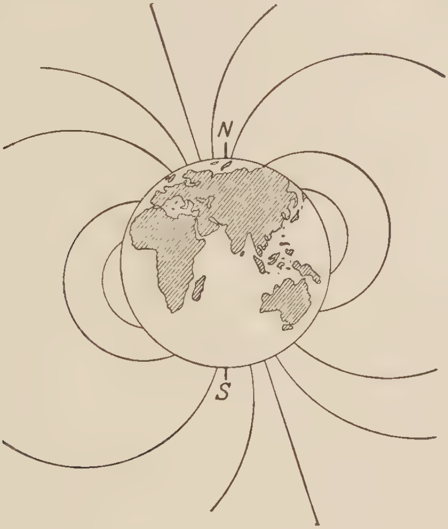


Fig. 10. The magnetic field of the earth.

The *magnetic field of the earth* is measured on a similar principle. Fig. 10 gives a schematic diagram of the earth's lines of force. The component of the field parallel to the earth's surface is called the horizontal component. At Greenwich it amounts to about 0·14 ampere-turns/cm.

3. Iron in the Magnetic Field of a Coil, and Molecular Currents.

On pp. 86—91 we became acquainted with a whole series of magnetic fields of various forms. For our further investigations we shall take the field of a coil of the kind shown in fig. 11.

We can follow up the lines of force emerging from the ends of this coil for a considerable distance. How far it is possible to do this in a particular case depends merely on the product of the intensity of the current and the number of turns, when the length of the coil is given. We call this product *in* the *ampere-turns* of the coil. Suppose that the magnetic field of the coil is weak. Let it be insufficient to arrange iron filings in a clearly visible way or to turn a compass-needle away from the north and south direction of the earth's field at the point *A*. Then we have only to increase the current intensity (amperes) or wind more turns on to the coil, or do both at once, in order to raise the intensity of the field at the point *A* until its existence can easily be demonstrated.

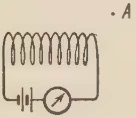


Fig. 11. Coil carrying a current: a compass-needle is placed at *A*.

The introduction of a bar of iron into the coil has exactly the same effect as increasing the number of ampere-turns. In the case of a coil with an iron "core" even quite large pieces of iron, such as nails or screws, can be used to show the lines of force, while compass-needles are deflected at great distances.

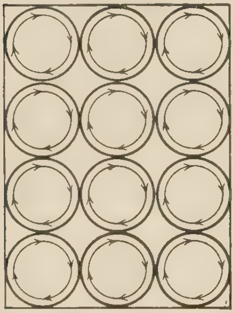


Fig. 12. Rough diagram of ordered molecular currents.

It necessarily follows from this that the iron inside the coil increases the number of ampere-turns. Now the iron increases neither the number of turns in the wire nor the number of amperes in the current in the coil as recorded by the galvanometer. There is only one way out of this difficulty, namely that *invisible currents must circulate in the iron in the same direction as the current in the coil.* It is their ampere-turns that are added to the original ampere-turns of the coil.

These invisible currents are called the molecular currents. We shall illustrate them by a schematic diagram. Let fig. 12 represent the rectangular cross-section of the iron bar. The molecules are drawn as circles in the rough naïve way to which we are already accustomed. In each molecule atoms of electricity circulate along the paths marked with arrows. They follow these paths as in the case of a current flowing through a ring-shaped conductor. In cross-section we have essentially the same diagram as we had in fig. 6 when we replaced a thick coil by a whole bundle of thinner coils.

The molecular currents exist even before the iron is brought into the magnetic field of the coil. Then, however, they are not arranged in any particular way; the atoms of electricity circulate in widely differing planes and directions. When the iron is brought into the magnetic field of the coil, the planes of the orbits of the atoms of electricity set themselves parallel to the turns of the coil. The path of a single molecular current behaves just like the movable coil in fig. 9d (p. 8).

We remove the magnetic field of the coil by taking out the iron core or breaking the current in the coil. The field of the iron then vanishes to a great extent, but not completely. Most of the molecular currents spring back into their old random positions. *But a few remain in the direction they acquired. The iron exhibits "residual magnetism": it has become a "permanent" magnet (a compass-needle).*

A very striking experiment may be adduced to show that the principle of this conception is correct and that the molecular currents do exist. The experiment depends on the idea that the atoms of electricity circulating as molecular currents are little *tops*.

In the mechanical case of the top the following experiment may be shown. A person stands erect on a revolving stool and holds a top in his hand. The axis of the top points in any direction oblique to that of the axis of his body. He then lets the axis of the top tilt so that it becomes parallel to the axis of his body (fig. 13). This tilting causes the person to receive an impulse and he begins to rotate about his long axis. The rotation gradually ceases owing to the friction of the stool on its bearings.

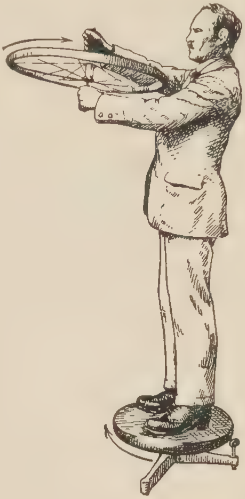


Fig. 13. Tilting of a top on a revolving stool (conservation of angular momentum).

We now imagine the person to be replaced by a bar of iron and the top by the electrons circulating at random. The bar of iron hangs in the long axis of a coil as shown in fig. 14. If a current is sent through the coil, the axes of the paths of the circulating electrons place themselves parallel to the axis of the coil or bar. The iron bar executes a rotation which is observed using a mirror and beam of light.

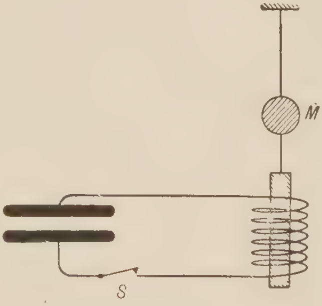


Fig. 14. Diagram of the experiment proving the existence of the molecular currents in iron.

In practice we let the current flow only for a very short time (by discharging a condenser, which takes 10^{-3} seconds): that is, we use only that small fraction of the electrons which retains a definite direction even after the current has disappeared and which produces the residual magnetism of the iron. If a current flowing all the time were used, the difficulty would arise that the magnetic field of the coil could not be made completely uniform, although it might extend to a great distance. Hence the bar of iron would gradually be attracted into the region of greatest intensity of the field if the current were allowed to flow for a longer time, as in the experiment shown in fig. 10 (p. 9). Unfortunately this fundamental experiment is not well suited for performance on a larger scale.

4. The Essential Difference between the Electric Field and the Magnetic Field.

The lines of force of an electric field and of a magnetic field bear a strong resemblance to each other externally. There are important differences, however, between the electric field and the magnetic field. In the electric field we can actually observe the *electric substances* at the ends of the lines of force. We can separate the positive and negative electric substances in space. Of any two bodies, one (K) may contain negative substance (in excess) and the other (A) positive substance (in excess). All the lines of force start from the one body (K) and end on the other (A).

It has not been found possible to make the analogous observations for the magnetic field. North and south magnetic substances have been much sought after. They have never been found, nor has it ever been found possible to separate them from each other in space.

An impartial observer would scarcely look for north and south magnetic substances in the case of magnetic fields in the neighbourhood of conductors carrying currents (figs. 1—4, pp. 86—88), for there nothing is to be seen but endless lines of force in the form of closed curves. As a matter of fact all the experiments for finding and separating the north and south magnetic substances have been carried out with *permanent steel magnets*. All efforts have remained fruitless. Nowadays we no longer see any difference between a permanent magnet and a bundle of coils carrying currents. The conception of closed molecular currents has become an experimental certainty, and since then the whole question of finding and separating the magnetic substances has lost all meaning. According to all the experimental results, there exist no north and south magnetic substances which are in any way comparable with positive and negative electric substances.

Nevertheless, the fiction of quantities of magnetism mentioned in connection with fig. 5b (p. 89) is frequently used in expositions of the subject. This is to suit the mathematical treatment. It is a misleading analogy which at the very least makes it more difficult for the beginner to understand the subject.

In the electric field a current can flow in a conductor, because atoms of electricity are capable of moving through many bodies. *There is no such thing as magnetic conduction.* Atoms of magnetism cannot move, seeing they do not even exist!

In the electric field the current in a conductor causes induction phenomena to appear. If we bring a conductor into an electric field, the atoms of electricity move separately to opposite parts of the surface. The interior of the conductor becomes free of electric lines of force.

The corresponding phenomenon is absent in the magnetic field because neither atoms of magnetism nor magnetic conduction exist. A body brought into a magnetic field is always penetrated by magnetic lines of force. Most bodies do not affect the course of the lines of force at all, at least to an extent observable without sensitive measuring instruments.

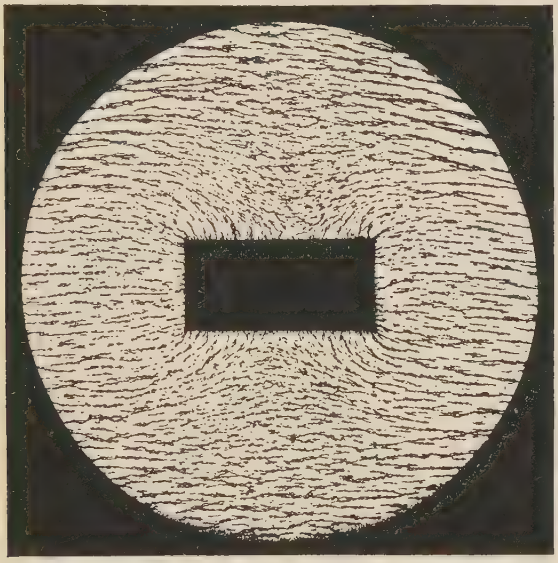


Fig. 15. Rectangular piece of iron in a previously uniform magnetic field. Apparent induced magnetism.

In the case of iron, however, there is a phenomenon which on casual inspection may simulate induced magnetism. We bring a cylinder of iron into a uniform field, as shown in fig. 15. The lines of force crowd together towards the ends of the iron and become correspondingly few in number along the longer sides.

There is no difficulty, however, in interpreting this phenomenon as an effect due to the molecular currents. In fig. 16 we give the original uniform field by itself. The outline of the iron is indicated by shading. The external field arranges the molecular currents in the iron. The molecular currents then give the dotted lines of force. As we already know, they are the same as the lines of force of a coil of the same size and shape as the iron cylinder. These two fields, the external field and the field due to the molecular currents (shown in dotted lines) overlap and combine to form the resultant field. In the interior of the iron and outside its ends a strong field is produced, because here both fields are in the same direction: note the directions of the arrows in fig. 16. Outside the lateral surfaces, however, the arrows point in opposite directions, and the fields practically cancel each other over a wide area.

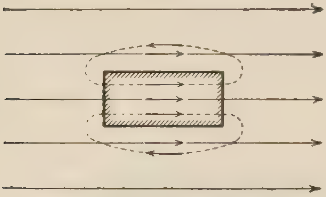


Fig. 16. To explain fig. 15.

The influence of pieces of iron of other shapes on magnetic fields can be explained in a similar way. Fig. 17 shows a thick iron ring in an approximately uniform magnetic field. The field inside the ring is greatly weakened. These iron rings or hollow spheres are used as so-called "magnetic screens" in order to prevent sensitive instruments from being disturbed by magnetic fields.

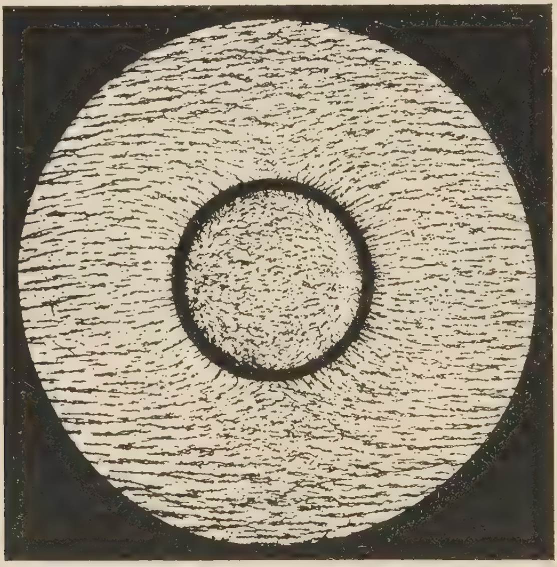


Fig. 17. Magnetic screen for minimizing the effect of a magnetic field.

The difference between the electric and magnetic fields is shown in practice by the ways in which the *directions* of the lines of force are settled in the two cases. **H** and **R**, the intensities of the magnetic and electric fields, are both *vectors* represented by arrows. In the *electric* field the two ends of the arrow correspond to actual physical differences. The feathers correspond to *electrons*, which are minute bodies weighing only $^{1}/_{1800}$ as much as an atom (cf. p. 212). The arrow-head corresponds to positive ions, which are formed when chemical atoms and molecules lose electrons. The direction of the arrow in the *magnetic* field cannot be settled by any physical difference, but only by the *formal* difference between right and left. The two ends of the arrow can be distinguished only in the same way as the two ends of an axis of rotation can be distinguished, i. e. by the clockwise or counterclockwise direction of the rotation. It is by no means an accident that the direction of a magnetic line of force is defined by the north-seeking compass-needle, that is, by its orientation with respect to the axis of rotation of the earth.

The more one pays heed to the essential difference between magnetic lines of force and electric lines of force, the simpler does the understanding of magnetic and electric phenomena become.

CHAPTER IV

The Connection between Electric Fields and Magnetic Fields

1. Preliminary Remarks.

Hitherto we have only known of the following comparatively loose connection between electric and magnetic fields.

Between the condenser plates K and A there is an electric field (fig. 1). K has an excess of negative atoms of electricity, A of positive. K and A are then connected by a conductor, say by a wire. As a result the electric field between K and A breaks down. The atoms of electricity move through the conductor, i. e. a current i flows in the conductor. This current is surrounded by closed magnetic lines of force. The magnetic field exists as long as the current does. So much for our present knowledge, which we acquired in Chapter II.

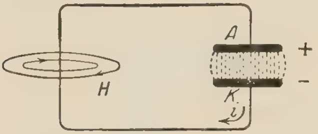


Fig. 1. Current in the conductor during the discharge of a condenser. We always call the direction of motion of the atoms of negative electricity the "direction" of the conduction current.

In reality, however, the connection between electric and magnetic fields is much deeper than our experiments would hitherto lead us to suppose. *There are two remarkably close connections between electric fields and magnetic fields.*

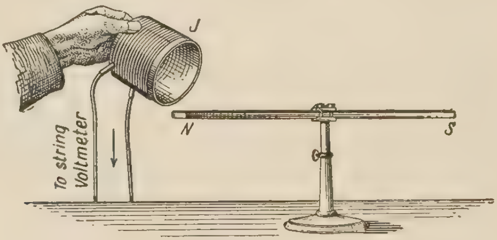


Fig. 2. Experiment on electromagnetic induction.

The discussion of these mutual connections forms the most important part of this chapter. We begin by stating the two *experimental facts from which we start:*

(1) the phenomenon of electromagnetic induction (pp. 101—106);

(2) the *magnetomotive force* of the conduction current (pp. 106—112).

The extension of the significance of these phenomena according to Maxwell then follows. We find that

(1) A changing magnetic field is surrounded by closed ring-shaped lines of electric force (fig. 13, p. 113) (Law of electromagnetic induction, Maxwell's second equation);

(2) A changing electric field is surrounded by closed ring-shaped lines of magnetic force (fig. 22a, p. 120) (Magnetomotive force of the displacement current as defined on pp. 118—120, Maxwell's first equation).

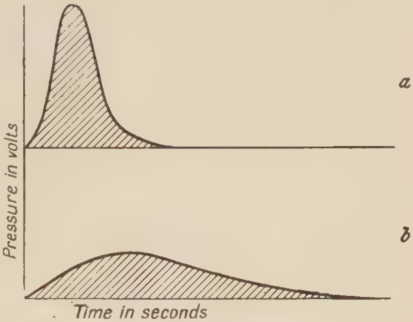


Fig. 3. Two equal "time-integrals of pressure", or "integral pressures", measured in volt-seconds.

2. The Phenomena of Electromagnetic Induction.*

Suppose that we are given any magnetic field, e. g. that of the bar magnet in fig. 2. In this magnetic field there is an easily movable coil J, the ends of which lead to a string voltmeter (unifilar voltmeter). By turning, tilting, or displacing the coil J we alter the number of lines of magnetic force passing through it. Any such movement if carried out suddenly enough causes a distinct deflection of the voltmeter, which lasts for a short time: we observe that there is an electrical pressure, measurable in volts, between the ends of the coil J. This pressure is said to be "induced". *In future we shall call the coil J the "secondary coil" for short.*

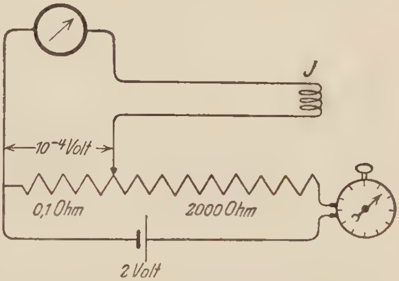


Fig. 3c. Graduation of a moving-coil galvanometer with a secondary coil J in series with it (cf. figs. 2, 4, et seq.) to measure integral pressures in volt-seconds.

We repeat one of the movements we have just mentioned many times in succession, but each time with a different *velocity*. We find that the electric pressure between the ends of the secondary coil J exists throughout the whole duration of the motion. If e. g. we rotate the coil suddenly the deflection of the voltmeter is somewhat as sketched in fig. 3a: high pressures for a short time. If the motion is slower, we obtain the diagram of fig. 3b, say: low pressures for a longer time.

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* [Discovered by Faraday in 1831.]

The shaded area is called the "time-integral of the pressure" ($\int E\,dt$) or the "integral pressure", measured in volt-seconds. Here we have an analogy with the time-integral of the current, measured in ampere-seconds or coulombs, which we discussed in detail on pp. 32—36.

In order to measure an integral pressure in volt-seconds we use the ballistic deflection of a long-period voltmeter. We proceed just as we did in the determination of the number of ampere-seconds on p. 34. For the long-period voltmeter we can use either a quadrant electrometer (p. 82) or a moving-coil galvanometer. We prefer the latter on account of its being more convenient to manipulate. The graduation of the galvanometer so as to record volt-seconds is very simply carried out in a way analogous to the graduation of a galvanometer in ampere-seconds described on p. 34 (cf. fig. 3c). *We connect known pressures with the galvanometer during short but accurately*

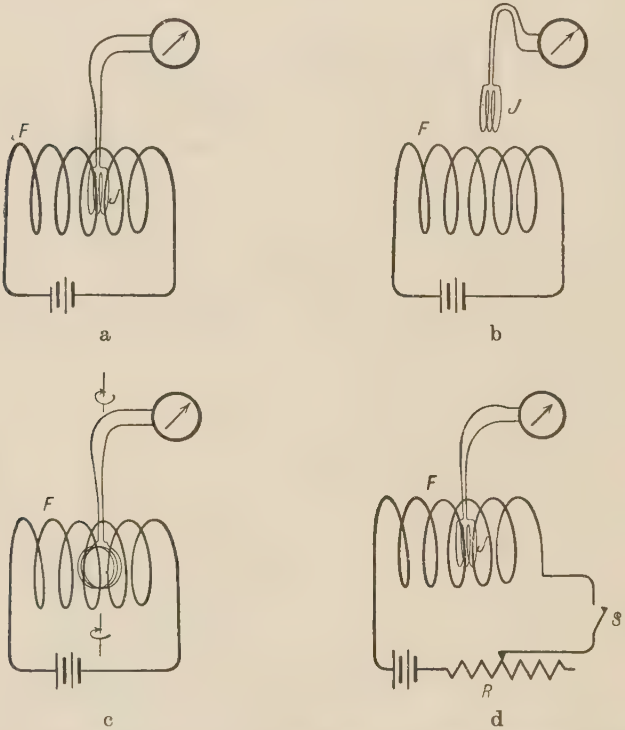

Fig. 4a—d. Various forms of the experiment on electromagnetic induction: the secondary coil J is smaller in diameter than the primary coil F.

measured periods of time t. We again do this by means of the stop-watch switch which we met with in fig. 24, p. 35.

A known pressure of suitable magnitude is obtained by dividing up a pressure as shown in fig. 3c. We observe the deflections α for various values of the product Et and we find that the quotient Et/α is constant. This constant value is the number of volt-seconds which correspond to an instantaneous deflection of one scale division. The galvanometer is now graduated to measure integral pressures in volt-seconds.

For the magnetic field causing the induction we take that of a coil F carrying a current (fig. 4a). Using the field of a coil like this, the number of lines of magnetic force passing through the secondary coil J can be conveniently altered by the same amount in various ways. To do this we take all the ways in which the secondary coil is alternately traversed *at right angles* by the lines of force or traversed by *no lines of force at all.*

(1) The secondary coil J is pulled out from or pushed into a slit between the turns of the "primary" coil F. F may remain fixed while J is moved, or vice versa (fig. 4b).

(2) One of the coils is rotated about the axis marked by arrows, in such a way that the turns of the secondary coil and the primary coil are alternately parallel and perpendicular to each other (fig. 4c).

(3) The current in the primary coil F is switched off or on. This can be done suddenly with a switch S or gradually by altering a resistance with a sliding contact (fig. 4d).

In all these cases we obtain ballistic deflections of equal amount. The number of induced volt-seconds is quite independent of the nature of the process employed and of its velocity. Its sign changes, however, if instead of a certain process that in the opposite direction is used.

The sign is determined by Lenz's law, which is as follows: *The electric field produced by electromagnetic induction is always in a direction such that it slows down the motion producing the induction. Here we must regard a current as a motion of the atoms of electricity. An increase of the current means an acceleration of the electrons, a decrease of the current means a retardation of the electrons.*

Lenz's law is a simple consequence of the principle of energy. If the effect had the opposite sign, the initial motion causing the induction would increase to an infinite extent, producing work out of nothing. It is not worth while to verify Lenz's law in the case of the above experiment by following out the directions of the currents in the primary coil and the secondary coil on a particular occasion. We shall come across many very obvious cases of Lenz's law later on.

We proceed to summarize the induction phenomenon as follows: *If the magnetic field traversing one coil is altered, electric pressures appear between the ends of the coil. If the same change is caused in various* *ways and at various speeds, the same number of volt-seconds is observed in all cases.*

3. The Law of Electromagnetic Induction.

The relation between the induced volt-seconds and **H**, *the intensity of the field. The permeability* μ.

We proceed to the quantitative investigation of the phenomenon of electromagnetic induction. We have to determine which quantities fix the number of volt-seconds induced in a coil. For this we require:

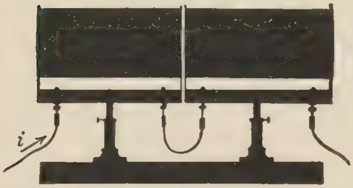

Fig. 5a. Primary coil F for the induction experiments of fig. 4, made in two parts; $n/l = 20$.

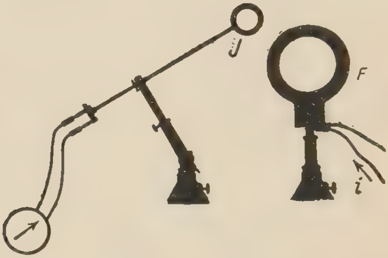

Fig. 5b. To illustrate the experiment diagrammatically represented in fig. 4b. The primary coil F is foreshortened owing to its being seen end-on.

(1) Secondary coils J of various cross-sections A_J and various numbers of turns n;

(2) Uniform magnetic fields from coils, whose intensity **H** can easily be altered by altering i, the current in the coil. By equation (1) (p. 92) we have

$$\mathbf{H} = \frac{\text{current } (i) \cdot \text{number of turns } (n)}{\text{length of the coil } (l)}.$$

The secondary coil may be of *smaller* cross-section than the primary coil, as e. g. in the case of figs. 4a—d. The reverse may also be the case. The secondary coil may, for example, completely *surround* the primary coil F of cross-section A_F, as in figs. 6 and 7.

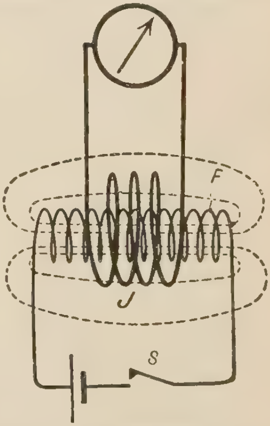

Fig. 6. Diminution of the induced volt-seconds owing to the lines of force of a short primary coil F returning through the secondary.

In this case, however, a source of error which is indicated in fig. 6 must be avoided. Attention must be paid to lines of force returning through external space. In very long coils this source of error may be neglected. The lines of force passing from pole to pole stray so far into external space that they do not pass through the secondary coil to any noticeable extent. A ring-shaped primary coil like that in fig. 7 is still more reliable. In its case there is certainly no field in the external space. In the ring-shaped coil the poles which form the starting-points of the lines of force passing into external space are absent (cf. p. 90).

With this apparatus we obtain a very simple result. *The integral pressure, or the number of volt-seconds, depends only on the change of the product* $n.A.\mathbf{H}$. *Here* A *means the cross-section (in a direction perpendicular to the lines of force) of the magnetic field or bundle of lines of force enclosed by the turns of the secondary coil.*

Examples: —Let the secondary coil, of cross-section A_J, be placed right inside the primary coil (fig. 4c). Then $A = A_J$ when the coils are parallel to each other. When they are inclined at an angle of 45° A is equal to $\frac{A_J}{\sqrt{2}}$. A is equal to $\frac{A_J}{2}$ if half the surface of the secondary coil projects through a slit at the side of the primary coil. Again, the secondary coil may be altogether outside the primary coil, surrounding it as in fig. 7. Then A is equal to A_F, the cross-section of the primary coil, independently of the inclination of the coils.

All the processes which alter the product $n.A.\mathbf{H}$ *by the same amount give the same integral pressure, measured in volt-seconds.* In what way or with what velocity this is done in a particular case is of no importance. In the form of an equation:

Induced integral pressure, measured in volt-seconds

$$= \mu\left[\underset{\text{at the beginning}}{(n_1 A_1 \mathbf{H}_1)} - \underset{\text{at the end}}{(n_2 A_2 \mathbf{H}_2)}\right], \quad (1)$$

where μ is a constant factor of proportionality. This is the fundamentally important law of electromagnetic induction. The constant μ is called the *permeability*. Its numerical value for empty space is

$$\mu = 1{\cdot}256.10^{-8} \text{ volt-seconds/ampere-centimetre;}$$

the value for air is practically the same.

The maximum value of the integral pressure S which can be induced using a given coil and a given intensity of field $\mathbf{H}$ is

$$S = \mu\, n\, A\, \mathbf{H} \text{ volt-seconds} \quad (1a)$$

by equation (1). Here the whole cross-section of the coil A must be penetrated by the lines of force $\mathbf{H}$ at right angles, and either A or $\mathbf{H}$ must be altered between zero and its full value.

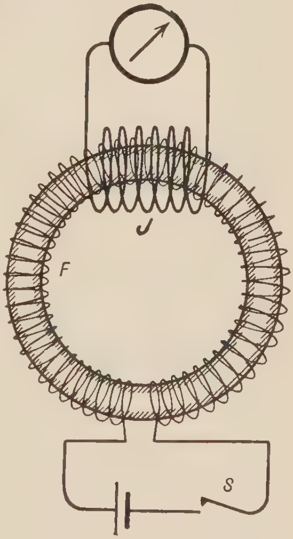

Fig. 7. Measurement of permeability.

In the numerous applications of the law of electromagnetic induction in later chapters we shall frequently use a given coil penetrated by a magnetic field $\mathbf{H}$ which alters with the "velocity" $\dot{\mathbf{H}}$. That is, the magnetic field $\mathbf{H}$ is to increase or decrease per second by $\dot{\mathbf{H}}$ ampere-turns per centimetre. In this case we observe an induced pressure between the ends of the coil amounting to

$$E_{\text{ind.}} = \mu n A \dot{\mathbf{H}} \text{ volts}. \tag{1b}$$

The permeability μ is a constant depending on the material. We have already given its value for a vacuum; the value for air is practically the same. In order to determine μ for different substances, the lines of force must be made to run their whole course in the substance in question. The simplest arrangement is that sketched in fig. 7, where the interior of the ring coil is filled with the substance. We usually measure relative values μ' only, taking the permeability of a vacuum as unity. The table below gives some of these values. Substances for which $\mu' > 1$ are called paramagnetic; substances for which $\mu' < 1$, diamagnetic. In all cases the values are so nearly equal to 1 that only the differences between μ' and 1 are given; this is done in the following table.

Table II.

Paramagnetic Substances			Diamagnetic Substances		
	$\mu' - 1$			$1 - \mu'$	
Nitrogen	0·013	$\cdot 10^{-6}$	Hydrogen	0·063	$\cdot 10^{-6}$
Air	0·38		Benzene	7·5	
Oxygen	1·9		Copper	8·8	
Vulcanite	14		Water	9·0	
Aluminium	23		Glass	12·6	
Platinum	360		Rock Salt	12·6	
Liquid Oxygen	3400		Bismuth	176	

We note these values of the permeability at this stage, just as we did previously with the relative values of the dielectric constant K'. We shall see later that the former can also be deduced from motions of electrons in the atoms of matter.

4. The Magnetomotive Force M*.

We know that a magnetic field is the inseparable companion of the electric current, and that any conductor carrying a current is surrounded by closed lines of magnetic force. *Hitherto, however, the quantitative connection between the current and the magnetic field has been wanting.* Now that we have the law of electromagnetic induction at our disposal we can obtain the required connection in a very simple way by experiment. We shall discuss this in the present section.

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* [German: *magnetische Spannung*, i. e. magnetic pressure.]

In the electric field the electric pressure is equal to the line-integral of the electric intensities:

$$E = \mathbf{R}_1 s_1 + \mathbf{R}_2 s_2 + \ldots + \mathbf{R}_m s_m = \sum \mathbf{R}_m s_m, \quad \text{(equation 4, p. 68)}$$

or, otherwise expressed,

$$E = \int \mathbf{R}\, ds. \qquad \text{(equation 4a, p. 68)}$$

The unit of electric pressure is the volt.

In an analogous way we define the magnetomotive force in the magnetic field as the line-integral of the magnetic intensities $\mathbf{H}$:

$$M = \mathbf{H}_1 s_1 + \mathbf{H}_2 s_2 + \ldots + \mathbf{H}_m s_m = \sum \mathbf{H}_m s_m, \tag{2}$$

or

$$M = \int \mathbf{H}\, ds. \tag{2a}$$

The unit of magnetomotive force is the *ampere-turn*.

Magnetomotive force can be measured by a simple piece of apparatus, the so-called *magnetic potentiometer*, using the principle of the law of induction.

The magnetic potentiometer essentially consists of a very long secondary coil, wound on a strap, say. It is wound in two layers, with the leads at the middle of the outer layer (fig. 9). (A coil of *one* layer might, when considered as a whole, represent a large plane secondary coil formed by *one* winding of a spiral wire.)

We shall proceed to explain the way in which this magnetic potentiometer works. Let the broken line $s_1\, s_2 \ldots s_m$ (fig. 8) represent the path s, between the ends of which the magnetomotive force is to be determined. Let the intensities of the fields acting on the separate elements of the line be $\mathbf{H}_1, \mathbf{H}_2, \ldots, \mathbf{H}_m$. Let the potentiometer enclose the whole of the path s, and let it have N turns per centimetre of its length. Then in the m^{th} line element, whose length is s_m, there are $N \cdot s_m$ turns. If the field $\mathbf{H}$ is created or destroyed, a definite integral pressure, say S volt-seconds, is induced in the potentiometer. This integral pressure is obtained by adding the amounts for the separate line-elements. Hence, if A denotes the (rectangular) cross-section of the potentiometer coil,

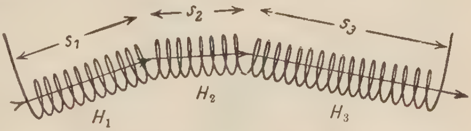


Fig. 8. Diagram of Rogowski's magnetic potentiometer.

$$\begin{aligned} S &= \mu A\, \mathbf{H}_1 N s_1 + \mu A\, \mathbf{H}_2 N s_2 + \ldots \mu A\, \mathbf{H}_m N s_m, \\ &= \mu A N (\mathbf{H}_1 s_1 + \mathbf{H}_2 s_2 + \ldots \mathbf{H}_m s_m) \\ &= \mu A N M, \end{aligned}$$

so that

$$M = \text{const.}\, S, \tag{2b}$$

the constant being $\frac{1}{\mu N A}$. That is, S, the integral pressure measured in volt-seconds, multiplied by the constant $\frac{1}{\mu N A}$, directly gives the required magnetomotive force in ampere-turns. The constant is determined once and for all; A and N are found by direct measurement, and μ is the universal permeability of a vacuum, or practically that of air, which is equal to $1.256 . 10^{-8}$ volt-seconds/ampere-centimetre, as we saw on p. 105.

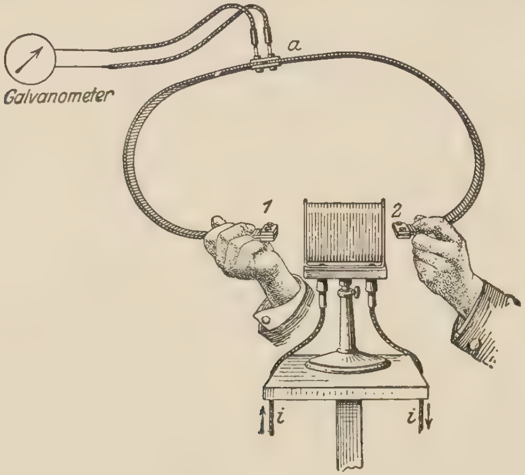


Fig. 9a. Manipulation of the magnetic potentiometer.

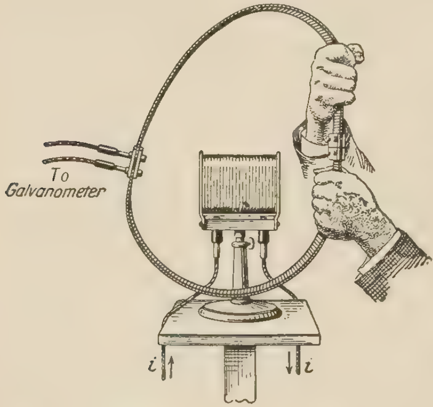


Fig. 9b. Closed path of a magnetic potentiometer enclosing no current.

We use a magnetic potentiometer 120 cm. long. Its constant is $5 . 10^5$ amps./volt-second (9600 turns in all, each with a cross-section of 2 sq.cm.). The graduation is to be carried out as shown in fig. 3a. (J is the potentiometer.) The volt-seconds induced are measured by a long-period voltmeter such as was mentioned on p. 102.

Fig. 9a explains how the potentiometer is manipulated. Suppose that the magnetomotive force M of the field of a coil between the points 1 and 2 counted along the path $1a2$ is to be measured. The potentiometer is made to take the shape of this path. We then change the magnetic field from zero to its full value by switching the current off and on, and observe the number of induced volt-seconds on the ballistic galvanometer.

In this way we can investigate the magnetomotive force in the neighbourhood of conductors carrying currents, by means of the magnetic potentiometer.

We thus establish the following facts:

(1) The magnetomotive force between two distinct points of the field is independent of the shape of the path between them; the path may even include *loops*, provided that these loops do not encircle the current.

(2) If the beginning and the end of the path coincide, the magnetomotive force is equal to zero, provided that, as e. g. in fig. 9b, the path does not encircle the current.

(3) On the contrary, the magnetomotive force has a finite value even for a closed path when the path goes *once* round the current. The shape of the path (circular, rectangular, etc.) makes no difference (fig. 9c). To prove this important fact we make the potentiometer surround a conductor through which a current of say 50—100 amperes is flowing. Currents of this magnitude can easily be produced by a 2-volt accumulator.

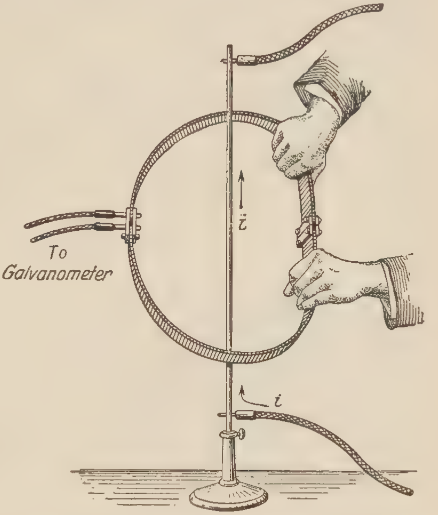


Fig. 9c. A current encircled once by a magnetic potentiometer.

(4) The numerical value of the magnetomotive force M turns out to be the same as that of i, the strength of the current encircled by the potentiometer. We have

$$M = \int \mathbf{H}\, ds = i\,. \tag{3}$$

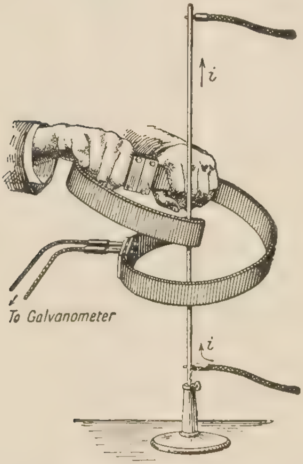
Numerical example: $i = 83$ amperes. A 12 cm. fling on the long-period voltmeter means that $S = 1{\cdot}7\,.\,10^{-4}$ volt-seconds. S, multiplied by the constant of the potentiometer, namely $5\,.\,10^5$, gives the magnetomotive force $M = 1{\cdot}7\,.\,10^{-4}.\,5\,.\,10^5 = 85$ ampere-turns.

(5) The magnetomotive force M is multiplied by n if the potentiometer encircles the current n times; that is, if we form the line-integral of **H** along a closed curve encircling the current n times (fig. 9d), we have

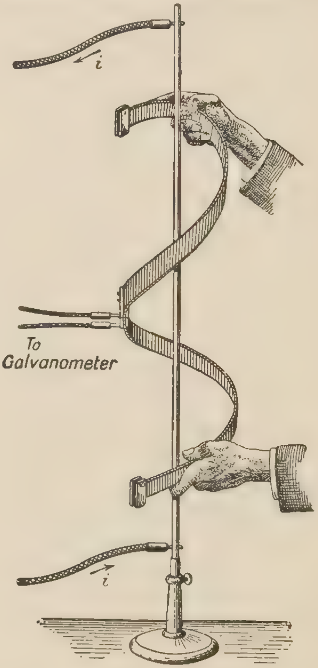
$$M = ni\,. \tag{3a}$$

In fig. 9d the path encircling the current i twice is *closed:* the beginning and the end of the potentiometer coincide This, however, is not necessary: the potentiometer may equally well encircle the current n times in the form of a spiral with open ends. For example, the same magnetomotive force is obtained in the experiment of fig. 9e as in that of fig. 9d. (In both cases $n = 2$.)

The following further experiment may serve to make this clear. We know that a magnetic field straightens out the molecular currents in iron and thus raises the number of ampere-turns in the field (pp. 94—96). We surround a copper wire carrying a current of about 50 amperes with a thick iron wire. A strong magnetic field, which is made visible by iron filings, appears between the ends of the iron wire. We repeat the experiment, encircling the current n times; e. g. let $n = 5$. The magnetic field between the ends of the iron wire is considerably stronger: it supports a much thicker bunch of iron filings.



d



e

Fig. 9d, e. A current encircled twice by a magnetic potentiometer, d by a closed path, e by an open path.

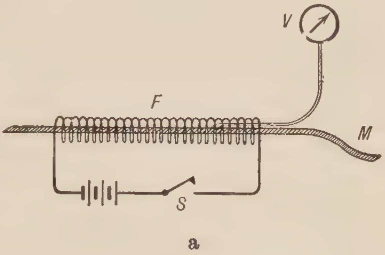
Thus we have experimentally obtained the required quantitative connection between current and magnetic field. In words it is as follows: *The magnetomotive force along an arbitrary closed curve which encircles the current i once is numerically equal to the number of amperes in the current. If the curve encircles the current n times the magnetomotive force rises to n times the above value.* This result is of fundamental importance. Equation (3a) is the most concise way of stating it.

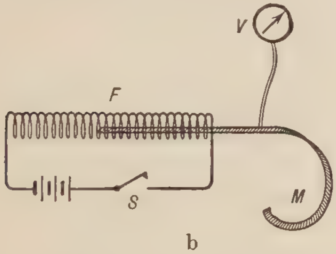
In order to emphasize these important facts, we bring forward some consequences of them which can easily be compared with experiment.

(1) We apply equation (3) to the *long coil traversed by a current*, which we have often used already. As before, let its length be l. By

fig. 4 (p. 88), **H**, the intensity of the field within the coil, is constant except at the short regions near the poles, so that the magnetic field is uniform. $\mathbf{H}_0$, the intensity of the field in the space outside, is vanishingly small compared with **H**, the intensity inside the coil. Hence the magnetomotive force M is simply the product $\mathbf{H}l$.

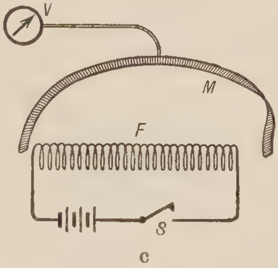
The potentiometer passes right through the whole length l of the primary coil F. Putting the switch S off and on gives an integral pressure of $1{\cdot}7\,.\,10^{-3}$ volt-seconds every time, i. e. $M = 850$ ampere-turns, by equation (2b). The length and position of the hanging ends of the coil are of no account in practice; that is, the field in the space outside the coil does not add any appreciable amount to the line-integral of the intensity **H**, i. e. to the magnetomotive force.





The potentiometer traverses the right-hand half of the primary coil only. We obtain an integral pressure of $0{\cdot}85\,.\,10^{-3} = {}^1/_2\,.\,1{\cdot}7\,.\,10^{-3}$ volt-seconds, or $M = 425$ ampere-turns. That is, **H**, the intensity of the field, is constant inside the coil.

The potentiometer is in an arbitrary position wholly outside the coil. The induced integral pressure only amounts to about $9\,.\,10^{-5}$ volt-seconds. In the space outside the coil, M is only about 45 ampere-turns, which is negligible compared with the 850 ampere-turns obtained inside the coil. M, the line-integral of the intensity **H**, is in fact practically zero, even in this case where the coil is by no means very long.



Figs. 10a—c. Distribution of magnetomotive force in the field of a long primary coil F. F has 900 turns, a length of 50 cm. and a diameter of 10 cm. It is traversed by a current of one ampere, and **H**, the intensity of the magnetic field within it, amounts by calculation to 18 ampere-turns per centimetre.

On the other hand, M must be equal to ni, by equation (3a) (p. 109). For the lines of force encircle the n turns, each of which carries a current of i amperes, i. e. they encircle ni amperes in all. Hence we have

$$\mathbf{H}l = ni \quad \text{or} \quad \mathbf{H} = ni/l\,.$$

But this is none other than equation (1), which we obtained empirically on pp. 91—92, and according to which **H**, the field intensity in a long coil, is merely determined by the number of ampere-turns per centimetre of the length of the coil.

We should not fail to verify the fact that the field of a long coil is distributed in the above way, by using the magnetic potentiometer. Figs. 10a—c illustrate suitable experiments.

(2) *A current i is flowing through a straight wire.* The magnetomotive force along one of its circular lines of force (fig. 4, p. 5) of radius a is, from symmetry,

$$M = 2\,a\,\pi\,\mathbf{H}_a = i\,,$$

or

$$\mathbf{H}_a = \frac{1}{2\pi}\cdot\frac{i}{a}\,. \qquad (4)$$

$\mathbf{H}_a$, the intensity of the magnetic field in the neighbourhood of a straight wire, is directly proportional to i, the current in the wire, and inversely proportional to a, the distance of the point of observation from the wire. This relation formerly played a great part in elementary instruction under the name of "Biot and Savart's law".

(3) *Measurements of magnetomotive force in the fields of permanent magnets.* In discussing the subject we have always emphasized the fact that the magnetic fields due to currents in conductors are essentially the same as those of permanent magnets. We can illustrate this once again by means of the magnetic potentiometer. Fig. 11 shows how we can find the magnetomotive force e. g. between the poles of a horse-shoe magnet. The measurement is carried out by removing the magnet suddenly. The magnetomotive force again turns out to be quite independent of the path. If the path is a closed one it is always zero, for, of course, it is quite impossible for the potentiometer to encircle the molecular currents; to do this, it would have to be taken through the middle of single molecules. Any hole bored in a permanent magnet goes *between* the molecules and not *through* them.

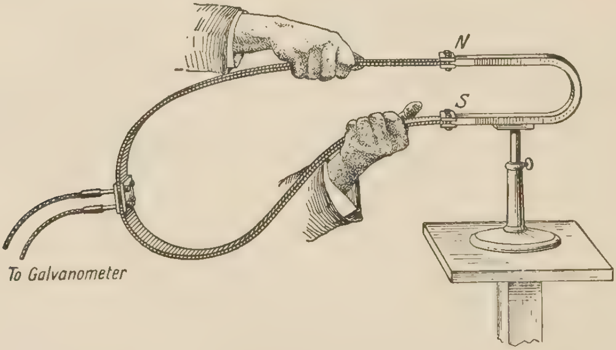


Fig. 11. Magnetic potentiometer in the field of a permanent magnet.

5. A More Thorough-going Interpretation of the Phenomenon of Electromagnetic Induction. Closed Lines of Electric Force. Maxwell's Second Equation.

We have deliberately discussed the phenomenon of electromagnetic induction quite empirically on pp. 101—106. We *observe* electric pressures between the ends of a coil while the magnetic field penetrating the coil is changing. All our previous experiments on electromagnetic induction can be reduced to this, for we know from experiment that all induction phenomena depend on *relative* motions only. For example, it is all the same whether we have a coil fixed in position and move a magnetic field away from it, or whether we move a coil away from a fixed magnetic field. Accordingly, in all our future discussions of induction phenomena we shall take the case of a *fixed secondary coil.*

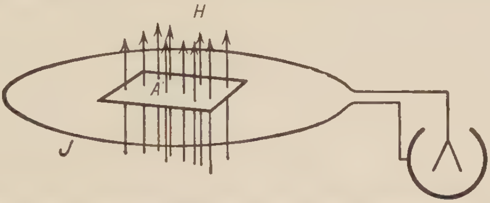


Fig. 12. Diagram of an induction experiment with a secondary coil of only one turn ($n = 1$).

We proceed similarly in mechanics. For example, when we are cycling we observe the motion of the point of one of our shoes with reference to the saddle and say that its path is a *circle.* From another point of view, looking from the street, say, the path appears to be a much more complicated curve, namely a *cycloid.*

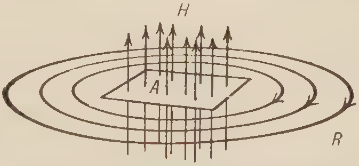


Fig. 13. The deeper significance of the induction phenomenon.

We imagine the induction experiment repeated in a particular case. Let a coil J, consisting of only one turn, encircle the surface A, which is cut at right angles by the lines of force, by any path s (fig. 12). Let the magnetic field change by an amount $\mathbf{H}$ per second, i. e. let $\dot{\mathbf{H}}$ be its "rate of change". Then the induced pressure observed is

$$E_{\text{ind.}} = \mu \dot{\mathbf{H}} A \text{ volts}. \tag{1b}$$

This experimental result will now be interpreted in a more thorough-going way as follows:

The conductor, namely the coil of wire, is of quite subordinate importance; in fact, it is quite irrelevant. The real phenomenon is quite independent of the accidental presence of the coil of wire. It consists *in the occurrence of closed ring-shaped lines of electric force surrounding the magnetic field which is being changed* (fig. 13).

Closed re-entrant lines of electric force are something quite new and unexpected. So far we have only known of electric lines of force with ends at which the atoms of electricity are situated. We now see that electric fields can exist even without atoms of electricity. Such electric fields are called "electromagnetic".

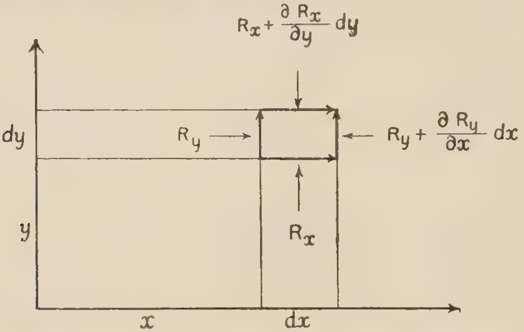

$R_x + \frac{\partial R_x}{\partial y} dy$
dy
R_y
$R_y + \frac{\partial R_y}{\partial x} dx$
y
R_x
x
dx

Fig. 14. Formation of the line-integral of the electric intensity **R** along the boundary of a surface element *dx dy*.

Further, according to this more thorough-going interpretation, the turn of wire is merely an indicator for showing the existence of the electric field. It serves to measure the line-integral of the electric intensity **R** *along its path, i. e. the electric pressure E.* That is, it completely corresponds to the magnetic potentiometer, which measures the line-integral of the magnetic intensity **H** along its path *s*, i. e. the magnetomotive force *M*.

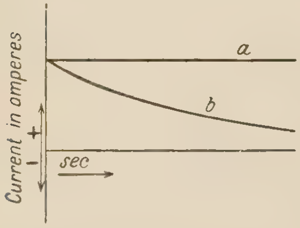


Fig. 15. *a* Constant, *b* decreasing, direct current.

The effect of the coil of wire is easily seen. The coil is situated within the electric field. But the wire is a conductor; hence the electric field in it is not permanent. The atoms of electricity move up to the ends of the conductor, along *both* directions of the line of force, on just the same principle as say in fig. 40 (p. 46) above. Atoms of electricity of opposite sign are heaped up at the ends until there is no field left in the conductor. At the same time these atoms of electricity cause the electric pressure between the ends which is recorded on the voltmeter. This pressure between the ends is equal to the pressure which previously existed along the whole path traversed by the wire. To this extent we can get a general idea of the phenomenon on the basis of the knowledge of electric lines of force which we acquired in Chapter II.

What follows is new, however. In the electric fields which we previously investigated the lines of force have beginnings and ends, and the line-integral of the intensity **R** along a *closed* path is zero. It is equal to the pressure between the beginning and the end of the path, independently of the path, of course. That is, it is equal to zero when the beginning and the end of the path are infinitely close together and in the limiting case absolutely coincident. Here, *in the region of the electromagnetic field* with its endless closed lines of electric force, things are different. *Here the electric pressure has a finite value even along a practically closed path.* It behaves just as the magnetomotive force does along a path completely surrounding an electric current.

In yet another point, moreover, the two fields with closed lines of force exhibit agreement. The magnetomotive force M is multiplied by n when the path of the magnetomotive potentiometer encircles the current n times. The corresponding fact holds for electric fields with closed lines of force. *If the indicator of electric pressure, i. e. the wire, encircles the region of the variable magnetic field n times, the pressure is multiplied n times.* We have already established this experimentally in detail (p. 104).

Hence on this interpretation the induced electric field is the essential feature of the induction phenomenon. The observed pressure is the line-integral of its electric intensity **R**, namely

$$E_{\text{ind.}} = \int \mathbf{R}\, ds\,.$$

Hence equation (1b) takes the form

$$\int \mathbf{R}\, ds = \mu A \dot{\mathbf{H}}\,. \tag{5}$$

This connects the line-integral of **R**, the intensity of the electric field, with the rate of change of **H**, the intensity of the magnetic field. This equation contains the gist of Maxwell's so-called second equation.

The latter is a differential equation, which can be obtained from (5) by taking the line-integral along the boundary of an infinitely small surface element $dx\,dy$. Fig. 14 illustrates this calculation. In this way we obtain

$$\frac{\partial \mathbf{R}_y}{\partial x} - \frac{\partial \mathbf{R}_x}{\partial y} = \mu \dot{\mathbf{H}}_z\,. \tag{5a}$$

6. Direct Current, Alternating Current, and Displacement Current.

Hitherto we have always taken the words "electric current" to mean a "conduction current", i. e. a movement of atoms of electricity through the conductor. *Once and for all we define the direction of motion of the negative atoms of electricity or electrons as the direction of the current**. The current flows out from the negative pole of a source of current and into the positive pole. This convention is a natural one now that we know that in conducting metal wires the motion is practically confined to the electrons and that the positive atoms, on the contrary, do not leave their places (see p. 249).

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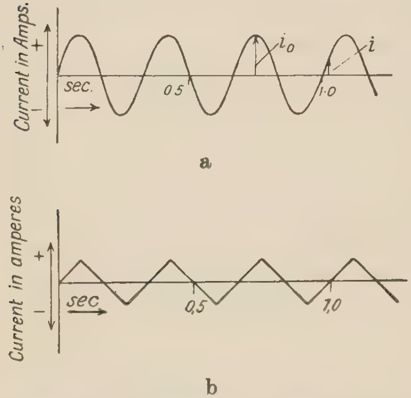
* [N. B. The student should note carefully that in the older textbooks the positive direction of the current is taken as the reverse of this.]

A conduction current which does not alter in direction is called a *direct current* (or a *continuous current*). Such a current deflects the pointer of a moving-coil ammeter in a definite direction, say to the right. In the graph of fig. 15, *a* denotes a direct current which remains constant as time goes on, *b* a direct current which decreases as time goes on.

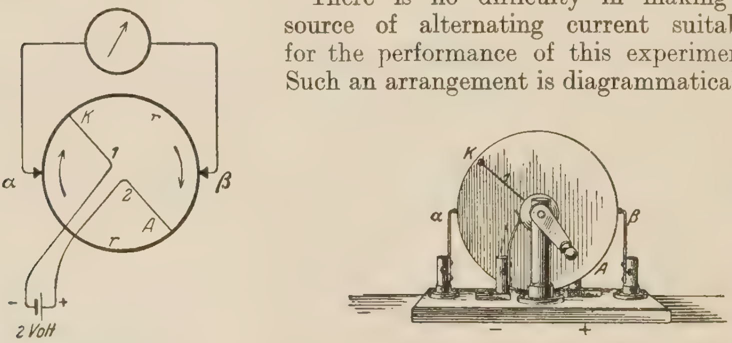


a

b

Figs. 16a, b. Alternating currents, *a* with a sine curve, *b* with a zigzag curve.

In contradistinction to direct current, a conduction current is said to be an *alternating current* if the pointer of the aforesaid moving-coil ammeter oscillates backwards and forwards in the way shown in fig. 16. In the case of fig. 16a we speak of a simple harmonic alternating current; its curve has the simplest wave form, that of the sine curve. An alternating current may also have a curve of any complicated shape; we refer to fig. 16b as an example of this.



a

b

Fig. 17. The production of an alternating current of very low frequency: *a* diagram, *b* practical apparatus. One of the two slip-rings which prevent the direct current leads getting twisted is shown in the latter.

There is no difficulty in making a source of alternating current suitable for the performance of this experiment. Such an arrangement is diagrammatically sketched in fig. 17. *R* is a closed ring-shaped resistance wire on the periphery of a circular disc mounted so as to be free to rotate. The direct current from an accumulator is introduced at two diametrically opposite points A and K. α and β are two fixed sliding contacts from which wires lead to the moving-coil ammeter. If we turn the disc with constant velocity (by means of a crank), the pointer of the ammeter executes simple harmonic oscillations. It passes the equilibrium position when the line joining K and A in fig. 17 stands vertical. The greatest deflections to right and left occur when KA is horizontal. n revolutions per second produce an alternating current of frequency n.

In the language of mathematical symbolism a simple harmonic alternating current is represented by the equation

$$i = i_0 \sin 2\,n\pi t \qquad \text{or} \qquad i = i_0 \sin \omega t.$$

The meanings of i and i_0 are clear from fig. 16a. i is the *instantaneous value*, i_0 the *maximum value* of the current. n and ω have the meaning familiar in mechanics: n is the frequency, i. e. the number of waves, periods, or cycles, in one second; ω is the number of waves or periods in 2π seconds. In fig. 16a $n = 3$, $\omega = 3 . 2\,\pi = 18{\cdot}85$.

The pointer of the moving-coil ammeter possesses mechanical inertia and hence it can only follow slow alternations of current. As the frequency n rises the deflections to right and left become smaller. With the frequencies usual in practice (e. g. $n = 50$ cycles per second) the pointer merely vibrates round about the equilibrium position. Hence it follows that in order to demonstrate and measure alternating currents we must use ammeters whose deflection is independent of the direction of the current. This is the case e. g. with the hot-wire ammeter: the direction of the current does not alter the heating effect. *Hot-wire ammeters are instruments suited to the measurement of alternating currents.*

They give us a certain average value of the current, known as the *effective current*, or *root mean square current*. The effective current is $1/\sqrt{2}$ times the maximum number of amperes (i_0), which is reached twice in each cycle or period (fig. 16a).

Using an alternating current and a hot-wire ammeter, we shall now establish a fact which usually lands the beginner in a peculiar difficulty. He has previously learned that a permanent current flows only in a closed circuit and is interrupted, if the conductor is broken anywhere. He is thinking of an electric light switch or the push of an electric bell.

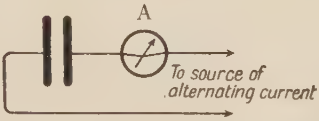


Fig. 18. Displacement current in a practical paper condenser, produced by an alternating current of 50 cycles per second, such as is used in practice.

With alternating current things are immediately changed. We connect up a practical source of alternating current (a generator at 150 volts) an ammeter, and a condenser, as shown in fig. 18. The circuit is certainly interrupted by the space between the plates of the condenser (in case of doubt, do a control experiment with direct current). Yet a permanent alternating current of about 1 ampere now flows freely through the leads and the ammeter.

This experiment often takes people aback at first sight. Yet there is really nothing new involved. The experiment merely exhibits the periodic repetition of an experiment with which we have long been familiar. It shows us once again that when the plates of a condenser are charged or discharged, a current must flow in the leads (see e. g. fig. 16, p. 30). This is conveniently illustrated by the apparatus sketched in fig. 19. By means of the switch S the plates of a condenser may be charged, discharged, charged again with electricity of opposite sign, and so on, as many times as we like. Every time we move the switch to and fro from one contact to another the lamp lights up. If we move the switch very quickly, the separate impulsive currents merge into each other and the lamp flickers. If the frequency of the motions is still higher it shines practically continuously. A conduction current alternating in direction, i. e. an alternating current, flows in the conducting circuit and in the current-indicator (electric lamp or ammeter), while in the condenser the electric field is continually changing.

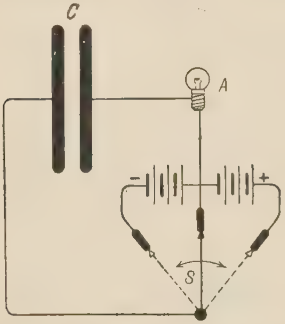


Fig. 19. Switch for alternately charging and discharging a condenser rapidly.

For practical reasons connected with their construction the condensers of figs. 18 and 19 of course always have the plates separated by a solid dielectric instead of by empty space. In theory, however, any condenser *with* a dielectric may be replaced by a similar condenser *without* a dielectric but with a smaller interval between the plates (figs. 56, 57, p. 58).

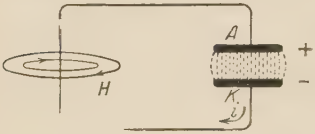


Fig. 20. Conduction current during the discharge of a condenser (cf. fig. 16, p. 30).

Thus the experiment of fig. 18, in which an alternating current flows continuously, reduces to the familiar diagram which is repeated once more in fig. 20. If two condenser plates separated by an empty space are joined by a conducting wire, then a conduction current flows in the conductor whenever the electric field between the condenser plates is changing.

In spite of the simplicity of the matter a new idea, namely that of the *displacement current*, has been coined in order to describe it.

By the term "displacement current" we denote an alteration of the electric field in time, i. e. the appearance and disappearance of lines of force. The term "current" has no doubt been historically adopted from the analogy with water. In the conduction current atoms of electricity *really do move or flow. In the term "displacement current" there remains no trace of the original meaning of the word "current".*

What, then, is the advantage of introducing this idea? The answer is that we can make this statement: "*Completely closed electric currents alone exist in nature*". In conductors they are conduction currents, but in the electric field (of the condenser) they are displacement currents. Electric currents can never have a beginning or an end in space. Where the conduction current stops the displacement current begins, and vice versa.

The beginner will scarcely see the use of this theorem and it will certainly fail to convince him of the value of the idea of displacement current. *A current where there is nothing actually flowing, but merely electric lines of force altering in the course of time* — this is indeed a hard saying, and one that may be positively misleading. We grant this without more ado. We shall see later, however, that the idea of displacement current reveals unsuspected relationships to us and vastly extends our physical conceptions of the world. Here we shall content ourselves with mentioning that the light which reaches our eyes from any source is from the point of view of physics nothing but a displacement current. The objections from the literary and didactic points of view to a current where there is nothing flowing will disappear of themselves later on.

Historically the term "displacement current" originated as follows. In condensers *with* a dielectric we have displacements of atoms of electricity *within* the individual molecules (fig. 59, p. 59), in addition to changes in the field *between* the molecules. The combined effect of these two phenomena was called the "displacement current in the dielectric"; i. e. the name arose from the one component, namely the displacement of the atoms of electricity within the molecules, and thus became attached to what is merely a subsidiary phenomenon. For the *main* phenomenon, namely the displacement current in a vacuum (which is of fundamental importance for our understanding of electrical theory), merely consists of changes in an electric field within which no atoms of electricity exist at all.

Hitherto we have only introduced the displacement current qualitatively as an alteration of the electric field in time. A quantitative statement will now follow. On p. 67 the ideas of displacement Q_D and displacement-density **D** were defined. Q_D is the quantity of electricity which an electric field can induce on a surface perpendicular to the lines of force (fig. 67, p. 66). Hence Q_D is measured in coulombs. **D**, the displacement-density, is equal to $\frac{Q_D}{A}$, i. e. the quantity of electricity induced per unit area (coulombs/cm^2).

The displacement current must be measured in amperes just like any other current. On the other hand, it must be the change in time of one of the quantities which determine the electric field. The latter must accordingly have the "dimensions" of a coulomb. This is the case only for the displacement Q_D. We thus obtain the following relationship. The displacement current i_d is equal to the change of displacement Q_D per second or "rate of change of displacement Q_D". Here, as is usually done, we shall denote the rate of change of a quantity by a dot over the appropriate symbol. That is,

$$i_d = \dot{Q}_D = A\dot{\mathbf{D}}\,, \tag{6}$$

where **D** is the displacement-density over the area A (coulombs/cm²). So much for the measurement of displacement current.

7. The Magnetic Field of the Displacement Current and its Magnetomotive Force. Maxwell's First Equation.

We know that a *conduction* current is invariably accompanied by a magnetic field. A conductor through which atoms of electricity are moving is surrounded by closed lines of magnetic force. This is again shown in fig. 21 by means of the circles **H** for the case of a rectangular circuit, through which the condenser discharges itself, giving rise to the conduction current i.

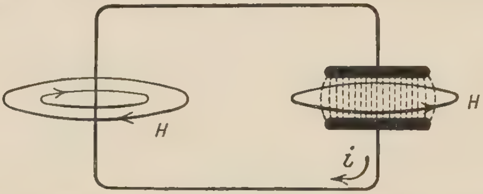


Fig. 21. Diagram of the magnetic fields of the conduction current and the displacement current.

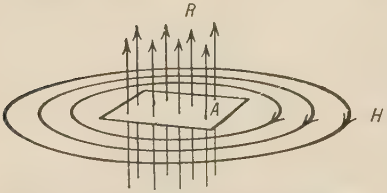


Fig. 22 a. Diagram of the magnetic field of a displacement current.

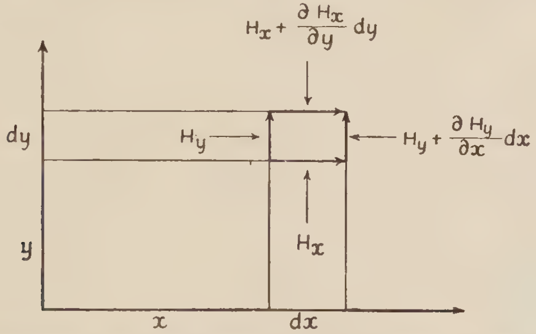
Now Maxwell's great idea was that the invariable association of the current and the magnetic field exists not merely for the conduction current i but also for the displacement current i_d. According to Maxwell, the region of the displacement current must also be surrounded with closed lines of magnetic force. This is shown by a circle on the right-hand side of fig. 21.

Unfortunately the lines of magnetic force of the displacement current in fig. 21 cannot be simply exhibited by means of iron filings like those of a conduction current. This would be very convenient for didactic purposes. In practice, however, it is impossible to produce the necessary number of amperes in the displacement current in electric fields with long lines of force. This is shown by a simple approximate calculation.

Let **R** in fig. 22a be a bundle of vertical lines of electric force. Let A be the cross-section of the bundle. Let the intensity of the field be **R** and its rate of change $\dot{\mathbf{R}}$. By equation (3), p. 67, the corresponding displacement current is

$$i_d = A\dot{\mathbf{D}} = AK\dot{\mathbf{R}} \text{ amperes .} \tag{6a}$$

Here K is the absolute dielectric constant and is of the order of magnitude of 10^{-13}. Hence $\dot{\mathbf{R}}$ in equation (6a) must be of the order 10^{13} if a surface A of 1 sq.cm. is to be penetrated by a displacement current of about 1 ampere. **R**, the intensity of the field, i. e. the pressure along 1 cm. of a line of force, can at most amount to 10^5 volts/cm. in an experiment like that in fig. 22. Otherwise we get pressures between the ends of the lines of force which are too great to control in practice.* Lines of force only 10 cm. long would give a million volts! Hence there remains only the possibility of altering **R** in a very short time, in our example about 10^{-8} second. We shall not come across methods for producing such rapid changes in the field until Chap. VII, so that this is not the place to enter into details of the experiment (see p. 192).



$H_x + \frac{\partial H_x}{\partial y} dy$
dy
H_y
$H_y + \frac{\partial H_y}{\partial x} dx$
y
H_x
x
dx

Fig. 22b. Formation of the line-integral of **H**, the intensity of the magnetic field, along the boundary of a surface element $dx\,dy$.

The carrying out of this experiment, however, would not really prove anything about the production of a magnetic field by the *displacement current*. The magnetic field observed in fig. 21 could always be ascribed to the conduction current which flows in the wire right up to the boundaries of the electric field.

The existence of the magnetic field of a displacement current can only be really proved by using closed ring-shaped lines of electric force. This is not done till Chap. XI, namely by means of the proof of the existence of electric waves progressing freely in space. There is no way of getting round this difficulty in an account of the modern theory of electricity. There is no help for it: we can only introduce the magnetic field of the displacement current as a *hypothesis* in the first instance. We can of course make its existence appear plausible from fig. 21, but we cannot prove it experimentally to begin with.

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* In the practical condensers of figs. 18 and 19 the lines of force are only about 2.10^{-7} mm. long, and besides, the areas are counted in square metres!

The quantitative treatment of the train of thought developed above leads to Maxwell's so-called first equation. The fundamental equation (3) (p. 109) for the magnetomotive force is

$$\int \mathbf{H}\, ds = i\,.$$

For the displacement current we have

$$i_d = KA\dot{\mathbf{R}}\,, \tag{6a}$$

whence it follows that M, the magnetomotive force of the displacement current, is equal to

$$\int \mathbf{H}\, ds = KA\dot{\mathbf{R}}\,. \tag{7}$$

This equation connects M, the line-integral of **H**, the intensity of the magnetic field, with $\dot{\mathbf{R}}$, the rate of change of the intensity of the electric field **R**. It contains the gist of Maxwell's first equation.

The equation itself is again a differential equation. We obtain it from fig. 22b, just as we did with equation (5a) above (p. 115). It is as follows:

$$\frac{\partial \mathbf{H}_y}{\partial x} - \frac{\partial \mathbf{H}_x}{\partial y} = K\,\dot{\mathbf{R}}_z\,. \tag{7a}$$

In this it is assumed that the surface $A = dx\,dy$ in fig. 22b is traversed by a displacement current only. If a conduction current i flows through the surface in addition, i/A must be added to the right-hand side.

8. Concluding Remarks.

The train of thought in this chapter is by far the most difficult in the whole book. We shall recapitulate it here in the form of a comparison*.

The experimental facts from which we start are these:

An induction experiment: A magnetic field perpendicular to the surface A changes at the rate $\dot{\mathbf{H}}$. We observe an electrical pressure between the ends of a wire encircling A (fig. 12, p. 113).	A conduction current i is surrounded by closed lines of magnetic force (fig. 4, p. 5).

Extended significance of this experimental result:

The wire is by no means essential for the induction experiment. The primary phenomenon is the appearance of endless closed lines of electric force **R** which encircle the changing magnetic field in A (fig. 13, p. 113).	The displacement current $i_d = KA\dot{\mathbf{R}}$ also has a magnetic field. $\dot{\mathbf{R}}$ is the rate of change of the electric field penetrating the surface A.

---

* [The analogy is made still clearer if we think of magnetomotive force as magnetic pressure.]

Measurement of the line-integral of the intensity of the field in the neighbourhood of the closed lines of force gives

the electric pressure	the magnetomotive force
$E = \int \mathbf{R}\, ds$.	$M = \int \mathbf{H}\, ds$.
Unit: the volt.	Unit: the ampere-turn.

To make the measurement we use

a wire in which the field breaks down (fig. 12, p. 113).	the long spiral coil of two layers (fig. 8, p. 117, in which a pressure is induced.

Results of the measurements:

$E = \int \mathbf{R}\, ds = \mu A \dot{\mathbf{H}}$,	$M = \int \mathbf{H}\, ds = K A \dot{\mathbf{R}}$,
independently of the path s. The electric pressure E has a finite value for a closed path s if the path encircles a changing magnetic field *once*. If the field is encircled n times, E is multiplied by n.	independently of the path s. The magnetomotive force M also has a finite value for a closed path s if the path encircles the displacement current *once*. If the current is encircled n times, M is multiplied by n.

Maxwell's equations (5) and (7) summarize the "classical" theory of electricity (as contrasted with Planck's quantum theory), which is distinguished by its great simplicity. It was fitting that we should show how these foundations of the theory of electricity are bound up with the results of experimental physics.

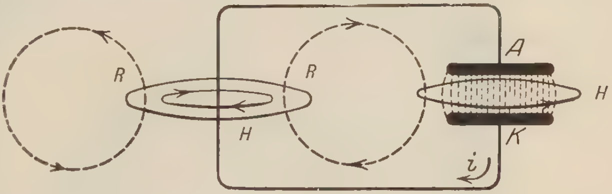

Fig. 23. The linking-together of electric and magnetic lines of force.

We shall not refer to the mathematical formulation in the chapters which follow. The reader, however, must master the accounts of the mutual relationships between the electric field and the magnetic field given in this chapter. Otherwise he will be unable to understand the mechanism of electric waves later on.

Let us refer back to the preliminary remarks at the beginning of this chapter (p. 100). Fig. 1 shows what we knew at that time about the connection between the magnetic field and the electric field: *the conduction current which appears in the conductor* when the electric field changes is surrounded by magnetic lines of force.

Instead of fig. 1 we now have fig. 23, which is quite a rough sketch. Magnetic lines of force are again shown by continuous lines, electric lines of force by dotted lines. This figure expresses two statements more than the earlier one did:

(1) The changing electric field in the condenser is encircled by magnetic lines of force. "The displacement current also has a magnetic field".

(2) The change of the magnetic lines of force in its turn produces closed electric lines of force which encircle the magnetic lines of force: "Electromagnetic induction".

CHAPTER V

Magnetic Flux, Self-induction, Magnetic Energy, Forces in Magnetic Fields

1. $\mathbf{B}$, the Magnetic Induction, and $B = \mathbf{B}A$, the Magnetic Flux. Their Magnification by means of Iron.

The experimental nucleus of Chapter IV is the experiment on electromagnetic induction. The discussion which we gave there was by no means an exhaustive one. Four matters have still to be taken up:

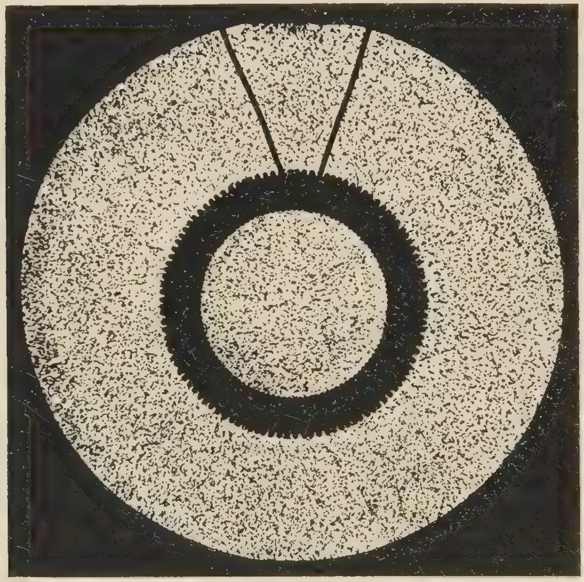

Fig. 1. Coil with iron core, both being ring-shaped and closed.

(1) the definitions of magnetic flux and of magnetic induction;

(2) the use of iron cores in the primary and secondary coils;

(3) a special case of the phenomenon of electromagnetic induction equally important in physics and electrical engineering, namely, the phenomenon known as self-induction;

(4) induction phenomena in conductors, some parts of which are in motion relative to other parts.

These points will be dealt with in this section and the four following. We shall then be in a position to evaluate the energy of the magnetic field in the volt-ampere system of units and calculate the magnitudes of the mechanical forces occurring in the magnetic field in a simple way. This paves the way for Chapter VI, which deals with some applications of magnetic fields, generators and electric motors in particular.

We begin by defining magnetic flux. *We call the product* $\mu \mathbf{H} A$, *which appears in the law of induction, the magnetic flux* B; *that is,*

$$B = \mu \mathbf{H} A \,. \tag{1}$$

The magnetic flux has a simple physical meaning: B *is the maximum integral pressure, measured in volt-seconds, which a magnetic field of intensity* $\mathbf{H}$ *can induce in a turn of a coil of area* A [cf. equation (1a), p. 105, with $n = 1$]. Here it makes no difference whatever whether the magnetic flux originates from conduction currents or displacement currents. It follows from the above that the magnetic flux B has the dimensions of a volt-second.

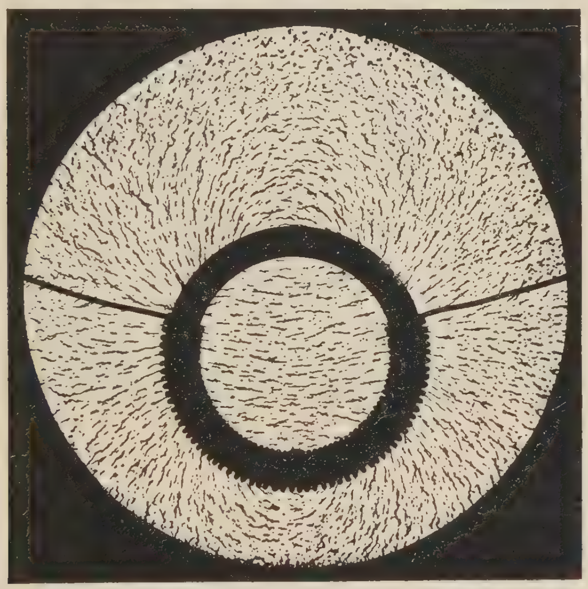

Fig. 2. Coil with iron core, the core being closed and the coil open.

We define the magnetic induction $\mathbf{B}$ as the product $\mu \mathbf{H}$ or B/A, i. e. the magnetic flux per unit area. $\mathbf{B}$ *is the maximum integral pressure, measured in volt-seconds, which a magnetic field of intensity* $\mathbf{H}$ *can induce in a "unit coil"*. The unit coil has one turn of area one square centimetre. $\mathbf{B}$ has the dimensions volt-seconds/cm^2.

In previous experiments we usually produced the magnetic fields we required by means of coils carrying currents; occasionally, however, we used permanent magnets. This is just what is done in practical applications of electricity. In practice, however, a further very effective artifice is used: in the majority of cases iron or steel is placed in the coils through which the currents flow. The purpose of this procedure is immediately obvious; the visible ampere-turns of the coil are to be reinforced by the invisible ones of the ordered molecular currents.

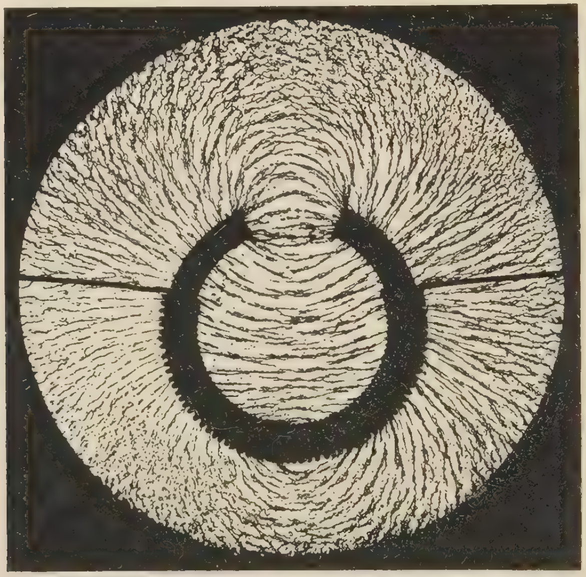

Fig. 3. Coil with iron core, both open.

H must be large in order that the magnetic induction $\mathbf{B} = \mu \mathbf{H}$ may reach a high value ($\mu = 1{\cdot}256 \, . \, 10^{-8}$ volt-sec./amp.-cm.).

The greater the extent to which the visible ampere-turns can arrange the circular molecular currents in parallel planes, the greater is the amount of invisible molecular current. The lines of force of the coil are most effectively employed for the carrying out of this arrangement when all the lines of force of the coil *and* of the molecular currents which have already been arranged run their whole course in iron. We then have the case of the coil shown in fig. 1, which is completely filled with iron. Here every part of every line of force takes its share in the arranging of the molecular currents.

If the winding of the coil is not completely closed (fig. 2), the lines of force begin to stray into the space outside the coil. This causes the extent to which the molecular currents are arranged in order to diminish somewhat; it decreases still further when a gap is made in the iron core (fig. 3) and the stray field increases.

The contribution of the molecular currents to the total ampere-turns determining **H**, the intensity of the field, is very small in the case of a cylindrical coil with a strong stray field. Fig. 4 gives an example of this.

All this can be observed without trouble. We have only to surround the field with *one* turn of wire. This is shown e. g. in fig. 5. The shape of the turn is of no importance, as the area A is of course determined only by the cross-section of the primary coil, just as in fig. 7, p. 105. The ends of the coil lead to the long-period voltmeter V. We interrupt the current in the coil (i). This gives rise to an instantaneous deflection corresponding to a definite number of volt-seconds. These volt-seconds represent the required magnetic flux B [equation (1)].

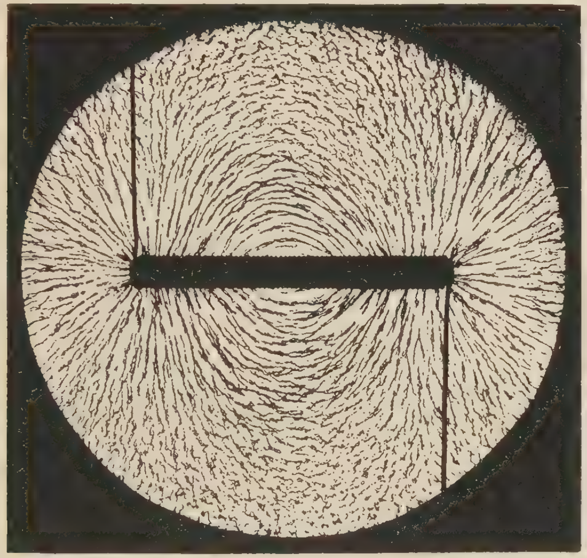

Fig. 4. Straight coil with iron core.

It follows that iron in the field of a coil carrying a current enables us to increase the magnetic flux B, *and hence also the magnetic induction* **B**, *greatly by using the molecular currents. At the same time we reap another advantage: we obtain a new method for altering the magnetic flux and the magnetic induction rapidly and conveniently.* This process is called "altering the iron circuit": it is illustrated in fig. 7.

In fig. 5 there is a considerable stray field and few of the molecular currents are arranged in order. We now bring a movable piece of iron, a "keeper" or "armature", near the coil by hand (fig. 7). The stray field diminishes; when the keeper is put on nearly all the lines of force run through the iron and the molecular currents contribute a considerable amount to the magnetic flux. Hence the latter increases while the keeper is being made to approach, and at the same time the voltmeter records a pressure. If we remove the keeper the reverse takes place: the magnetic flux decreases, and while it is changing the voltmeter again records a pressure.

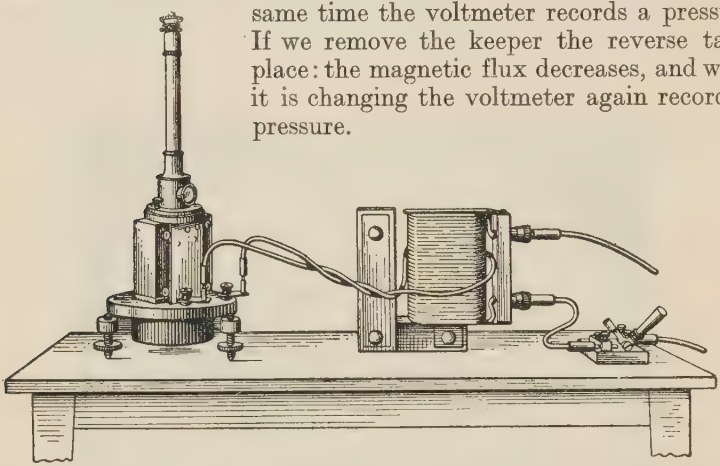

Fig. 5. Measurement of the magnetic flux of a coil with iron core. The galvanometer is again of the moving-coil type of fig. 8, p. 8. The window for admitting light to the mirror attached to the moving coil is seen on the right-hand side of the instrument.

It follows that introducing the iron core merely means increasing the number of ampere-turns. An iron core is nothing but a bundle of countless separate thin coils, whose ampere-turns are added to the visible external ampere-turns. The magnetic induction **B** increases because the field intensity **H** does so owing to the rise in the number of ampere-turns. From our point of view this is equally true for the parts of the field occupied by iron and by air.

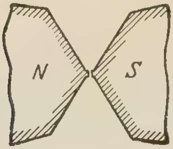

Fig. 6. Pole pieces of an electromagnet for producing large magnetic induction.

Without a quantitative knowledge of the molecular currents we cannot calculate in advance the amount by which the magnetic flux B is increased by the iron. As we see from the explanation of figs. 5, 7 just given, we can of course measure the magnetic flux and magnetic induction by means of a secondary coil, using equation (1). We thus obtain values of the magnetic induction **B** up to about $2 . 10^{-4}$ volt-sec./cm^2, the cross-section of the iron being arbitrary. We can go even further in the case of magnetic fields of very small cross-section. In order to diminish the cross-section of the iron we adopt pole pieces of a peculiar conical shape as shown in fig. 6. In this way we obtain values of the magnetic induction up to 6.10^{-4} volt-sec./cm^2. in the "air-gap", but seldom, unfortunately, in a region greater than one square centimetre in cross-section and a few millimetres in depth.

2. The Magnetic Circuit.

The part played by iron in the production and utilization of magnetic flux is theoretically a simple one (see last section), but the presence of iron brings in many difficulties in the practical construction of apparatus and machinery. In this and the following section the problems which in practice arise from the use of iron will be briefly illustrated. A knowledge of these technical matters is not necessary for the understanding of subsequent chapters.

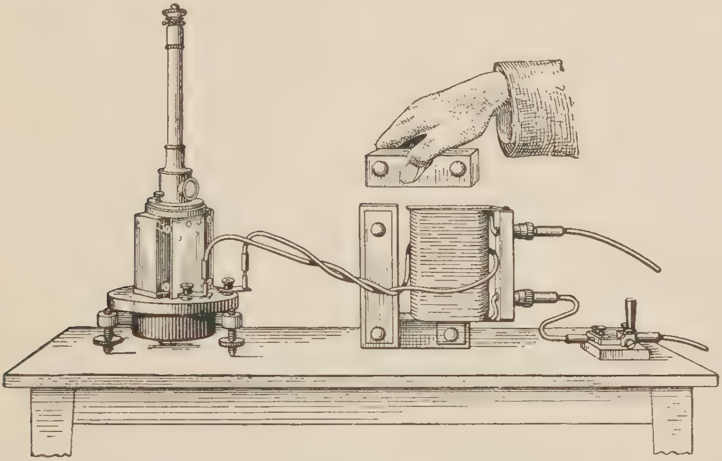

Fig. 7. Changing the magnetic flux of a coil with iron core by altering the iron circuit.

For the construction of machinery in practice numerical values of the magnetic flux are required. The electrical engineer cannot always be going round testing and measuring; nor can he wait till the mechanism of molecular currents is thoroughly understood. What he needs is a convenient method for rapidly calculating the values of B for a combination of ordinary fields due to coils and molecular fields due to iron. The method of the magnetic circuit serves this purpose.

It is as follows: the existence of the molecular currents is disregarded. The field intensity **H** is taken to be the same for the coil filled with iron as for the coil without an iron core. **H** is simply calculated e. g. for the ring coil of fig. 7, p. 105 by dividing the number of ampere-turns by the length of the coil.

Suppose that we observe a magnetic flux say 100 times that of air. This observation is interpreted by equation (1a), p. 105, to mean that iron has a permeability 100 times as great as that of air or most other substances. There is no intrinsic objection to this, but it is pure convention.

Once we have mastered this way of looking at the matter, we can transform the equation

$$B = \mu \mathbf{H} A\,, \tag{1}$$

which serves to define the magnetic flux, for the case of the ring coil as follows:

$$B = \mu \frac{i\,n}{l} A$$

or

$$B = \frac{i\,n}{\left(\dfrac{l}{\mu A}\right)}, \tag{2}$$

where in = number of ampere-turns, l = length of coil. By equation (3a), p. 109, in is equal to the magnetomotive force M. The denominator of the expression is called the *magnetic resistance* or *reluctance*, i. e.

$$R_m = \frac{l}{\mu A}, \tag{2a}$$

in obvious, but purely formal, analogy with Ohm's law (cf. equation 9, p. 236).

Equation (2) is then stated in words as follows: *The magnetic flux is equal to the magnetomotive force divided by the reluctance.*

The essential point is that in fairly uniform fields the reluctance can be found by *adding* the values of $l/\mu A$ for the separate parts of the magnetic circuit. If the "circuit" includes an air-gap of length d, as in fig. 3, say, the total reluctance is given by

$$R_m = \frac{1}{A}\left(\frac{l_{\mathrm{Fe}}}{\mu_{\mathrm{Fe}}} + \frac{d_{\mathrm{air}}}{\mu_{\mathrm{air}}}\right). \tag{2b}$$

For μ_{Fe} we gave the value 100 μ_{air} above, i. e. $\mu'_{\mathrm{Fe}} = 100$. If $d = {}^1/_{100}\, l$, i. e. if the iron circuit is interrupted by air along ${}^1/_{100}$ of its length only, the reluctance is already doubled and the available magnetic flux reduced by half. Hence air-gaps are made as narrow as possible in practice when it is important that the magnetic flux should be high.

Considerations of the magnetic circuit can be extended to magnetic fields of complicated geometrical forms; equation (2b), however, is sufficient for many approximate calculations.

3. The Permeability of Iron and the Hysteresis Loop.

According to pp. 124—128, **B**, the magnetic induction of a coil, is greatly increased by the presence of iron. The same is true for the other ferromagnetic substances, nickel, cobalt, and "Heusler's alloys", which contain manganese. The facts can be interpreted in two different ways:

According to the first interpretation, the value of the permeability remains practically the same for all substances, but the field intensity in the coil is greatly increased by the numerous ampere-turns due to the molecular currents. This interpretation is preferable for qualitative investigations only.

In the second interpretation the existence of molecular currents is disregarded. The field intensity **H** is calculated from the *visible* ampere-turns of the coil, and a permeability of an order of magnitude quite different from that of other substances is ascribed to iron. In the examples of last section iron is taken to have a permeability of about 100, relative to air or a vacuum. It is this second point of view that has been adopted by electrical engineers. They thereby reap the advantage of being able to make more or less accurate calculations about magnetic circuits.

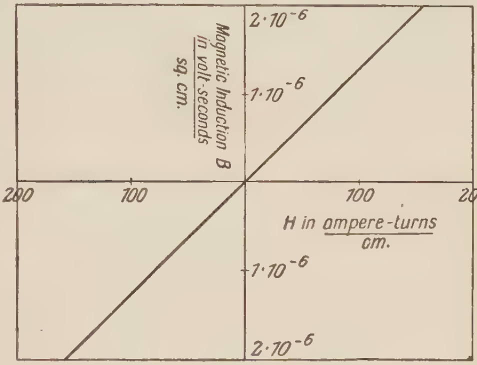

Fig. 8. The connection between the magnetic intensity **H** and the magnetic induction **B** for a paramagnetic or diamagnetic substance.

Unfortunately, however, this by no means overcomes the difficulties brought in by the use of iron in practical applications of magnetic fields. There is another troublesome complication: *the conventionally-defined high permeability is not even approximately constant.* It depends on **H**, the intensity of the field being used, and even on the past history of the iron. This is illustrated by contrasting figs. 8 and 9.

In both figures abscissae denote values of **H**, the field intensity, as calculated e. g. for the ring coil of fig. 7, p. 105, from the strength of the current, the number of turns, and the length of the coil. In both figures ordinates denote the values of **B**, the magnetic induction, i. e. the integral pressure induced in the unit coil (see p. 125), measured in volt-seconds.

Fig. 8 refers to any substance, say wood or brass, whose permeability is practically equal to the universal value $1{\cdot}256 . 10^{-8}$ volt-seconds/ampere-centimetre (cf. Table II, p. 106). The relationship between **B** and **H** is represented by a straight line. **B** is proportional to the field intensity **H** in every case. The equation $\mathbf{B} = \mu \mathbf{H}$ is rigorously true. The straight line is unaltered if **H** is periodically varied by altering the size and direction of the current in the coil, that is, if the field of the coil is produced e. g. by an alternating current.

Fig. 9 is quite different. It holds for iron and shows the results of measurements on two different kinds of iron. Instead of the straight line we have a complicated loop, which is called the *hysteresis loop*. To each value of **H** there corresponds not one value of **B**, but two. Which of them we are concerned with depends on the previous magnetic history of the iron. If we are measuring the successive values of **B** for increasing positive values of **H**, we are concerned with the right-hand branch of the curve. In the molecular picture of the phenomenon this means that more and more of the molecular currents are being placed parallel to each other. When **H** is about 20 ampere-turns per centimetre, **B** reaches a saturation value. All the molecular currents have been arranged parallel to each other. **B** no longer increases to a noticeable extent when **H** is further increased.

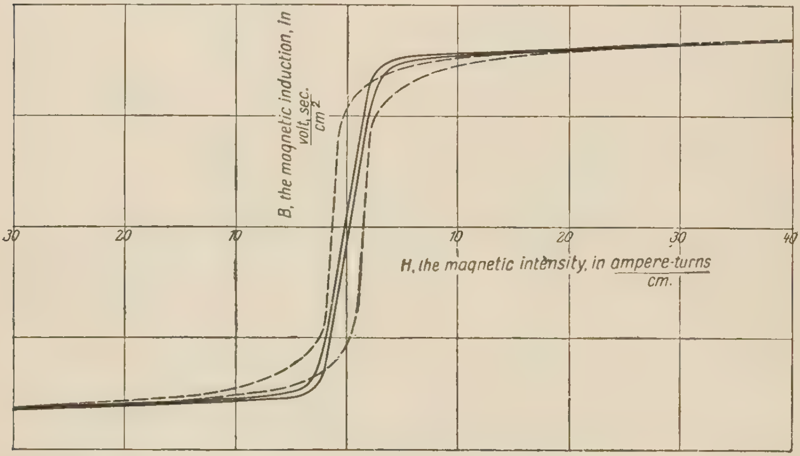

Fig. 9. Hysteresis loops, the continuous curve being for electrolytic iron annealed twelve times, the dotted curve for unannealed dynamo steel.

If we now continue the measurements with decreasing values of **H**, the observed values of **B** lie on the left-hand branch of the curve. **B** retains a finite value even for the field **H** = 0. Some of the molecular currents retain the orientation which they previously received, even when there is no field in the coil. The iron has become a permanent magnet. If we then continue with slowly increasing negative values of the field intensity **H**, we obtain the lower left-hand part of the curve. We follow it until saturation occurs, and so on as many times as we like.

It follows from fig. 9 that it is quite out of the question for iron to have a constant value for the quotient **B**/**H** which in practice is defined to be μ_{Fe}.* From fig. 9 we can merely obtain an average value corresponding to a definite field intensity **H**. Hence in calculations connected with magnetic circuits we have to substitute a different value for μ_{Fe} for every available field intensity **H**. This is not exactly convenient, of course, but after all it is a matter of subordinate importance.

The shape of the hysteresis loop for iron, however, involves a further drawback, which often raises difficult problems for the electrical engineer. The engineer has frequently to work with magnetic fields which are continually altering both in magnitude and direction. This is the case e. g. when the magnetic fields are produced by alternating currents.

Within every complete period the whole curve of fig. 9 is traversed once. Within every period the molecular currents or tops are turned round through 360°. This motion of the molecular currents requires a supply of work to overcome frictional resistances of some kind or other, and this work must be continually supplied by the current in the coil. We observe it through the iron in an alternating magnetic field becoming warm and often even strongly heated.

The amount of this work for every separate period follows directly from fig. 9. For a cubic centimetre of iron it is equal to the area enclosed by the hysteresis loop, this area having the dimensions of the product $\mathbf{B} \cdot \mathbf{H}$. This gives

$$\frac{\text{volt-seconds}}{\text{cm.}^2} \cdot \frac{\text{number of ampere-turns}}{\text{cm.}} = \frac{\text{volt-coulombs}}{\text{cm.}^3},$$

i. e. the number of volt-coulombs per cubic centimetre of iron.

The amount of energy represented by the area enclosed by the hysteresis loop is used up during each complete period. Hence the greater the number of periods per second (i. e. the greater the frequency), the greater are the loss of energy and the disturbing heating effect. There is only one way of combating this disturbance, namely, the use of kinds of iron or alloys of iron that possess a very slender hysteresis loop, say as in fig. 9. In electrotechnics the magnetic properties of the kinds of iron and steel used have to be considered in detail as well as their mechanical properties.

We shall conclude with another theoretical remark. Iron and the other ferromagnetic substances are occasionally said to be substances with highly developed paramagnetic properties. This mode of expression, however, is correct only within limits. In paramagnetic substances the permeability is a quantity which is characteristic of the molecules of the substance in question. In the case of iron, steel, etc., its value on a particular occasion definitely depends on the internal arrangement of the molecules in the crystal structure of the solid body. Even very trifling changes in the crystal lattice or crystal structure, e. g. small expansions due to heating, alter the permeability and the shape of the hysteresis loop considerably. Thus, for example, there are iron-nickel alloys which are immediately attracted by the poles of a bar magnet at room temperature but which fall off when the magnet is dipped into boiling water.

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* The numerical values given in the literature are, almost without exception, relative values of the permeability, taken with reference to the permeability of air as unity (cf. p. 106).

The influence of the crystal and lattice structure on the permeability of ferromagnetic substances has been the subject of very important investigations, which, however, are by no means complete.

4. Self-induction.

Self-induction is the name given to a particular case of electromagnetic induction. A knowledge of this phenomenon is of the very greatest importance for the understanding of the modern theory of electricity.

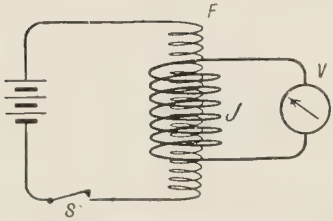


Fig. 10. Diagram of an induction experiment.

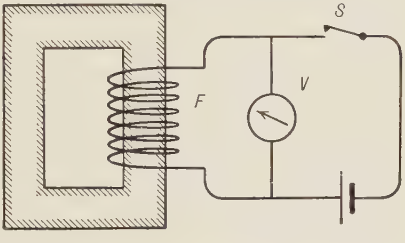
In the course of our account of induction phenomena we performed among others the experiment sketched in fig. 10. A current flows through the coil F and produces a magnetic flux. The changing of the latter, e. g. when the current is interrupted, induces an impulsive pressure, which may be measured in volt-seconds, in the secondary coil J.

Now the magnetic flux penetrates not only through the turns of the secondary coil J, but also through those of the primary coil F. Accordingly, every change in the magnetic flux ought to induce pressures in the turns of the primary coil also. This is actually the case. This induction phenomenon is called self-induction. *That is, in the phenomenon of self-induction the changing magnetic field induces a pressure in its own conductor.* We give two simple experiments to prove the existence of self-induction.

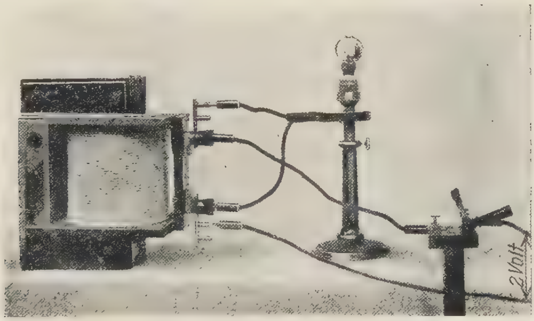
Fig. 11a shows a coil of wire F with say 300 turns, which contains a closed rectangular iron core to increase the magnetic flux. The ends of the coil are connected with an accumulator and a small moving-coil voltmeter V. The voltmeter records the 2 volts due to the accumulator. If we interrupt the current by means of the switch S, the magnetic flux suddenly disappears.

At the same time the voltmeter shows an instantaneous deflection as far as the scale division corresponding to 20 volts. That is, the pressure due to self-induction momentarily reaches a value about ten times as great as the original pressure of the accumulator. If a moving-coil voltmeter of sufficiently low inertia is not available, we use a small six-volt electric lamp instead. As long as the circuit is closed the filament emits only a dull red glow. It flashes up brightly, however, as soon as the self-induction phenomenon is set going by breaking the circuit. The white-hot filament is a visible sign of the energy which was stored up in the magnetic field of the coil.

A slightly different form of the experiment is shown in fig. 11c. The sketch shows the well-known electric bell with "hammer break". *K* is the spring contact which breaks the current at every swing of the clapper. Two wires, with handles which the experimenter holds, are connected to *a* and *b*, the ends of the coil. When the bell is ringing every interruption of the current produces an impulsive pressure, owing to self-induction, which causes a current which can be felt strongly by the muscles. We may remark in passing that in the older literature a current caused by self-induction is called an "extra current". Such superfluous words maintain their ground for an astonishing length of time.



a



b

Fig. 11. Demonstration of the impulsive pressure in the phenomenon of self-induction, a with the voltmeter, b with an electric lamp. (The coil has an iron core as in figs. 7, p. 129, 4, p. 171, and 8, p. 176.)

S, the integral pressure induced in a coil, depends on two quantities: firstly, on the change of the magnetic flux, and secondly, on the shape of the coil.

In the magnetic field of a coil or of any conductor the change of the magnetic flux is determined by $i_1 - i_2$, the difference between the currents at the beginning and the end of the phenomenon. Hence we write

$$\text{Integral pressure } S = L(i_1 - i_2) \text{ volt-seconds}, \qquad (3)$$

and we call the factor of proportionality L the *coefficient of self-induction.** As we already know, the coil plays a double part in connection with self-induction; in the first place it acts as the primary coil which produces the magnetic flux, and in the second place as the secondary coil, in which this magnetic flux induces an integral pressure measurable in volt-seconds. L depends on both functions of the coil.

This is conveniently shown by calculating L, the coefficient of self-induction, for a simple case, namely that of a long coil with a uniform magnetic field.

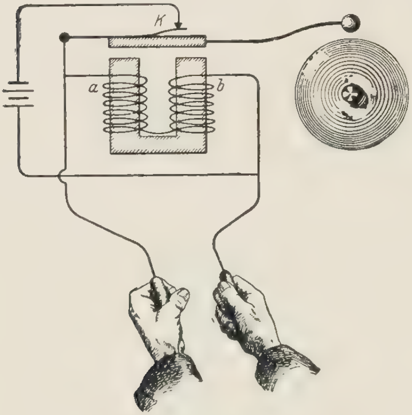


Fig. 11c. Demonstration of self-induction with an electric bell.

Let A be the cross-section of the coil, l its length, and n the total number of turns in it.

We first consider the coil as a primary coil. The current i_1 produces the magnetic field

$$\mathbf{H}_1 = \frac{n\, i_1}{l} \qquad \text{[equation (1), p. 92]}$$

in it. By equation (1), this field has the magnetic flux

$$B_1 = \mu\, \mathbf{H}_1 A = \frac{\mu\, n\, i_1 A}{l}\,.$$

If i_1 alters by the amount $(i_1 - i_2)$, the change in the magnetic flux is

$$B_1 - B_2 = \frac{\mu\, n A}{l}\,(i_1 - i_2)\,.$$

We now consider the coil as a secondary coil. All its n turns are penetrated by the change of magnetic flux $(B_1 - B_2)$. Hence in its n turns there is induced an integral pressure S equal to

$$n\,(B_1 - B_2) = \frac{\mu\, n^2 A}{l}\,(i_1 - i_2) \text{ volt-seconds}\,.$$

A comparison with (3) gives us the required coefficient of self-induction of the coil, namely

$$L = \frac{\mu\, n^2 A}{l}\,. \tag{4}$$

According to equation (3), L has a simple physical meaning: L, *the coefficient of self-induction, is the integral pressure, measured in volt-seconds, which is induced in the conductor when the current changes by one ampere.*

It follows that L is measured in volt-seconds/amperes. *Instead of volt-seconds/ampere we often use the abbreviation "henry".* For example, the coil with closed iron core which we used in fig. 11a had a self-induction of about $^1/_3$ henry.

We shall conclude this section as we began it, namely by emphasizing the extreme importance of self-induction. The reader must not let himself be misled by the unpretending nature of the experiment.

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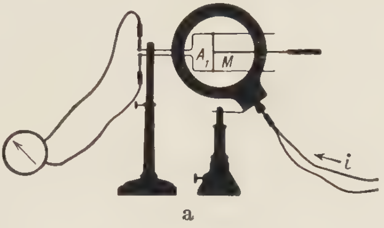
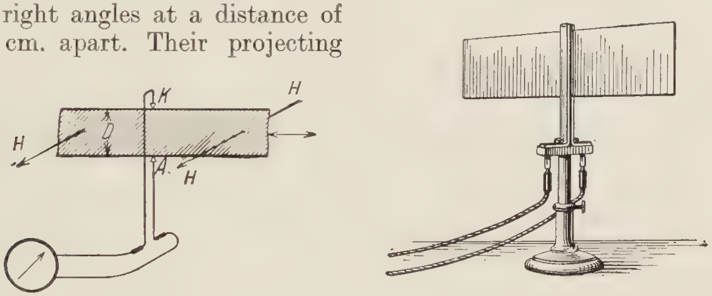
* [The term *inductance* is often used.]

5. Induction Phenomena in Conductors, Some Parts of which are in Motion relative to the Other Parts.

In all the induction phenomena considered so far the secondary coil was rigid, i. e. its shape did not alter while the phenomenon was taking place. In contradistinction to this we shall now meet with an induction phenomenon in which the change of magnetic flux is caused by a change in the shape of the secondary coil only. In fig. 12a we are looking towards the magnetic field of a long coil (with $\mathbf{H} = 50$ ampere-turns/cm., for example) in a direction parallel to the lines of force. The coil consists of two halves separated by a narrow slit. To the left of the circular field of view we see two metal wires bent at right angles at a distance of D cm. apart. Their projecting ends are connected with a galvanometer graduated in volt-seconds. M is a piece of metal which forms the right-hand boundary of the rectangular "area of the coil", A_1, and which slides on the two horizontal wires. We move this slider through an arbitrary distance x by means of a handle, thereby altering the area of the rectangle by $A_1 - A_2 = xD$. At the same time we observe an integral pressure, the magnitude of which turns out to be $\mu \mathbf{H}(A_1 - A_2)$, i. e. in agreement with equation (1), p. 105.



a



b c

Fig. 12. To illustrate induction in conductors, some parts of which are in motion relative to the others. The primary coil in fig. 12a has 10 turns per centimetre, i. e. $n/l = 10$. Fig. 12c shows a practical form of the apparatus shown diagrammatically in fig. 12b.

By systematically varying the conditions of experiment we obtain a surprising result: it is not the *change of area* $A_1 - A_2$ that is essential for the occurrence of the induction phenomenon, but only the motion of the slider, i. e. *the motion of a part of the rectangular secondary coil relative to the other parts*. This is shown by means of the apparatus sketched in fig. 12b.

Here the secondary coil takes the form of a narrow loop of wire which is interrupted between K and A by a strip of metal of breadth D. The strip of metal can be moved in the directions of the double arrow without breaking metallic contact with the loop of wire at K and A.

The loop of wire is placed in the uniform magnetic field of a long coil, as nearly parallel to the lines of force as possible. None of the lines of magnetic force may pass through the area enclosed by the loop. (The experimenter should make sure that no impulsive pressure is induced when the current in the primary coil is made or broken.) The strip of metal projects sideways from a slit in the primary coil. Now for the actual experiment. We displace the strip of metal through the distance x with any velocity we please. This makes an area of metal $A = xD$ square centimetres pass the line joining K and A. At the same time we observe an integral pressure of $\mu \mathbf{H} A$ volt-seconds.

This time the numerical agreement with equation (1), p. 105, is a purely formal one. For although A still means a surface penetrated at right angles by the lines of magnetic force, it is no longer enclosed by a wire.

Another point is, however, of much greater importance. Previously we were in a position (thanks to the relativity of the motions causing induction) to consider all induction phenomena from the point of view of a fixed secondary coil and, following Maxwell, to reduce them to the diagram which we explained on p. 113: a magnetic field altering with time is surrounded by closed lines of electric force. But this point of view is inadmissible when some parts of the coil or loop are in motion relative to others. The argument developed on p. 113 is no longer applicable. This we must candidly admit. The mechanism of these induction phenomena is a different one, involving closer acquaintance with the mechanism of conduction; we shall discuss it on p. 260. Here we shall provisionally content ourselves with the facts assured by experiment.

It is easy to replace the limited motion of the strip of metal in fig. 12b by a continuous motion. We have only to join the strip outside the magnetic field, like the belt of a band saw, and lead it over pulleys. Then if the band advances uniformly (with velocity $\dot{x}$) we observe a constant pressure $E = \mu \mathbf{H} D\dot{x}$ volts (shown by the permanent deflection of the galvanometer).

Numerous variants of the experiment may be carried out in a similar way. For example, one variant is derived from fig. 59a, p. 247, by replacing the source of current shown in the figure by a galvanometer and turning the disc in the direction opposite to that of the bent arrow. The old names of "rotational induction" and "unipolar induction" have arisen from this arrangement and similar ones. These names, however, do not express the essential fact, namely that some parts of the secondary coil are in motion relative to other parts.

6. The Energy of a Magnetic Field.

When we previously calculated the energy of the electric field, we started with the uniform field of a plate condenser: the electric field disappears while work is being done, the two plates of the condenser attract each other and thereby raise a load.

We shall calculate the energy of a magnetic field in a similar way, using the uniform field of a long coil: the magnetic field spontaneously disappears when work is being done, the turns of the coil attract each other and thereby raise a load.

This experiment on the mutual attraction of the turns of the coil (a fact we have not previously mentioned) is a variant of that shown in fig. 9, p. 8. The experiment is arranged as shown in fig. 13, namely with the turns of the coil vibrating vertically up and down, because when the coil contracts the current is then automatically interrupted at *a* (mercury break).

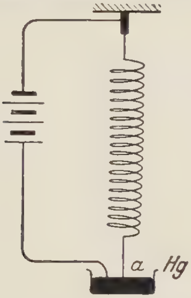


Fig. 13. Contraction of a coil carrying a current. (In this case the energy of the magnetic field *increases* at the expense of the source of current while work is being done; cf. p. 79.)

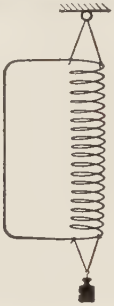


Fig. 14. Illustrating the calculation of the magnetic energy in the field of a coil. (In this case the energy of the magnetic field *decreases* while work is being done.)

The remarkably high quality of the insulating materials available in practice enables the electric field of a condenser to be maintained without the plates being permanently connected with a source of current. If we let the plates of such a condenser attract each other till they touch, the energy of the condenser is completely transformed into work. In this way we previously obtained

$$W_e = \frac{1}{2} C E^2 \qquad \text{(equation 13a, p. 75)}$$

for the energy of the electric field in the condenser, or in general

$$W_e = \frac{K}{2} \mathbf{R}^2 \cdot V \qquad \text{(equation 14, p. 76)}$$

for the energy of a uniform field **R** of volume V.

Unfortunately the quality of even the best conducting materials available in practice does not enable the magnetic field of a coil to be maintained unless the ends of the coil are permanently connected with a source of current. The current in the conducting wire of the coil consists of a motion of atoms of electricity (cf. Chapter VIII).

This motion is continually retarded by resistances of a frictional nature, as is shown by the heating which inevitably takes place in all conductors traversed by a current. As a result of these continual losses of energy a current very soon comes to rest even in a closed conducting path.

Ignoring this practical defect in our conducting material, we proceed to more abstract considerations. *Let fig.* 14 *represent a coil in which a current i, which has been produced* (say by induction) *once and for all, circulates indefinitely without loss of energy.* Let the whole magnetic energy of the coil be transformed into work done in raising a weight. When the coil raises the weight it is to contract into a vanishingly small ring with no current. We can immediately calculate the magnetic energy of the coil from this imaginary experiment.

During the contraction the phenomenon of self-induction appears in the coil. This produces an electric (electromagnetic) field which by Lenz's law acts as a *brake* on the current of atoms of electricity, until i becomes equal to zero. It is not difficult to find the work done in this retardation of the induced electric field: the average value of the current which falls from its initial value i to zero is $i/2$. By equation (3), self-induction gives Li volt-seconds.

Li is, of course, the time-integral of the pressure (measured in volt-seconds) which can be produced when the magnetic field of the current i disappears completely and its own turns act as the secondary. The way in which the field is made to disappear is of no importance in this connection.

The product $\frac{i}{2} \cdot L\,i = \frac{1}{2} L\,i^2$ watt-seconds accordingly gives us the work which is required to retard the quantity of electricity $\frac{i}{2}t$ coulombs passing through the pressure of the induced electric field, $E = \frac{L\,i}{t}$ volts.

$$W_m = \frac{1}{2} L\,i^2 \tag{5}$$

is the total magnetic energy of the coil in fig. 14 which we can obtain in the form of work done in raising a weight when the field disappears. Note the formal agreement between this relationship and equation (13a) (p. 75 and above).

A mechanical analogy will make it easier to understand this. Let a flywheel have moment of inertia Θ and rotate with angular velocity ω. Its kinetic energy is $\frac{1}{2}\Theta\omega^2$. In order to measure or utilize it, a string with a weight is attached to the shaft. The flywheel winds the string round the shaft and thereby brakes itself, and we obtain the work done in raising the weight.

L, the coefficient of self-induction of the long coil, is known from equation (4). We have

$$L = \mu\frac{n^2 A}{l},$$

where A is the cross-section of the coil. Further, by equation (1), p. 92, the intensity of the magnetic field is

$$\mathbf{H} = \frac{n\,i}{l}.$$

Hence it follows that

$$W_m = \frac{\mu}{2}\,\mathbf{H}^2\,V \text{ volt-coulombs} \tag{6}$$

is *the energy of a uniform magnetic field of intensity* $\mathbf{H}$ *of volume* V. Note once more the formal agreement between this relationship and equation (14) (p. 76 and above).

Electric and magnetic energy overlap without affecting each other. Hence the energy of both fields can be summed up by the term "electromagnetic" energy: the latter is given by

$$W_{em} = \frac{1}{2}\,(K\,\mathbf{R}^2 + \mu\,\mathbf{H}^2)\,V. \tag{7}$$

In lax verbal usage, however, we usually say "electric energy" instead of "electromagnetic energy".

In coils traversed by currents, a field intensity $\mathbf{H}$ of about 500 ampere-turns per cm., i. e. a magnetic induction $\mathbf{B}$ of about 6.10^{-6} volt-seconds/cm.^{2}, can easily be maintained permanently. For such a value of the magnetic flux, energy amounting to about 1·6 watt-seconds is contained in a litre ($V = 1000$ c.c.), by equation (6). This is no more than what is normally stored up in electric fields (cf. p. 76). But the magnetic induction $\mathbf{B}$ can be greatly increased by using iron: we can obtain values of $\mathbf{B}$ of about $1{\cdot}5.10^{-4}$ volt-seconds/cm.^{2} (fig. 9, p. 9). A litre then contains about 1 kilowatt-second, by equation (6) $\left(\text{put } \mathbf{H} = \frac{\mathbf{B}}{\mu}\right)$.

The magnitudes of the forces which can be produced in the magnetic field depend on the amount of the energy present in the field. Hence the application of these forces in the construction of modern apparatus and machinery also depends on the energy of the field.

7. Forces in Various Magnetic Fields.

1. *The attraction between two magnetic poles.* The experiment represented diagrammatically in fig. 14 is not a purely imaginary experiment which it is quite impossible to carry out. We shall see on p. 253 that coils of wire, e. g. of lead, can be produced in which a current which has once been introduced will circulate practically indefinitely, i. e. for days on end, at a temperature of about — 270° C. So far, at least, we have not been able to make these resistanceless superconductors for ordinary temperatures. Whether we shall ever be able to do this the future alone can tell. But we do know of currents which circulate indefinitely even at ordinary temperatures without

a source of current, namely, the molecular currents in permanent magnets.

If N and S in fig. 15 are the poles of two bar magnets, they attract each other and do work, in raising a weight, say. This work can only arise at the expense of the energy of the molecular currents. The energy of the magnetic field diminishes owing to work being supplied, as in the train of thought used in connection with fig. 14.

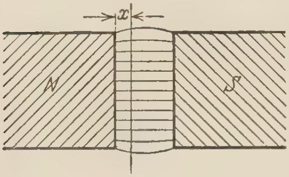


Fig. 15. Two magnetic poles a short distance apart.

If the distance between the poles is sufficiently small and their area sufficiently large, the field between them is uniform. A disc-shaped volume of thickness x contains magnetic energy

$$W_m = \frac{\mu}{2}\mathbf{H}^2 A\,x\,,$$

by equation (6). This energy is transformed into work when the distances between the poles is diminished by the small amount x. We then have

$$\text{Work} = \text{Force } F \cdot \text{Length of path } x = \frac{\mu}{2}\mathbf{H}^2 A\,x\,,$$

or

$$F = \frac{\mu}{2}\mathbf{H}^2 A = \frac{1}{2}\frac{\mathbf{B}^2}{\mu}A\,. \tag{8}$$

This, then, is the force with which two magnetic poles, between which there is a uniform field, attract each other.

Here, as always in the volt-ampere system of units, the force appears as a multiple of the nameless unit 10·2 kilograms weight. If we want the result in kilograms weight, the right-hand side of equation (8) must be multiplied by 10·2. If the force is to be in dynes, the right-hand side must be multiplied by 10^7, and so on (cf. p. 73).

2. *The electromagnet and its keeper.* In engineering and laboratory practice the poles of *electromagnets* are extremely often used instead of the poles of permanent magnets. Electromagnets are coils in which a current is maintained by some source. The coils almost invariably contain iron cores, in order to increase the visible external ampere-turns by the ampere-turns of the invisible molecular currents. Equation (8) holds for electromagnets as it stands. We have already emphasized that there is no difference between the fields due to permanent magnets and those due to coils traversed by a current.

A numerical example may be useful. The iron core of the coil together with the keeper should form as complete an iron circuit as possible. This is the case e. g. in the iron-clad electromagnet drawn on half scale in fig. 16. Its cross-section in iron is only 9 sq.cm. When the iron circuit is good, it is not difficult to obtain a magnetic induction $\mathbf{B} = 1{\cdot}5\,.\,10^{-4}$ volt-seconds/cm.^{2} (fig. 9, p. 132). According to p. 105, μ practically universally has the value $1{\cdot}256\,.\,10^{-8}$ volt-seconds per ampere-centimetre. Hence this little iron-clad electromagnet will bear a weight of about 80 kilograms, by equation (8); that is, it will easily carry an adult.

3. *The force on a wire perpendicular to the magnetic field and carrying a current.* We recall the experiment illustrated in fig. 7, p. 7. A conductor traversed by a current is deflected in a direction perpendicular to its length and to the magnetic lines of force. This is the characteristic feature of the phenomenon. The question of sign is settled by the diagrammatical sketch of fig. 17.

The corresponding lines of force diagram is shown in fig. 18. It is produced by the overlapping of the ring-shaped lines of force of the straight conductor (fig. 4, p. 5) and the straight lines of force of the uniform field. The conductor traversed by the current is forced out from the region where the lines of force are close together (this is opposite to the Magnus effect in aerodynamics).

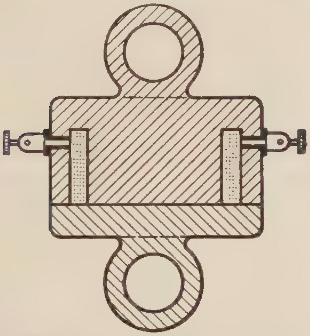
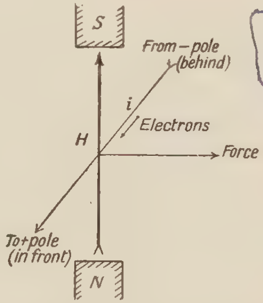


Fig. 16. Electromagnet with good iron circuit, for lifting weights.



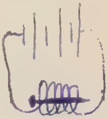


Fig. 17. The force on a straight conductor carrying a current in a magnetic field.

The reader will do well to get a thoroughly clear idea of this phenomenon. This will make it easier for him to understand electric motors later on.

Let the magnetic field in fig. 19 be perpendicular to the plane of the paper. Let 1 be a long bent piece of wire open on the right-hand side and let 2 be a straight wire (called the "slider" for short) which can be slid along it without friction. Let r, the resistance of the whole rectangular wire circuit, be very small. Let a source of current of pressure E be included in the rectangle. As long as the slider is at rest the "short-circuit current" $i = E/r$ flows through it. The magnetic energy of the latter is $\frac{1}{2} L i^2$, if L denotes the coefficient of self-induction of the rectangle. If the slider is free to move, it proceeds with increasing acceleration in the direction of the arrow. The area of the rectangle increases and hence, by equation (1), p. 105, a pressure $E_{\text{ind.}}$ is induced in it. By Lenz's law this must be in such a direction that it acts as a brake on the cause of the motion of the slider, i. e. on the motion of the electrons. The higher the velocity of the slider rises, the nearer $E_{\text{ind.}}$ approaches E in value and the smaller i becomes. If in the final limiting case $E_{\text{ind.}} = E$, i becomes equal to zero. The slider is no longer accelerated but moves on with a constant final velocity u. The whole of the magnetic energy, $\frac{1}{2} L i^2$, is transformed into $\frac{1}{2} m u^2$, the *kinetic energy* of the slider, and the current in the slider henceforth remains zero.

It is otherwise if the slider *does work*, work against friction in the most primitive case. Then the velocity of the slider does not rise quite high enough for $E_{\text{ind.}}$, the pressure induced in the slider, to reach E, the pressure of the source of current. Hence a current flows all the time, but it is much smaller than the short-circuit current.

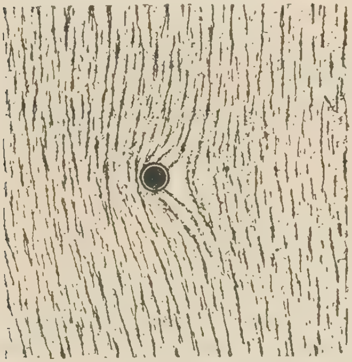


Fig. 18. A straight conductor traversed by a current and perpendicular to the plane of the paper, in a uniform magnetic field. The lines of force are shown by iron filings.

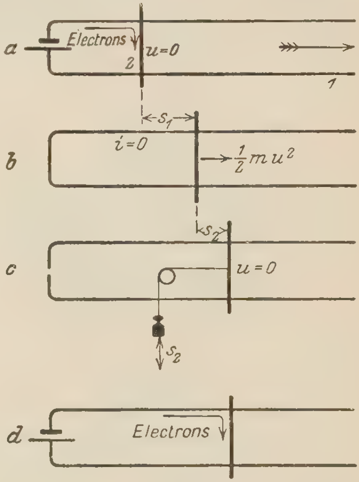


Fig. 19a—d. To illustrate the work done by a straight conductor 2 (the slider) in a uniform magnetic field perpendicular to the plane of the paper.

It is not difficult to understand why a current should continue to flow while the slider is doing work. It is best to think of the phenomenon as split up into three successive stages as follows:

(i) The source of current of pressure E is connected with the slider at rest and electrons amounting to q coulombs are accelerated (fig. 19a). This causes magnetic energy $W_m = qE$ volt-coulombs to appear in the slider.

(ii) The source of current is removed and the slider is accelerated along the path s_1 in the external magnetic field **H** (fig. 19b). The pressure which is thereby induced acts as a brake on the electrons until the current in the slider becomes equal to zero. Then when the path s_1 has been traversed, W_m, the total magnetic energy of the slider, has been transformed into the kinetic energy of the slider (fig. 19c).

(iii) The slider circuit is broken (fig. 19c) and the whole kinetic energy of the slider in the path s_2 is transformed into work, say in raising a weight (fig. 19c). (This is apart from heat due to friction of all kinds, i. e. including the heating effect of the current.)

When the slider comes to rest after having traversed the path s_2, the source of current is again switched on (fig. 19d) and the process begins again. If it takes place n times a second, nq is equal to i, the current in the slider when it is moving continuously. Ei is the electromagnetic energy which the loaded slider transforms into work (and heat due to friction of all kinds, i. e. including the heating effect of the current) per second.

So much for the mechanism of the phenomenon. We have still to calculate the force exerted on a straight wire of length l and carrying a current i when it is placed perpendicular to a magnetic field of intensity $\mathbf{H}$.

We again use energy considerations:

If the slider is displaced through a distance x in one second, the pressure $E = \mu \mathbf{H} x l$ volts per second is thereby induced in it, by equation (1), p. 105.

This pressure E acts as a brake on the electricity, which is moving at the rate of i amperes. This requires that the source of current should supply energy (available as work) amounting to $i\mu \mathbf{H} x l$ volt-coulombs. If we divide this energy by x, the length of path, we obtain the force which accelerates the conductor along the path x. Hence

$$F = i\mu \mathbf{H} l = i \mathbf{B} l \tag{9}$$

is the force exerted on a straight wire of length l in the magnetic field if it is placed perpendicular to the lines of force and is traversed by a current i. As regards units, the reader should refer to what we have said in paragraph (1).

(4) *The couple exerted by a magnetic field on a coil perpendicular to it and traversed by a current.* We recall the experiment illustrated in fig. 8, p. 8. The coil turns so that its surface is penetrated at right angles by the lines of force of the magnetic field. The question of sign is settled by the diagrammatical sketch of fig. 20a.

The corresponding lines of force diagram is shown in fig. 20b. It is produced by the overlapping of the lines of force of a turn of the coil (fig. 1, p. 86) and the straight lines of force of the uniform magnetic field which is used.

The moment of the couple amounts to

$$C = 10{\cdot}2 . \mathbf{B} i n A \text{ kilograms weight} \cdot \text{centimetres} \tag{7}$$

where A is the area and n the number of turns of the coil. [As regards the numerical factor see paragraph (1) of this section.]

This follows at once from equation (9). For each of the n turns we have twice the force $\mathbf{B}il$ multiplied by $2d$, the arm of the couple. The product $2dl$ gives A, the area of the coil.

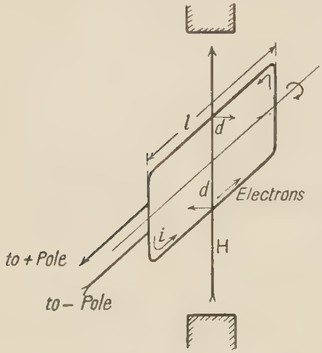


Fig. 20a. The couple exerted on a coil traversed by a current in a uniform magnetic field.

Instead of equation (7) we often write

$$C = \mathbf{B}\mathbf{M}\,, \tag{7a}$$

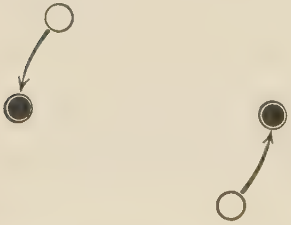
defining inA to be $\mathbf{M}$, *the magnetic moment* of the coil. Similarly, we may speak of the magnetic moment of a permanent bar magnet. By this we mean the product inA for a coil which can be used to replace the bar magnet (p. 91), i. e. the sum of the magnetic moments of the oriented molecular currents.

(5) *The motion of paramagnetic and diamagnetic bodies in the magnetic field.* μ, the permeability of a vacuum (which is practically that of air also) is $1{\cdot}256\,.\,10^{-8}$ volt-seconds/ampere-centimetres, as we saw on p. 105.

For paramagnetic bodies μ is somewhat greater than the above value, and for diamagnetic bodies somewhat smaller (cf. Table II, p. 106).

We establish the following facts by experiment:

(a) In magnetic fields between conical pole-pieces *paramagnetic bodies* are attracted into the region of *greatest* field intensity, *diamagnetic bodies* into the region of *least* field intensity.



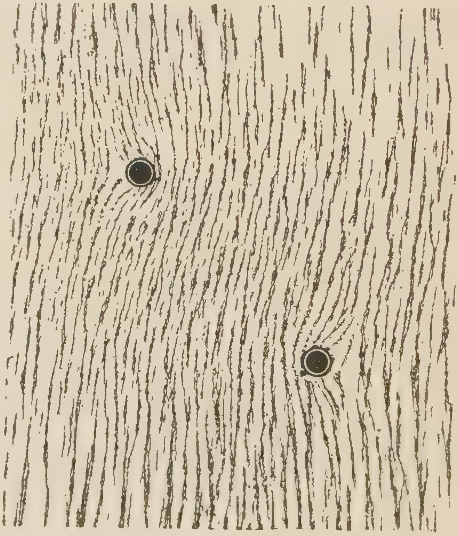


Fig. 20b. A coil perpendicular to the plane of the paper and traversed by a current in a uniform magnetic field.

(b) In uniform fields elongated paramagnetic bodies set themselves *along* the lines of force, elongated diamagnetic bodies *across* the lines of force.

The phenomena in paramagnetic bodies are perfectly analogous to those which we have already discussed in the case of insulators in the electric field. Diamagnetic phenomena, however, can also be imitated in the electric field, by bringing bodies with small dielectric constants into a condenser containing a dielectric with a high dielectric constant.

It is easy to explain case (a). The magnetic field is maintained by a source of current. Hence the motions take place in such a way that the energy of the field *increases* at the expense of the source of current (cf. p. 139). By equation (6), energy amounting to

$$W_m = \frac{\mu}{2} \mathbf{H}^2 \cdot V$$

is found in the volume element V of the field intensity $\mathbf{H}$. The energy of the whole magnetic field is increased by the greatest amount when a volume element of maximum intensity $\mathbf{H}$ is occupied by a paramagnetic body. On the other hand, a diamagnetic body is forced out of the field or at least moved to a region where the intensity is as small as possible; for the region freed from diamagnetic material thereby has its energy increased.

Case (b) comes to the same thing at bottom. In calculating the total energy of the field, however, the distortion of the field must be taken into account. A paramagnetic body, for example, alters the direction of the field in just the same way as is shown for the "strongly paramagnetic" (or "ferromagnetic") substance iron in figs. 15 and 16, pp. 97, 98.

CHAPTER IV

Electrical Generators and Motors

1. Generators.

The utilization of magnetic flux forms the greater part of modern electrical engineering. In this section and those following we shall only consider a few examples, and those only in broad outline.

The modern sewing-machine has two characteristic features: the needle with its eye at the point and the simultaneous use of two separate threads. In the very same way the essential features of a piece of electrical apparatus or electrical machinery may be depicted in a few bold strokes. The physical principles involved and the distinctive artifices which enable us to make use of them are always of a simple nature. The tremendous services rendered to the community by electrical engineering belong to the sphere of practical affairs, not to that of physics. A beginner would never suspect how much ingenuity, industry, and perseverance must be expended in order to work out practical applications of physical ideas, even when the latter are perfectly easy to understand.

We shall begin with generators, which are our most important sources of current, and which transform mechanical work into electrical energy*. In all generators the magnetic flux penetrating a coil is altered by periodic motions. The pressures thereby induced in the coil are used to produce currents. The magnetic flux in the coil is altered by the periodic motions in two ways which are already familiar to us:

(1) by rotating or displacing the coil relatively to the magnetic field (fig. 2, p. 100);

(2) by altering the iron circuit (fig. 7, p. 129).

As a rule both methods are employed simultaneously. We shall illustrate this in connection with some practical types of generators.

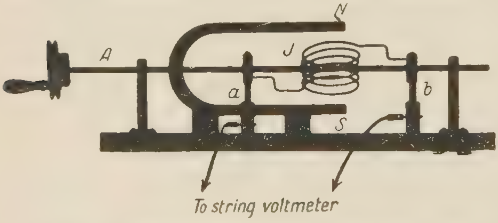


Fig. 1. Alternating current generator with external poles.

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* [The first generator working on the electromagnetic principle was made by Michael Faraday in 1831.]

(a) *The alternating current generator with external poles* (fig. 1). A coil J is rotated about the axis A in a magnetic field produced in any way we like. The ends of the coil lead to two slip-rings on which two contacts a and b slide. The supports of these contacts form the terminals of the machine, to which the leads are connected. There is an alternating pressure between the terminals. *The rotation of the coil J is merely a periodic repetition of our simple induction experiment* (pp. 101—103).

We connect the two poles of the machine across a voltmeter and follow the variations of pressure as we rotate the coil slowly.

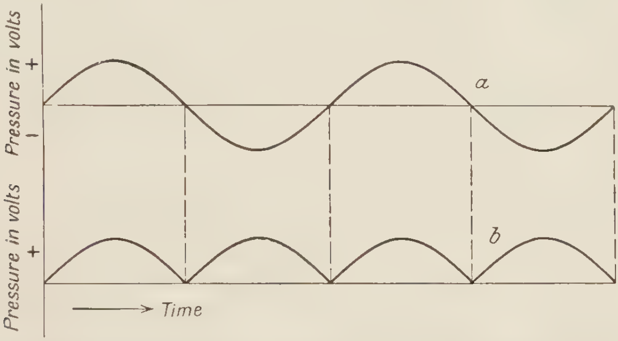


Fig. 2a. Simple harmonic alternating pressure of an alternating current generator. b. Pressure graph of a direct current generator with a simple ring armature.

In the particular case where the magnetic field and the rate of rotation are uniform the alternating pressure is simple harmonic (fig. 2a). The frequency n is equal to the number of revolutions per second.

In practice the coil is provided with an iron core shaped as in fig. 3. The coil and core together form the *armature*. This type of construction is used e. g. in the magnetos of internal combustion engines and in telephony for calling-up by turning a handle.

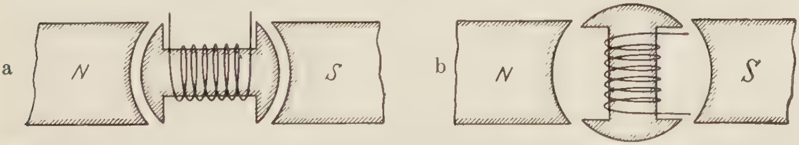


Fig. 3. Iron core and rotating coil of a generator: in the position a the magnetic flux is large, in b small.

(b) *The direct current generator.* Fig. 4 again shows a demonstration model in silhouette. The slip-rings of the alternating current generator are replaced by a "commutator" K. This is a simple piece of apparatus which reverses the connection between the ends of the coil and the poles of the machine at the end of each half revolution. The effect is to fold over the lower half of the curve of fig. 2a, so that we obtain

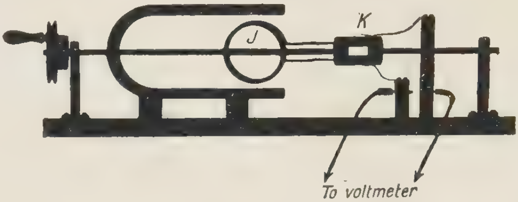


Fig. 4. Direct current generator with simple ring armature and permanent field magnets.

the pressure graph of fig. 2b. The pressure varies between zero and a maximum value, but its sign always remains the same.

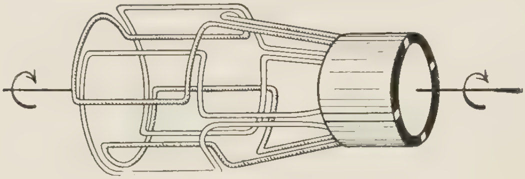


Fig. 5. Drum armature with two pairs of coils.

(c) *The direct current generator with drum-wound armature.* If, as usual, the arched form of the pressure graph of fig. 2b is inconvenient,

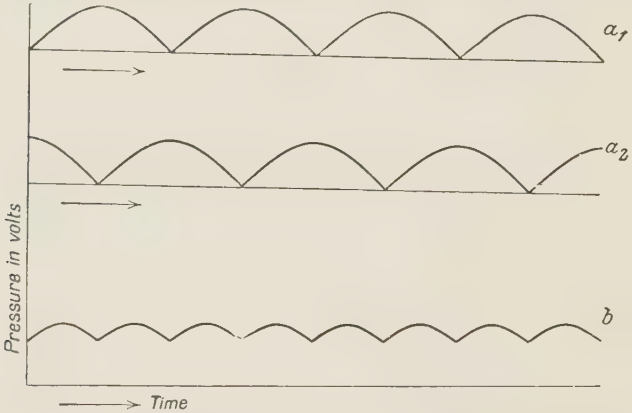


Fig. 6. Pressure graph of a drum armature with two pairs of coils, and the way in which the pressure arises.

several coils placed at equal angles apart are used instead of one. Instead of the "ring armature" we now have a "drum armature." Fig. 5

shows an arrangement with two pairs of coils and a collector divided into four. In this case two of the curves drawn in fig. 2b overlap in the way illustrated in fig. 6. As a result we obtain a direct pressure which, as shown in fig. 6, is already much more nearly constant. Henceforth we shall symbolize a drum armature and its sliding contacts or brushes by the diagram of fig. 7.

Fig. 7. Symbolic diagram of a drum armature.

Fig. 18 shows a type of direct current generator with drum winding which is widely used for demonstration purposes. This is an old-fashioned model. The drum armature consists of nine pairs of coils and the magnetic field is supplied by 2 × 25 steel horse-shoe magnets. In the small "high-pressure generators" for experimental work (giving e. g. 1000 or 2000 volts and 40 watts) we have a modern form, in which there is only *one* horse-shoe magnet. These little machines have the advantage that their construction closely resembles that of fig. 4, even in externals.

(d) *The direct current dynamo.* In the generators we have hitherto considered the magnetic flux is produced by permanent magnets. The permanent magnets may, however, be replaced by coils carrying a current, the so-called primary or field coils *F* in fig. 8. The current in the field coils may be taken from any auxiliary source of current. Fig. 8 is a diagram of this external excitation arrangement (invented by Wilde, Manchester 1866). *But we can actually use the pressure produced in the rotating armature to maintain a current in the field coils. This is the method used in the dynamo of Werner Siemens* (who was once a Prussian artillery officer). The great industry of high power electrical engineering sprang from the invention of the dynamo.

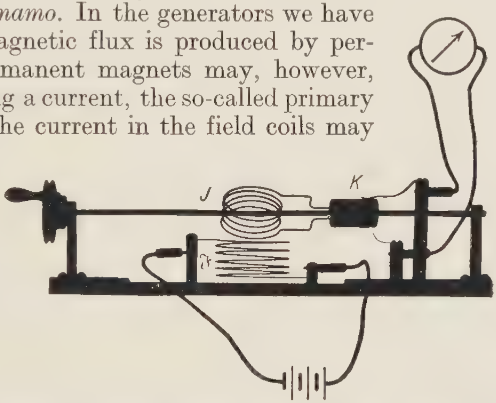


Fig. 8. Direct current generator with external excitation.

Fig. 9 shows a possible practical form: it is the diagram of a "series-wound dynamo". The whole current of the machine taken from the poles *a* and *b* flows through the field coils *F*.

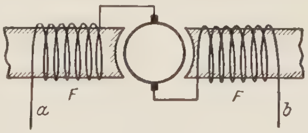


Fig. 9. Diagram of the connections of a series-wound dynamo.

In this type of machine we take for granted that there is iron in the coils. Unless this is so the machine cannot excite itself. The weak permanent magnetic field of the iron is indispensable for the production of an induced pressure in the armature at the beginning of the rotation.

In the series-wound dynamo the pressure between the poles rises as the current-load in the machine does. For the greater the current, the stronger is the magnetic field causing the induced pressure Fig. 10 gives the corresponding current-pressure graph, known as the *characteristic*, of a series-wound dynamo.

Except in special circumstances, generators are in practice required to produce a pressure which is within wide limits independent of the current-load. The *shunt-wound dynamo* fulfils this condition to a certain extent. Fig. 11 gives its characteristic and fig. 12 the diagram of its connections. Only a fraction of the current induced in the armature is used to excite the field coils. Mixed forms are a further development: they are known as "compound-wound dynamos".

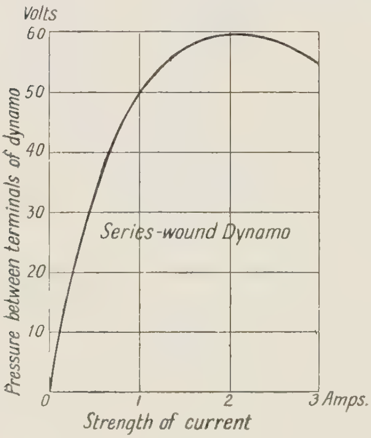


Fig. 10. Characteristic of a series-wound dynamo.

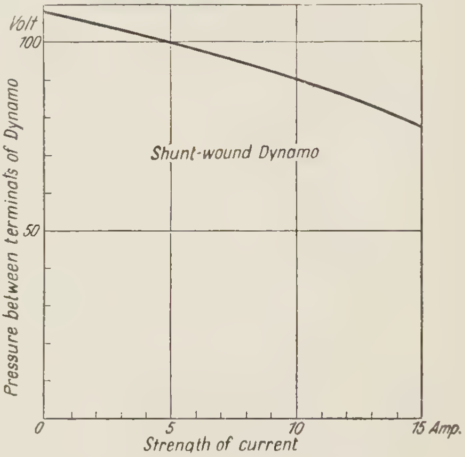


Fig. 11. Characteristic of a shunt-wound dynamo.

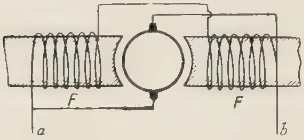


Fig. 12. Diagram of the connections of a shunt-wound dynamo.

Direct current dynamos as now constructed differ from the type of fig. 8 as regards an external matter. In the diagram only *one* pair of field coils and *one* pair of sliding contacts or "brushes" on the commutator are provided. In practice several pairs (3—5) are usually arranged in a centrally-symmetric way.

(e) *The alternating current generator with internal poles.* In the machine with external poles which we described under (a) the inducing magnetic field remains fixed. The coil J in which the pressure is induced is the moving part. The reverse is true for the machine with internal poles.

The rotating part carries the coil through which the direct current flows, while the fixed coil J, from which the alternating current is taken, is in the fixed part. In practice the coils are arranged in a centrally-symmetric way and repeated many times. The rotating part often consists of a flywheel which carries the coils traversed by the direct current on its rim. The direct current is supplied by an auxiliary machine on the shaft of the main machine.

(f) *Alternating current generators with no coils in the armature.* In the generators we have hitherto considered, the rotating part of the machine always carried a coil. But we can change the magnetic flux penetrating a coil even without making the coil itself revolve. Fig. 13 shows a machine on this principle. It is derived from fig. 7, p. 129 in a way which can easily be understood. In this model the rotating armature consists of a rectangular piece of iron *E*. As its position varies the magnetic flux penetrating the coil is altered. In fig. 13a the magnetic flux has its maximum value, in fig. 13b its minimum value.

In practice the permanent field magnets are often replaced by electromagnets, i. e. by iron-cored coils traversed by a direct current. Besides, all the separate parts are repeated many times and arranged at regular intervals, somewhat as in fig. 14. The rotating part then takes the form of a toothed wheel.

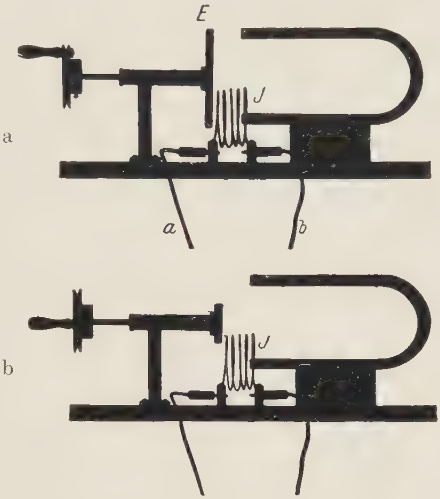


Fig. 13 a, b. Alternating current generator with no coils in the rotating part. In the position *a* the magnetic flux is large, in *b* small.

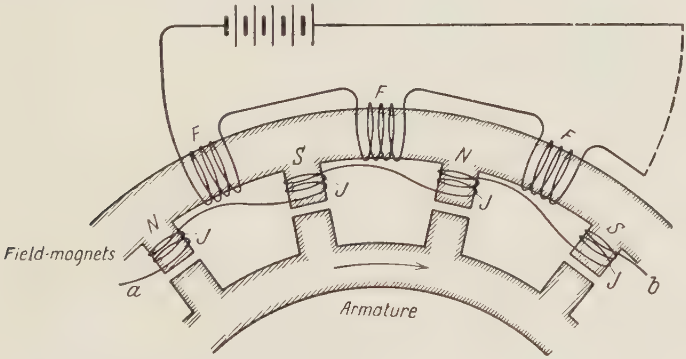


Fig. 14. Alternating current generator with toothed armature.

(g) *The telephone as a generator of alternating current.* The essential feature of the alternating current generator with no coils in the armature is the periodic alteration of the magnetic circuit, caused by the approach or withdrawal of the teeth of the armature.

The rotation may be replaced by vibration backwards and forwards. In fig. 15 J is again the secondary coil, in this case divided into two halves. This time the iron cores which serve to increase the magnetic flux have also been included in the figure. M is an iron membrane which can be made to vibrate and which takes the place of the revolving armature. This is only a practical variant of the experiment sketched in fig. 7, p. 129.

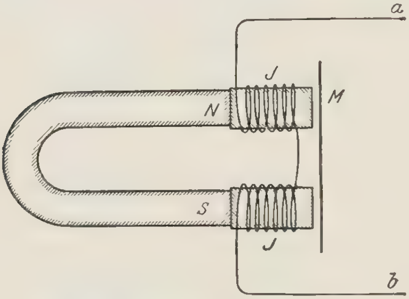


Fig. 15. Diagram of a telephone.

The whole apparatus is well known to everybody as the telephone. Here we are interested in it only as a generator of alternating current. We proceed to let sound-waves fall on the membrane M, where their energy is transformed into electrical energy. In order to demonstrate this, we connect any ordinary telephone (fig. 16) with a practical ammeter for use with alternating current. When we sing to the membrane currents of about 10^{-4} ampere are readily observed. These alternating currents have the same rhythm as the voice. Formerly they were conducted by line to the receiving station and there transformed back into mechanical vibrations (cf. fig. 17, in which the permanent magnets are represented by the bar type). This method is now obsolete. The vocal cords are no longer used as a motor for driving an alternating current generator. Nowadays we use our voices only to impress the rhythm of speech on to strong pre-existing currents (as in the microphone, pp. 282—284).

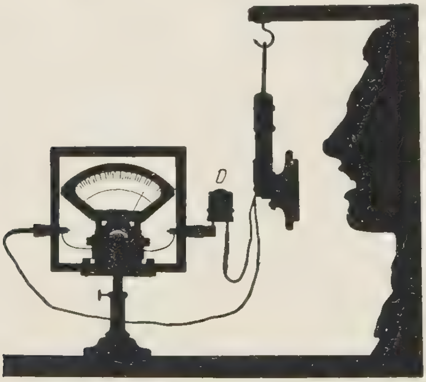


Fig. 16. The telephone as a generator of alternating current.

The most convenient alternating current ammeter of suitable sensitivity for demonstration purposes is a moving-coil ammeter with a small practical rectifier (p. 284); for example, we may include a small "crystal detector" in the circuit. This is a contact between two suitably chosen metals or minerals, say gold and silver. It is shown at D in fig. 16. The resistance of the contact depends very largely on the direction of the current, hence the rectifying action.

2. The Transformation of Energy in Generators; Lenz's Law and Rotary Fields.

Generators transform mechanical work into electrical energy. Here we shall illustrate the exchange of energy in a simple particular case. We consider the direct current generator shown in fig. 18.

If we turn the handle of the machine, a pressure E of 10–20 volts, according to the number of turns, is induced between the poles a, b. As long as the poles are not joined by a conductor there is no current, but only a pressure. Hence we only obtain the negligible energy of the electric field between the poles. The small amount of work put into the machine by our muscles while turning the crank is used up in overcoming the friction on the bearings and gearing.

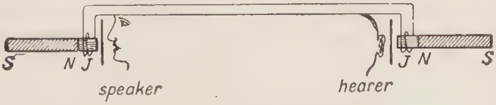


Fig. 17. Old-fashioned arrangement of two telephones for telephonic communication.

Things are quite different whenever we connect the two poles of the machine by a conductor and an ammeter. In fig. 18 we have taken for our conductor a 150 candle-power incandescent lamp with a thick tungsten filament. A current i of about 8 amperes now flows through the lamp, the ammeter A and the coil of the drum armature. There is a pressure of about 12 volts between the lamp leads.

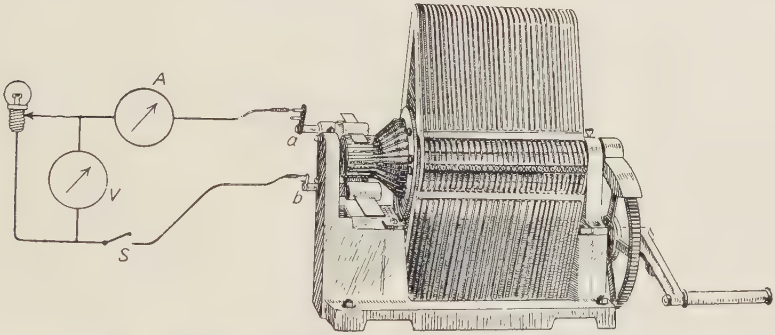


Fig. 18. Direct current generator with permanent field magnets and drum armature.

By Ohm's law (equation 1, p. 16), i is equal to $E/(r + r_a + r_i)$, where r denotes the resistance of the lamp, r_a that of the ammeter, and r_i that of the armature coil and leads.

The machine is stiff to drive and we have to work hard; the electrical energy produced (cf. p. 74) is transformed into heat, and the lamp becomes white-hot. After a few seconds we break the current by means of the switch S: we now meet with very little resistance in turning the handle; the change is very noticeable.

In this experiment the quantity of electricity $Q = i$ coulombs is moved through an electric field with a pressure of E volts every second. By equation 12, p. 74, this process yields energy amounting to $8 . 12 =$ about 100 volt-coulombs or watt-seconds. That is, we supply the lamp with about 100 watts or 0·1 kilowatt, which corresponds to about $^1/_8$ horse-power. In this way we can get a clear idea of the magnitude of 1 kilowatt from our own muscular exertion. We can then better appreciate the amount of energy in a kilowatt-hour, which, thanks to modern electrical engineering, is supplied to our houses at the cost of a few pence.

As long as the induced pressure is producing a current, i. e. as long as the poles of the machine are joined by a conductor, the machine is stiff to drive. It follows that the current produced by the induced pressure *retards* the revolution of the armature, that is, it retards *that* motion which brings about the induction phenomenon. This is a good experimental illustration of Lenz's law (p. 103).

This mechanical opposition to the motion in accordance with Lenz's law may be demonstrated in many other ways. We may mention three instructive examples:

(1) We take a "secondary" coil with only *one* turn, e. g. a silver ring several centimetres in diameter on a short handle, and place it between the poles of an electromagnet, approximately at right angles to the lines of force; we then try to move the ring about in its own plane. In the *uniform* parts of the magnetic field the magnetic flux through the ring remains constant; there is no induced current and no mechanical opposition. It is not so, however, in the *non-uniform* border regions of the magnetic field, or if we try to pull the ring out of the magnetic field; for then the magnetic flux through the ring is altered. A strong induced current flows in the silver ring, which is a good conductor, and the hand experiences a peculiar tough resistance which is often surprisingly large.

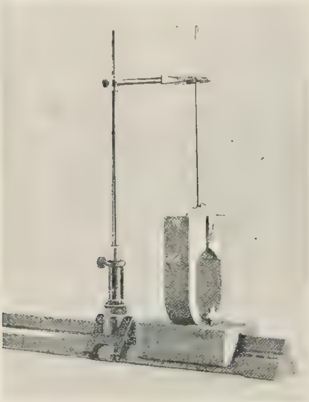
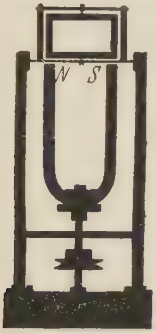


Fig. 19. A ring-shaped secondary coil suspended so as to swing between the poles of a movable horse-shoe magnet.

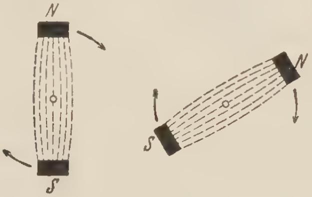
(2) We reverse the experiment. The ring is suspended so as to swing between the poles of a horse-shoe magnet, in such a way that it can move only in a plane perpendicular to the lines of force (fig. 19). The magnet is moved sideways in the direction of the rail, carrying its field with it. Then the ring or secondary coil is also displaced in the same direction, since action and reaction are equal.

Conversely, the ring avoids the magnetic field approaching it. It behaves, although only for the time being, like a diamagnetic substance, say a disc of rock-salt (cf. p. 147).

(3) In the two cases just mentioned the magnetic flux through the ring was altered by lateral *displacement*; changes in the magnetic flux, however, may equally simply be caused by *rotation.* We have only to alter the angle between the lines of force and the area enclosed by the ring. This is most simply done by means of a *rotary magnetic field.* By a rotary field we mean a field whose lines of force rotate like the diameter of a wheel (cf. fig. 20b). In order to produce a rotary field, we set a horse-shoe magnet *N S* in revolution about its long axis (fig. 20a). Into this rotary field we bring a secondary coil with only *one* turn; this time the coil is suspended so as to be free to *rotate* (the reader will easily understand how this is done). The secondary coil is protected from currents of air by a plate. If the magnet *N S*, and thereby the magnetic field, is set in rotation, the secondary coil begins to rotate also. The experiment is analogous to that represented in fig. 19. Soon the secondary coil is moving almost as quickly as the rotary field. It cannot move with exactly the same velocity as the rotary field, otherwise the alteration of magnetic flux and with it the induced current would cease. The percentage difference of velocity between the coil and the rotary



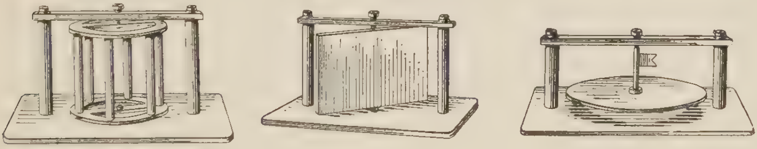
a



b

20a. Production of eddy currents by a rotary field.

20b. Diagram of the rotary field produced by the apparatus of fig. 20a, in two positions 60° apart. The small circles represent the axis of rotation of the horse-shoe magnet and of the lines of force between its revolving poles *N S*, as it appears to an observer looking down vertically.



c d e

Fig. 20, a—e. Rotary magnetic field with various forms of "rotor".

field is called the *slip*. In practical applications of this experiment the single secondary coil is replaced by a series of copper bars combined to form something like a cage. We then speak of a "squirrel-cage rotor" (see fig. 20c).

3. Eddy Currents produced by Alterations of Magnetic Flux. The Permeability of Diamagnetic Substances.

We have just read of three experiments demonstrating Lenz's law. The magnetic flux through a ring (fig. 19) or frame (fig. 20a) was altered in order to produce induced currents. The essential point is the change of the magnetic flux through the ring or frame, not the accidentally chosen shapes of the ring or frame. The shape of the ring or frame may vary greatly. The border can be made so broad that only discs with small holes in the middle are left. Even the holes may be dispensed with and the ring or frame replaced by a circular or rectangular disc. For example, we bring a silver coin into the non-uniform field of an electromagnet (fig. 21) and let it go. It does not fall with the velocity usual in air, but sinks quite slowly to the ground, as if it were in a viscous liquid. It may take several seconds to fall through a distance of a few centimetres in the field.

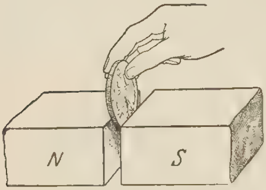


Fig. 21. Eddy currents retarding the fall of a silver coin.

This phenomenon is used in practice for damping troublesome mechanical vibrations e. g. in scientific instruments. The apparatus shown in fig. 19 is a suitable one for showing the effect, if an aluminium disc is substituted for the ring; or we bring a rectangular aluminium plate, mounted so as to turn freely, into a rotary magnetic field to act as a "rotor" (fig. 20d). The plate rotates if the magnetic flux passing through it alters periodically. Curiously enough, the metal disc shown in fig. 20e likewise revolves in the rotary field, forming a "rotor" (cf. p. 261).

The currents induced in these solid discs are called *eddy currents* (or *Foucault currents*). The disc shape, however, is not essential, for eddy currents occur in all solid pieces of metal penetrated by a changing magnetic flux. Apart from their mechanical effects, these eddy currents cause heating effects which are often very troublesome, especially in the case of iron cores used to increase the magnetic flux through coils carrying currents. Injurious results may, however, be effectively avoided in the following way. Instead of solid iron cores we use cores which are extensively subdivided, such as bundles of thin iron wires or packets of thin metal plates, with the wires or plates separated by insulating layers. The latter usually consist of coats of varnish on the wires and layers of paper pasted on to the plates. This device is used in machinery and apparatus wherever a changing magnetic flux passes through iron. As one out of many examples we may mention the armature of a direct current generator.

Apart from their practical importance, eddy currents are of considerable interest in another physical connection. They enable us to get a clear understanding of what is meant by the *permeability of diamagnetic substances.*

Substances which increase **B**, the magnetic induction of a primary coil, are called *paramagnetic.* μ, the proportionality factor **B**/**H**, appears greater for paramagnetic substances than for a vacuum, for which its value is given by $\mu = 1{\cdot}256 . 10^{-8}$ volt-seconds/ampere-centimetre. That is, μ', the value relative to the vacuum as unity, appears to be greater than unity (cf. Table II, p. 106).

The high permeability of *paramagnetic* substances is explained by means of *molecular currents.* It is assumed that the molecular currents exist beforehand, independently of the magnetic field, and that the latter merely *arranges* the molecular currents parallel to each other. We explained this in detail on pp. 94—95 for the particular case of iron. The field of the invisible molecular currents is to be added to the external field of the coil, thus raising the magnetic flux.

Bodies which diminish the magnetic flux of a primary coil are called *diamagnetic.* μ appears smaller than $1{\cdot}256 . 10^{-8}$ volt-seconds/ampere-centimetre, that is, the relative value μ' appears to be less than unity (cf. Table II, p. 106).

Now the low permeability of diamagnetic substances can likewise be explained in terms of molecular currents, if the latter are considered as *eddy currents in the molecules.* Eddy currents do not exist apart from the magnetic field. They do not arise until the substance is being brought into the magnetic field, during which process the induced electric field accelerates electrons in the molecules. Once set going, these electrons keep on circulating without hindrance, just as in any molecular current. By Lenz's law, their direction is opposite to that of the current in the coil producing the external magnetic field. The magnetic field of these invisible molecular currents is to be deducted from the external field due to the coil; hence the magnetic induction is diminished.

The behaviour of diamagnetic substances cannot be imitated on a large scale by means of ordinary conductors. Eddy currents in the latter do not persist; the electrons are rapidly retarded by friction (this causes the heating effect of the current). The imitation can, however, be carried out by means of "superconducting" metals, in which eddy currents persist practically undiminished for hours on end (pp. 253—254).

According to this conception of molecular currents every substance must be diamagnetic. Any substance may also contain electrons in permanent circulation independently of the field. If *their* influence preponderates in the magnetic field, the substance appears paramagnetic. When we have a more exact knowledge of the structure of the atom we shall be able to calculate the numerical values of the permeability from other atomic constants. At present we are very far from being able to do this.

4. Electromotors.

All electromotors can be reduced to the following simple principle. The magnetic flux penetrating a coil is periodically altered by means of an electric current. This periodic change in the magnetic flux is used to produce periodic motion, in particular, rotation. As a rule, a coil carrying current (the "rotor") rotates in the magnetic field of a "stator". It is merely a case of periodically repeating the rotation of the coil illustrated in fig. 20b, p. 146. We shall confine ourselves to a few examples:

(a) *The alternating current synchronous motor.* This motor resembles an alternating current generator in principle. Fig. 22 shows the same

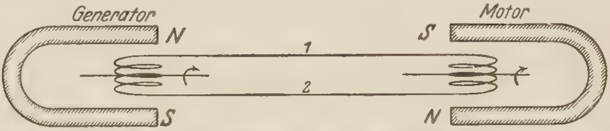


Fig. 22. Alternating current synchronous motor connected to an alternating current generator with external poles.

machine as a generator on the left and as a motor on the right. Let the rotating coil of the generator revolve n times per second. Then the generator supplies an alternating current of frequency n cycles per second. This reaches the rotating coil of the motor through the leads 1, 2. The current produces a turning moment or torque acting on the rotating coil. The direction of rotation depends on the direction of the current for the time being. Hence it is essential that the torque for every position of the rotor should be in the right direction for continuing the revolution. This is easily provided for in the following way:

Let the current in the rotating coil of the motor flow in such a direction at the instant under consideration that the torque produced by it acts in the direction of the arrow (fig. 22). $1/n$ of a second later the current again has exactly the same direction and strength. If it finds the rotor again in the same position, the torque again acts in the same direction. Hence we have only to make the rotor run at the right speed. Thereafter it will continue to run "in synchronism" with the alternating current of the generator.

For a demonstration experiment we wind a string round the shaft of the motor and pull it off, thus setting the rotor going as we do a child's top. The alternating current used has 50 cycles per second. It is obtained from any large practical generator.

A number of convenient ways for starting up synchronous motors are used in practice. Perhaps the simplest is to start the machine as an induction motor (p. 166), in which case a reduced pressure is applied at starting by means of a small auxiliary transformer (p. 175). Alternating current synchronous motors are widely used in practice because they can be excited so as to take the minimum current from the electric mains. This is obviously an economical arrangement.

They compensate the wattless components of the currents from the asynchronous machines on the same network.

(b) *The direct current electromotor.* Externally this resembles the direct current generator. A simple diagram of a motor is given in fig. 23. The torque acting on the rotating coil is likewise produced as in the arrangement of fig. 20b (p. 146). The armature turns on its axis till the area enclosed by the turns of the armature coil is perpendicular to the plane of the paper. The direction of the current in the armature is then reversed, and so on after each half revolution. This is done automatically by the commutator *K* with its sliding contacts or "brushes", which is rigidly fixed to the shaft.

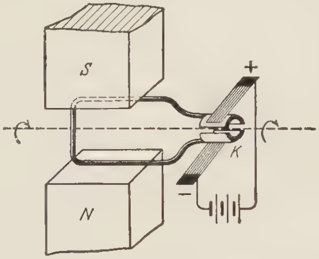


Fig. 23. Diagram of a direct current motor.

In this simple arrangement, which is still used in children's toys, the motor has a "dead point". It will not start if the coil is placed perpendicular to the lines of force. Besides, the torque is not constant during a revolution. These defects are avoided by the use of the drum armature, with which we became acquainted in connection with the direct current generator (fig. 5, p. 150) and which is almost universally employed in the electromotors in use nowadays. The fields of the stator are, moreover, always produced by coils carrying a current (electromagnets).

It is easy to show what factors determine the number of revolutions and the torque of the armature in such a machine. For this we use an "externally excited" field (cf. fig. 24a). We take an ordinary electromotor giving about 1 kilowatt. *F*, the fixed field coil, receives its current from a source of pressure E_1, the town mains in the simplest case. The current in the field coil can be adjusted by means of the regulating resistance *R* and the ammeter A_1. The current in the armature is supplied from a source of pressure E_2, say a battery of accumulators.

What we stated on p. 144 now holds for the current in the armature. If E_2 is connected to the armature when the latter is at rest, a short-circuit current amounting to many amperes flows through it; for r_i, the resistance of the armature coil, is but small. In large motors the turns of the coils and the leads would be endangered. This is prevented by means of a starter R_a (cf. fig. 24b), consisting of a resistance which is cut out as the machine starts to run and thus always keeps the current within limits which the apparatus can stand.

What we stated on p. 144 likewise holds for the moving conductor or armature. The armature should be adjusted to a velocity, i. e. number of revolutions per second in this case, such that E_i, the pressure induced in it, is equal to E_2, the pressure of the source of current. (That is, the turning of the armature is to have the same effect as if a battery of accumulators of pressure E_2 were inserted in the armature *at rest.*)

In fact, the number of revolutions per second increases as soon as we raise the pressure E_2 (by switching on more accumulators).

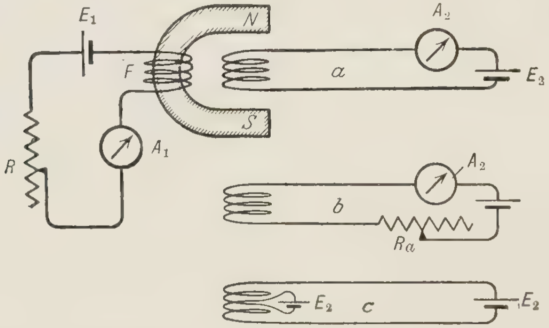


Fig. 24. Illustrating induction in the armature of a motor.

On the contrary, the number of revolutions per second decreases if we increase the magnetic flux of the field coils by increasing the current through them. When the magnetic flux of the field is increased, a slower motion of the armature suffices to make E_i equal to E_2.

The limiting case $E_i = E_2$ requires that no current should flow in the armature once the number of revolutions per second has become constant. In actual fact, the current sinks to very low values when the motor is running without a load. The limiting case $i = 0$ cannot be reached in practice. $i = 0$ would of course mean that the source of current would supply no more energy to the armature, once the latter had received the kinetic energy of its motion. In actual fact, however, even an armature with no external load must always do the work required to overcome the inevitable friction (of the bearings and the air), to which there is added the heating effect of the current. Hence, even when it is running light, the armature requires a certain supply of energy in order to maintain the same number of revolutions per second. There must be a current flowing through the armature, though it may be a small one. When a load is put on the motor, e. g.

by making it raise a weight or by braking the shaft with the hand, the current i in the armature increases.

Finally, we must not omit to mention the effect of making E_2, the pressure applied to the armature, very small. We take *one* 2-volt accumulator, say. The armature then attains a constant number of revolutions per second, even though it is running quite slowly. One complete turn may last more than a second. Let us now turn the armature *faster* by hand; the moving-coil ammeter A_2 now shows a current in the *reverse* direction. E_i, the pressure induced in the armature, has become greater than that of the source of current, E_2. The work done by our hand flows into the accumulator as electrical energy. The machine acts as a generator and charges the accumulator.

This experiment is very striking. It reduces the explanation of the machines which are of such tremendous practical importance in the transmission of electrical energy to two simple physical facts:

(1) the induction phenomenon in a moving conductor, which gives rise to the induced pressure E_i between the ends of the coil of the armature;

(2) the forces which are exerted by the electric field associated with the pressure E_i on the electrons in the coil of the armature.

If E_i *accelerates* electrons, a current is produced and mechanical work is transformed into electrical energy. If E_i *retards* or acts as a brake on moving electrons, a current is weakened and electrical energy is transformed into mechanical work. In both cases the fundamental equation (12), p. 74, holds quantitatively.

The considerations sketched here form the basis of the practical construction of direct current motors. External excitation of the field coils is not usual in practice. The field and armature coils are connected to the same source of current. As in the case of generators, we distinguish between series-wound and shunt-wound electromotors [cf. the connection diagrams, figs. 9, 12 (pp. 151, 152) given in connection with generators].

The series-wound motor adapts itself to widely-varying loads by large alterations in its rate of revolution. It develops its greatest torque when it is starting up. A decreasing load means a decreasing current in the armature. Since the armature and the field are traversed by the *same* current, the inducing magnetic flux of the field decreases at the same time. Hence the number of revolutions increases. When there is no load the machine runs away. The series-wound motor is the most suitable machine for electric railways and all kinds of hoisting arrangements, such as cranes, lifts, etc. Like the series-wound dynamo, it is externally recognizable by the thickness and small number of the turns in its field coils.

The characteristic feature of the shunt-wound motor is that the current in the field remains constant, independently of the current in the armature. With increasing load the number of revolutions sinks to some extent. When skilfully constructed even a small reduction in speed suffices to increase the current in the armature greatly. In this way the torque adapts itself to the new load. The shunt-wound motor maintains an approximately constant rate of revolution within certain limits of load. This is the most suitable motor for machine-tools of all kinds, which, like e. g. turning lathes, do not require a high driving torque to start them up. Like the shunt-wound dynamo, the shunt-wound motor is externally recognizable by the large number of fine turns in its field coils.

Mixed forms with both series and shunt winding produce an even greater degree of constancy in the rate of revolution than the shunt-wound machine does with a varying load. A few turns traversed by the armature current provide for a weakening of the inducing field when the armature current rises.

The direction of rotation of a direct current motor with field coils (as opposed to *permanent* field magnets) is independent of the sign of the pressure applied to its terminals. This is true both for the series-wound and the shunt-wound machine. In order to reverse the direction of rotation we have to reverse the direction of the current either in the field alone or in the armature alone.

(c) *The commutator motor for alternating current.* The direction of rotation of direct current motors is independent of the sign of the pressure applied. Hence direct current motors can theoretically be used with alternating currents. In practice the construction of these machines is modified in some respects to adapt them for use with alternating current. To avoid losses due to eddy currents the iron is subdivided to a great extent. The commutator motor is an asynchronous motor. It is used largely in Europe on main line electric railways, but in England and the United States direct current is preferred for railway electrification.

(d) *Rotary field or induction motors for alternating current.* In mechanics the combination of two perpendicular frequencies of equal frequency is discussed in detail. Fig. 25 shows a simple example. Two long leaf-springs a and b each carry a plate on their free ends. Each spring has a slit along the direction of its length. At the point of intersection of the two slits we can see through the springs; when we look in the direction of a source of light we see a bright spot. We strike the horizontal spring; the spot of light executes a practically rectilinear vibration a in a vertical direction (fig. 26). Similarly the other spring by itself executes a horizontal vibration b.

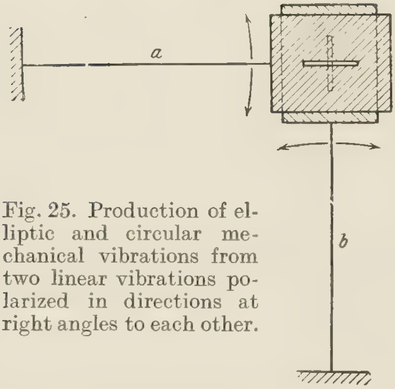


Fig. 25. Production of elliptic and circular mechanical vibrations from two linear vibrations polarized in directions at right angles to each other.

If we excite the springs skilfully, both vibrations may be made to begin at the same moment, and the spot of light vibrates backwards and forwards in a line inclined at 45°. The two "linearly polarized" vibrations a and b at right angles to each other combine to form a vibration c which is likewise linearly polarized (fig. 26).

We now let one spring begin its vibration a short time after the other (fig. 27). The spot of light describes an elliptic path. The vibration is "elliptically polarized". The distance of the spot of light from the equilibrium position is the "amplitude" of the vibration. That is, in the elliptical vibration the amplitudes vary periodically not only in size, but also in direction. The shape of the ellipse, whether narrow or wide, depends on the difference of path or the "phase-difference" of the two vibrations. In fig. 28 the difference of path amounts to a quarter-period (i. e. the phase-difference is 90°). In this particular case the ellipse degenerates into a circle and the vibration is *circularly polarized*. The amplitude remains constant as time goes on, but its direction rotates like the spoke of a wheel. So much for the mechanical case, which clearly illustrates a perfectly general relationship of a geometrical nature.

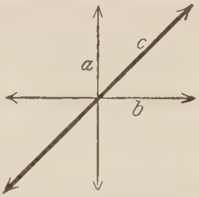


Fig. 26. When there is no phase-difference between the two separate vibrations the resultant vibration is also linearly polarized.

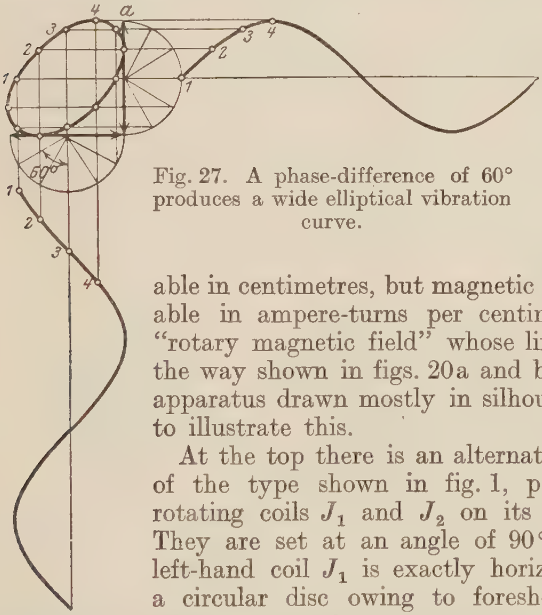


Fig. 27. A phase-difference of 60° produces a wide elliptical vibration curve.

We now apply this train of thought to the case of the vibrating magnetic fields of alternating currents. The amplitudes in figs. 27 and 28 no longer mean distances from the position of equilibrium measurable in centimetres, but magnetic intensities **H**, measurable in ampere-turns per centimetre. We obtain a "rotary magnetic field" whose lines of force rotate in the way shown in figs. 20a and b. The demonstration apparatus drawn mostly in silhouette in fig. 29 serves to illustrate this.

At the top there is an alternating current generator of the type shown in fig. 1, p. 148, but with two rotating coils J_1 and J_2 on its axis instead of one. They are set at an angle of 90° to each other. The left-hand coil J_1 is exactly horizontal and looks like a circular disc owing to foreshortening. In contradistinction to fig. 1, p. 148, the separate turns of the coils are not visible. Two alternating currents are taken from the pairs of slip-rings *ab* and *a'b'*. They are displaced from each other by 90°, just as in fig. 28.

On the right of the figure there are two perpendicular magnet coils divided in the middle, carried by a ring. The rotary field is to be produced in the common space in the middle. In order to do this, the horizontal coil is connected with the rotor J_1 and the vertical coil with the rotor J_2 (cf. the diagram drawn above fig. 29).

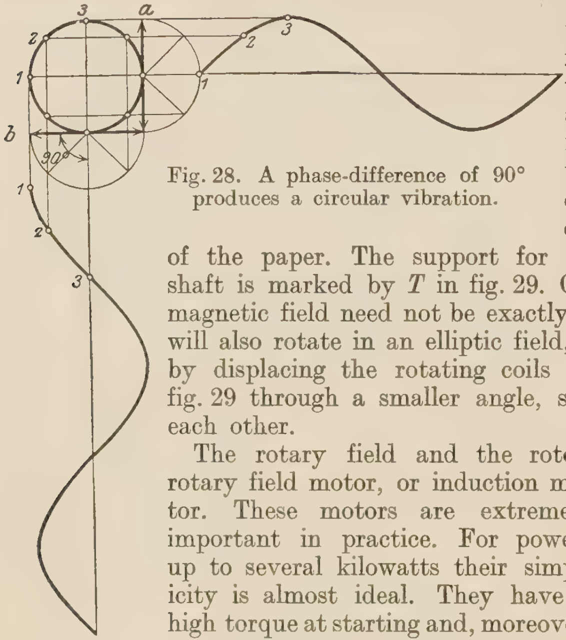
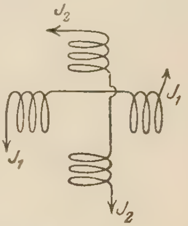


Fig. 28. A phase-difference of 90° produces a circular vibration.

One of the rotors which we met with in fig. 20 above, e. g. that in the form of a disc, serves as an indicator of the rotary field. The shaft of the rotor is perpendicular to the plane of the paper. The support for the bearing of the shaft is marked by T in fig. 29. Of course the rotary magnetic field need not be exactly circular. The rotor will also rotate in an elliptic field, such as is obtained by displacing the rotating coils of the generator of fig. 29 through a smaller angle, say 60°, relative to each other.



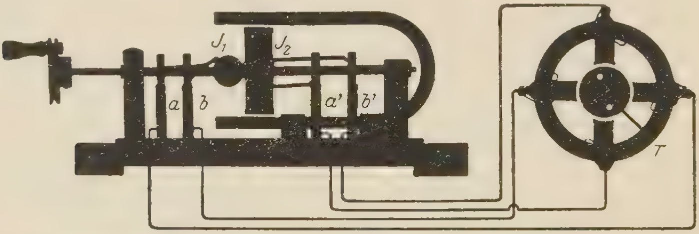


Fig. 29. Demonstration model of a two-phase rotary field generator and a rotary field motor with a copper disc acting as rotor.

The rotary field and the rotor together form a rotary field motor, or induction motor. These motors are extremely important in practice. For powers up to several kilowatts their simplicity is almost ideal. They have a high torque at starting and, moreover, do not require a starting resistance. (The slip is initially very large; see p. 158.) The number of revolutions is to a great extent independent of the load. Except for the slip, the number of revolutions is equal to the frequency of the alternating current used, or, if the motor is constructed in a suitable way, equal to the latter frequency divided by an integer. For high powers the advantage of having a rotating part without slip-rings or brushes has to be sacrificed. It is now necessary to include starting-resistances in the rotor circuit, and this cannot be done without using slip-rings. — We distinguish between one-phase, two-phase, and three-phase induction motors.

A two-phase motor is shown in fig. 29 above. This type requires four leads and is not much used.

A three-phase motor also works by means of a rotary field. Imagine that in fig. 29 there are *three* rotating coils J on the generator shaft, set 120° apart. Corresponding to these we set *three* coils 120° apart in the right-hand part of the apparatus of fig. 29. In this way we obtain a rotary field, or circularly polarized magnetic field, using three alternating currents displaced through 120° relative to each other. There are six leads, but they can easily be combined in pairs to form three. These three wires can be seen in many places in the overhead conductors of long-distance transmission installations.

The one-phase motor requires only two leads. Ordinary alternating current, such as is supplied by a machine of the type shown in fig. 1, p. 148, is led to the motor. The motor is not self-starting, so an auxiliary starting coil is usually provided in the stator, displaced by approximately 90° relative to the running coils. The two coils produce *at starting* a rotary field sufficiently powerful to start the motor. The starting coil is usually cut out automatically when the motor has reached running speed. The principle of the method is illustrated below by fig. 6c, p. 175.

CHAPTER VII

The Inertia of the Magnetic Field and Electric Oscillations

1. The Inertia of the Magnetic Field as a Result of Self-induction.

We discussed the fundamental experiment on self-induction on p. 134. There we deliberately omitted to consider the sign of the induced pressure. In dealing with this point now we shall be led to interpret self-induction in a more thorough-going way. We repeat the experiment, using the arrangement shown in fig. 1. In fig. 1a the moving-coil voltmeter indicates the two volts from the accumulator by a deflection to the left. The small fraction of the current flowing through the voltmeter has the direction shown by the bent arrow. In fig. 1b the accumulator has just been cut out. There is a large instantaneous deflection of the pointer of the voltmeter to the right. That is, a current now flows through the voltmeter in the reverse direction. By the figure, this is possible only if the current in the coil *F* continues to flow for some little time in its original direction after the current has been switched off, so that negative atoms of electricity are heaped up at *a*. That is, the current and its magnetic field exhibit inertia. They behave analogously to a mass in motion or a running flywheel.

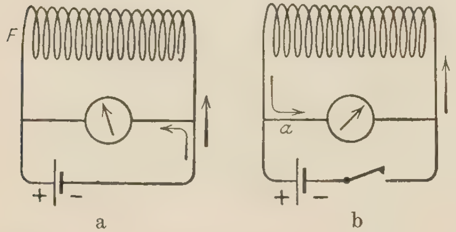


Fig. 1a, b. Inertia of the electric current in a coil.

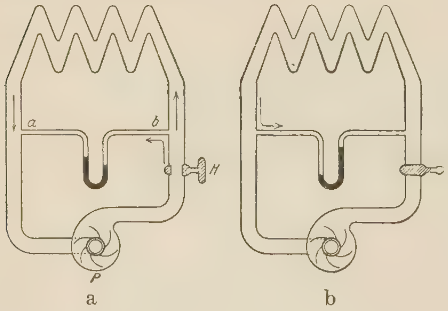


Fig. 2a, b. Inertia of a current of water in a pipe.

We briefly recall an example of mechanical inertia. In fig. 2a a current of water driven by a pump *P* is circulating through a tube. A mercury manometer inserted between *a* and *b* shows a displacement to the left, corresponding to the direction of the current and the resistance of the pipe. In fig. 2b the pump has been disconnected by means of the cock *H*. On account of its inertia, the column of water continues to flow for some little time in the direction of the arrow, and the mercury in the manometer shows a large displacement to the right. (The principle of this experiment is put to practical use in the construction of the machine for raising water known as a "ram").

A mass or a flywheel does not only exhibit inertia when it is being retarded but also when it is being set in motion. The same is true of a current or a magnetic field. This will be shown by a striking and very important experiment (fig. 3a). *E* is again a 2-volt accumulator. *A* is a good moving-coil ammeter with a pointer of small inertia (period of coming to rest under 1 second). *F* is a large coil of thick wire with a closed iron core (note the dimensions given in the diagram).

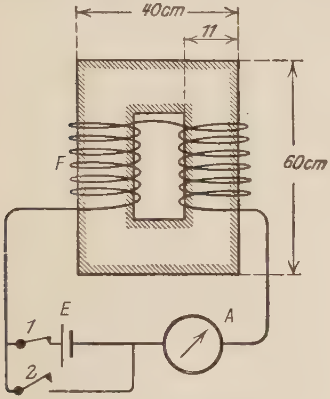


Fig. 3a. The slow beginning of a current.

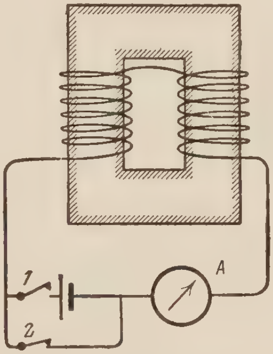


Fig. 3b. The slow dying away of a current after the source of current has been removed.

If we close the switch 1, the pointer of the ammeter moves immediately. But it only advances slowly; even after a minute it is still creeping on to a noticeable extent. It is a minute and a half before the magnetic field and the current finally reach their full values, so slowly are they developed.

When the maximum value has at last been reached, we first close the current circuit by means of the switch 2 (fig. 3b) and then immediately switch off the accumulator at 1. We have another illustration of the power of persistence, or inertia, of the magnetic field and current. Even after a minute the ammeter *A* still shows a definite current. The experiment has all the more surprising an effect on us owing to the fact that in ordinary life we always associate the idea of something instantaneous and timeless with electrical phenomena.

The result of these experiments is extremely important. The experiments show that self-induction, the inductive effect of a current on its own conductor, is externally revealed as an inertia of the current and magnetic field. As a result of self-induction, the current and magnetic field sluggishly oppose every change, just like an inert mass in mechanics.

Here we have purposely discussed this fundamental state of affairs from a purely empirical point of view. But it is also easy for us to recognize it as a simple consequence of Lenz's law. Consider the case of fig. 3a, in which the accumulator is connected to the coil in which no current was previously flowing. The current begins to flow, or the electrons begin to move. This motion causes electromagnetic induction to take place. By Lenz's law the electric field thereby produced is in such a direction that it slows down the motion which sets the process going, i. e. in this case it slows down the motion of the electrons.

Consider, again, the case of fig. 3b. After the source of current has been short-circuited the electrons would continue to move for an infinite time if the wire were an ideal conductor with no resistance. In reality, however, even the best actual conductor has a finite resistance r, which retards the electrons by forces resembling friction (thereby causing the heating effect of the current). This diminution of the velocity of the electrons is the cause of the induction phenomenon. Accordingly, by Lenz's law the induced electric field is in a direction such that it slows down the diminution in the velocity of the electrons. Thus part of the kinetic energy of the electrons which has been lost owing to friction is replaced at the expense of the energy of the magnetic field, and the decrease of the current with time is thereby checked. The inertia of the current and magnetic field thus depends on the inductive effect of the magnetic field. We know that the magnetic field and the current are inseparable. Hence in future it will suffice if we merely speak of the inertia of the magnetic field.

As we have just seen, the inertia of the magnetic field appears when the current *alters* as time goes on. Hence the inertia of the magnetic field plays an important part in practical applications of *alternating* current.

We begin with a simple experiment. The small coil with a closed iron core (cf. p. 135) which we have often used previously has a resistance of 0·33 ohm. It is to be traversed by a direct current of 3 amperes. For this we require a pressure $E_0 = ir = 3 . 0{\cdot}33 = 1$ volt, by Ohm's law. This is often called the "ohmic" pressure for short; it is sufficient to enable the electricity carriers to move along the spiral wire in spite of the resistances of a frictional nature which retard their progress.

We then try to send an alternating current (of frequency $n = 50$ cycles per second) of 3 amperes through the coil (fig. 4). In order to do this we must produce a pressure of about 100 volts between the ends of the coil.

This experimental result is scarcely surprising. In the case of alternating current it is impossible that the ohmic pressure E_0, which overcomes the "friction", should be sufficient. In addition there must always exist a further pressure E_i which keeps on alternately setting the magnetic field in motion and bringing it to rest. The magnitude of this pressure E_i depends on L, the coefficient of self-induction of the coil. Hence we call E_i the "induced" pressure.

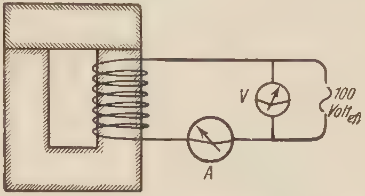
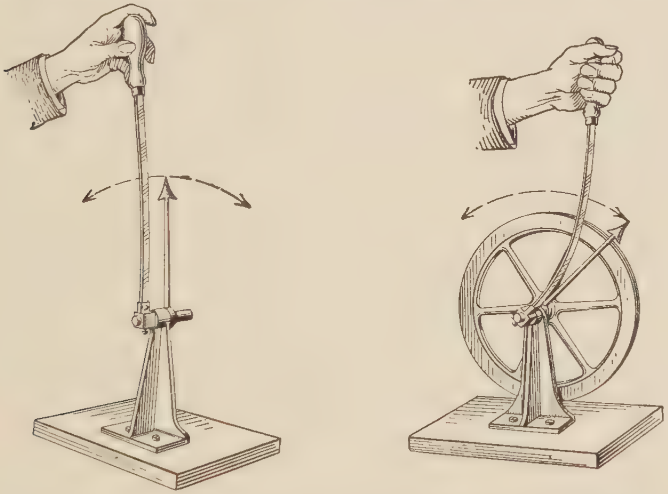


Fig. 4. Apparent resistance or impedance in the case of alternating current. The coil is the same as in figs. 11 b (p. 135) and 8.

There is a very good mechanical analogy to this phenomenon. Fig. 5a shows a shaft mounted horizontally, with moderate friction. We move it alternately backwards and forwards, n times per second. The (angular) velocity of the shaft at any moment corresponds to the velocity of the atoms of electricity, i. e. to i, the strength of the current A light pointer makes the velocity at any moment visible at a distance. The arm of the lever also serves as a dynamometer, as the bending at any moment is an easily visible measure of the applied force F_0. The force F_0 is always small; it merely serves to overcome friction. It corresponds to the ohmic pressure E_0. We



a b

Fig. 5a, b. To illustrate "ohmic" and "induced" pressure in alternating current circuits.

now fix a large inert flywheel on to the shaft and repeat the experiment; that is, we produce the same deflections and velocities (currents) at the same frequency as before. The small force F_0 is now insufficient. The muscles of our arm have to provide a large additional force F_i. The lever is considerably bent (fig. 5b). The force F_i has to overcome the inertia of the flywheel; it has to keep on alternately setting the inert flywheel in motion and bringing it to rest again. It corresponds to the induced pressure E_i.

We can infer considerably more from further observation of this model:

Case I. We have only friction to overcome. We see that the velocity (current) and force F_0 (ohmic pressure) are in phase: both are zero at the turning points, and both attain their maximum values as they pass through the central position. The force F_0 (pressure E_0) does work against friction. The work is transformed into heat (heating effect of current) in the bearings.

Case II. When the large flywheel is attached (large self-induction) we have practically only the inertia of the wheel (of the magnetic field) to overcome ($F_i > F_0$). The force F_i (induced pressure E_i) is now a quarter period (or 90°) ahead of the velocity (current): the force F_i reaches its maximum value when the velocity is zero, at the turning-points. In all, the force F_i (pressure E_i) does *no* work during a complete motion backwards and forwards (a period). Twice during a period kinetic (magnetic) energy is put into the inert flywheel (magnetic field); but twice the flywheel (magnetic field) gives up the same amount of work during the retardation. The velocity (current) is "wattless": on the average, it does not use up energy (which is measurable in watt-seconds).

Case III. We increase the friction (ohmic resistance) by means of a band brake (variable resistance) not shown in the figure. The force F_0 without the flywheel (E_0 when there is no self-induction) reaches the order of magnitude of the sum of the forces F_0 and F_i when the flywheel is attached ($E_0 + E_i$ when there is self-induction). φ, the difference of phase between the velocity (current) and the total resultant force $F_0 + F_i$ (total pressure $E_0 + E_i$) varies between 0° and 90° according to the relationship between the inertia and the friction (between the self-induction and the ohmic resistance).

So much for our mechanical model. A particularly good verification for the electrical case of the results inferred from the experiments with the model is obtained if we use an alternating current of very low frequency (about 1 cycle per second). We already know how to produce an alternating current of this kind (see fig. 17, p. 116). The maximum pressure need not exceed 2 volts. We pass this alternating current through the small coil with closed iron core, as shown in fig. 6a. A string voltmeter V and a string ammeter A enable us to observe how the pressure and current vary with the time. The two circular fields of view of the instruments are projected one on top of the other. In the position of equilibrium both fibres or "pointers" lie in the middle line.

We first use the coil *without* its iron core. This is case I, where the self-induction is very small, so that the resistance is practically the ohmic resistance only. We find that the current and the pressure are in phase. We then insert the iron core; the self-induction of the coil is now large, and we find that the pressure is 90° ahead of the current (case II). Finally, we insert a variable resistance of several ohms in the circuit (case III). We now see that the pressure is definitely ahead of the current, but no longer by as much as 90°. These experiments with alternating current of very low frequency are very striking and instructive.

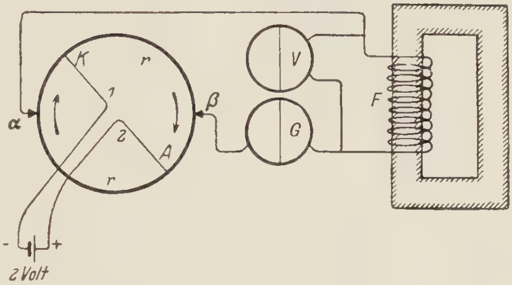


Fig. 6a. Difference of phase between current and pressure with an alternating current of very low frequency. On the left there is a source of alternating current like that shown diagrammatically in fig. 17, p. 116; *G* is a string galvanometer (fig. 17b, p. 12) for measuring the current (with a shunt of about 10^{-2} ohm); *V* is a unifilar voltmeter (fig. 81, p. 81) (or a string galvanometer acting as a voltmeter, with a resistance of 10^3 ohms in series).

In particular, they are very satisfactory from the point of view of measuring technique. The fibres of the instruments are damped so as to be practically aperiodic and their frequencies are far greater than the frequency of the alternating current under observation. If we use direct current instead of alternating current the apparatus shown enables us to carry out the experiment illustrated by fig. 3, p. 169. Once more we emphasize the fact that the formation and decay of the magnetic field take several seconds.

The induced pressure E_i may easily be calculated for the case of simple harmonic alternating current (p. 117). From equation (3), p. 135, which defines the coefficient of self-induction, we have

$$E_i = L\frac{\partial i}{\partial t}.$$

$\partial i/\partial t$, the rate of change of the current lagging 90° behind the pressure, is equal to $2\pi n i$, so that

$$E_i = 2\pi n L i.$$

The induced pressure E_i differs in phase from the ohmic pressure E_0 by 90°. Hence the two pressures may be graphically combined to form the total resultant pressure E by the parallelogram law familiar to us from mechanics (fig. 6b). For the ratio E/i, which we previously defined as the *resistance*, but which in the case of alternating current is called the *impedance*, we obtain the value

$$E/i = \sqrt{r^2 + (2\,n\,\pi\,L)^2}. \tag{1}$$

It follows that the quotient E/i is in general by no means constant in the case of alternating current, but depends on n, the frequency of the alternating current.

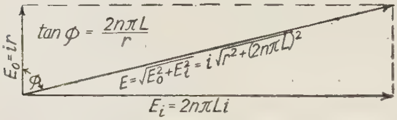

$\tan\varphi = \frac{2n\pi L}{r}$
$E_0 = ir$
φ
$E = \sqrt{E_0^2 + E_i^2} = i\sqrt{r^2 + (2n\pi L)^2}$
$E_i = 2n\pi Li$

Fig. 6b. The addition of two pressures differing in phase.

When L, the coefficient of self-induction, has a high value, the "impedance" may be greater than the ohmic resistance r for direct current by powers of ten, by equation (1). The experiment illustrated in fig. 4 exemplified this. In such cases we may neglect r as compared with $2\,\pi\,n\,L$*. Equation (1) then takes the simpler form

$$E/i = 2\,n\,\pi\,L; \tag{1a}$$

and we then indicate the conductor by a spiral instead of a zigzag line.

Coils of variable self-induction (e. g. with a variable iron circuit) enable us to regulate the strength of alternating currents as we please. For this purpose they have an important advantage over ordinary sliding resistances. They do not get heated and do not weaken the current by friction (Ohm's law), but "choke" it down by the inertia of the magnetic field without wasting energy ("choking coils").

The matters which we have only touched on here are dealt with in detail in the many excellent textbooks on alternating current practice. At present we shall content ourselves with briefly mentioning one special case. The difference in phase between current and pressure which occurs as a result of the inertia of the magnetic field in coils of high self-induction is often a source of disturbance both in physics and electrical engineering. This phase-difference may be got rid of by means of a condenser. The alternating current passes through the condenser as a displacement current (p. 118). The applied alternating *pressure* is 90° behind this displacement current. This may likewise be shown in a very striking way with an alternating current of very low frequency, merely by replacing the coil in fig. 6a by a condenser of capacity about 10^{-5} farad. We may also exhibit a simple experiment, in which we make the occurrence of a rotary field indicate the difference of phase. We let alternating current from the town mains flow through the primary coils arranged in the form of a cross (fig. 6c). For the purpose of our experiment we make the current divide itself between two branch circuits, in one of which we insert our secondary coil with its closed iron core. The difference of phase causes a rotary field to arise and the rotor revolves e. g. *clockwise.* We then replace the coil by a condenser (with a capacity of about 10^{-5} farad). The current in the horizontal primary coil is now ahead of that in the other and the rotor revolves *counterclockwise.* If we insert both coil and condenser in the circuit of the horizontal primary coil the phase-differences cancel each other and the rotor remains at rest.

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* [The latter quantity is sometimes called the *reactance*.]

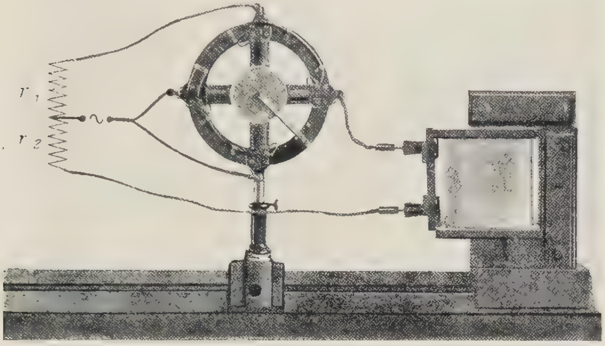


Fig. 6c. Demonstration of phase-difference by the production of a rotary field (currents a few tenths of an ampere, r_1, r_2 incandescent lamps acting as resistances in series, ~ alternating current of 50 cycles per second).

2. Transformers and Induction Coils.

Our interpretation of self-induction as inertia enables us to understand transformers for alternating current, which are important alike in engineering and in the laboratory.

A transformer consists of two coils linked by the same magnetic flux. Two practical forms are sketched in figs. 7a and b. Let one coil, F, called the primary coil, have n_p turns. Its ends are connected to the source of alternating current. Its resistance for direct current, or "ohmic" resistance, may be neglected, so that equation (1a) holds. The current and the magnetic flux in the primary coil only increase until the pressure caused by self-induction is equal to that of the source of current.

As we saw above, the same magnetic flux penetrates the other coil, i. e. the secondary coil J, and induces the secondary pressure E_s in its n_s turns. As the magnetic flux is the same in both cases, the pressures must be proportional to the numbers of turns in the coils, by equation (1b), p. 106, so that

$$E_s : E_p = n_s : n_p \,.$$

Hence by choosing the value of n_s/n_p, i. e. by choosing the value of the *transformer ratio*, any arbitrary step-up or step-down of the alternating pressure may be obtained. Transformations up to several hundreds of thousands of volts are nowadays carried out for many purposes in connection with physics and engineering. In such transformers the insulation of the turns of the coil from each other and from the iron core has to satisfy very stringent requirements. It is often useful to embed the whole transformer in oil, in which there is much less danger of a spark passing than in air.

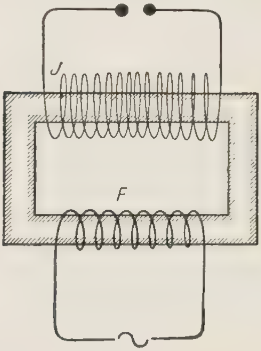


Fig. 7a. Alternating current transformer with closed iron core: ~ represents the source of alternating current.

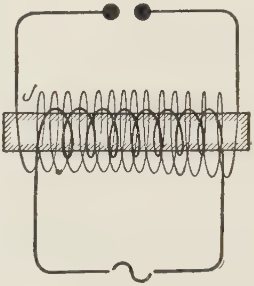


Fig. 7b. Alternating current transformer with open iron core.

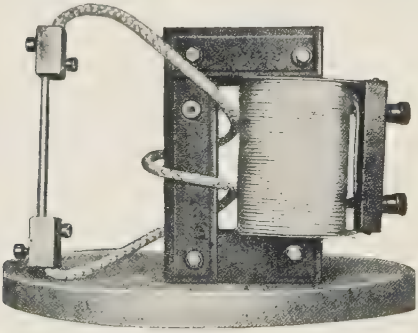


Fig. 8. Transformer for producing large currents.

A step-up of the pressure involves a step-down of the current in the secondary. Stepping-down the pressure has the reverse effect. Extremely large currents can be obtained in the secondary coil if the latter is composed of only a few thick turns. In contradistinction to the "high-pressure transformer", we may proceed in a very primitive manner in the present case of the "low-pressure transformer". Fig. 8 shows a transformer of this type which was improvised in the laboratory. It enables us to raise an iron wire 5 mm. thick to bright red heat.

From the point of view of the lecturer on physics, a small low-pressure transformer is the most convenient apparatus for producing currents up to several thousand amperes. Among other practical applications, "induction furnaces" for melting steel etc. are constructed on this principle. In this case the secondary circuit consists of *one* turn only. It is a ring-shaped pipe built into stonework which is not easily melted. The pipe is filled with the material to be melted. The current may reach ten thousand amperes.

Alternating current transformers have no moving parts and require no attention. On the other hand, the pressure of a direct current cannot economically be altered without the use of rotating machinery; the current is passed through an electric motor, which drives another dynamo supplying current at a different pressure. This is the reason for the extensive and continually increasing use of alternating current in practice. It provides the user with a convenient means of adapting the pressure to the various purposes for which it is employed. In particular, the present-day long-distance transmission of electrical energy could not be carried out at all without repeated transformations of the pressure. Only pressures of one or two hundred volts may be supplied to the consumer; apart from gross carelessness, such pressures do not endanger life. On the other hand, modern long-distance transmission plant pays only when the energy is transmitted at high pressure, the current being relatively small; the cross-sections of the conductors would otherwise be too large. It should not be forgotten that many modern installations transmit over 10000 kilowatts and have to carry currents of over 100 amperes even when the pressure is 10^5 volts.

The apparatus known as the "induction coil" is a variety of transformer. The primary and secondary coils are arranged coaxially and the iron core is not closed, i. e. we have the scheme of fig. 7b. Of the practical forms we may mention the induction apparatus used by physiologists and doctors. Cheap forms are frequently used as playthings. Besides these, fig. 9 shows a type of induction coil which is much used in laboratory work. It gives sparks 50 cm. long between SS, the ends of the secondary coil.

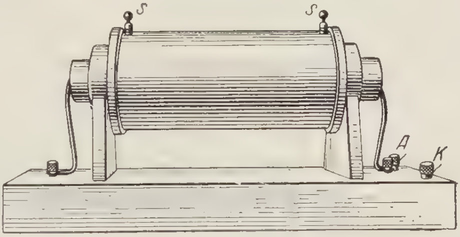


Fig. 9. Induction coil.

In ordinary transformers the periodic change in the magnetic flux is produced by an alternating current in the primary. In induction coils an intermittent direct current is used instead. Numerous automatic arrangements for interrupting the direct current have been devised. We shall describe three types. In the figures F always denotes the primary of the induction coil with its iron core.

(1) The hammer break shown in fig. 10. It works on the same principle as the hammer of an electric bell. The armature A is attracted towards the core and the current is broken at u.

(2) The jet break (fig. 11). A hollow vertical shaft A, the foot of which dips into mercury, is driven by a motor. As a result of a centrifugal effect a jet of mercury is ejected from a nozzle at the side. If the jet meets one of the zinc sheets marked a, the circuit is closed; if it comes into a gap, the current is broken. The whole piece of apparatus is enclosed in an iron case.

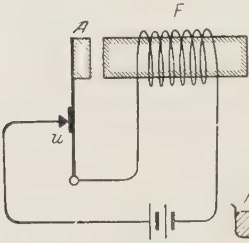


Fig. 10. Hammer break.

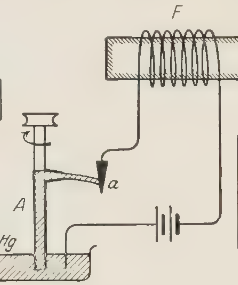


Fig. 11. Mercury break.

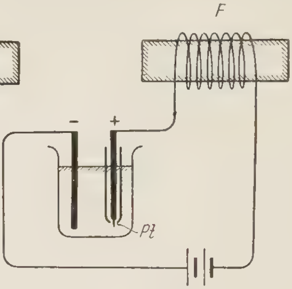
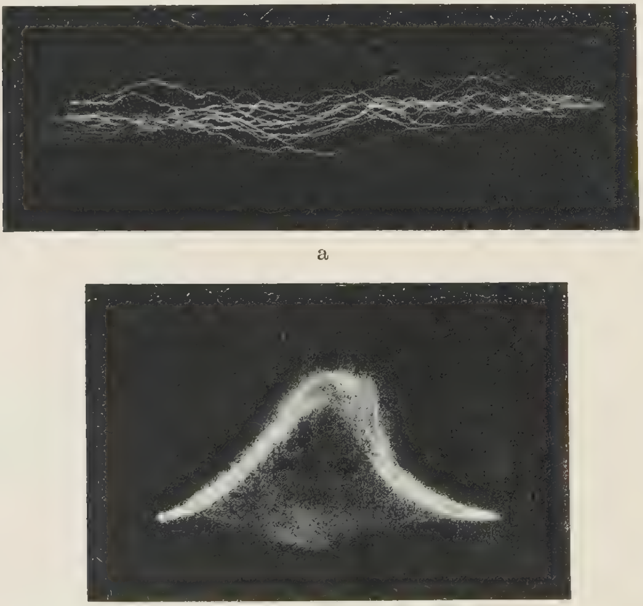


Fig. 12. Electrolytic break.



a

b

Fig. 13a, b. Bundles of sparks from a large induction coil.

(3) The electrolytic break (fig. 12). The primary current flows through a glass vessel full of dilute sulphuric acid. A sheet of lead

acts as negative electrode (leading the current in). The positive electrode consists of a platinum wire, about 1 mm. thick and 10 mm. long, protruding from a glass or porcelain tube. There is a vigorous evolution of gas, and the wire glows. A skin of gas is formed, which periodically bursts and interrupts the current. A discussion of the details would lead us too far out of our way at present.

Fig. 13 shows the sparks from a 40 cm. induction coil. These are photographs each with an exposure of 1 second. The picture *a* is obtained with the hammer or jet break, the picture *b* with the electrolytic break. With the latter several hundred interruptions per second can easily be obtained.

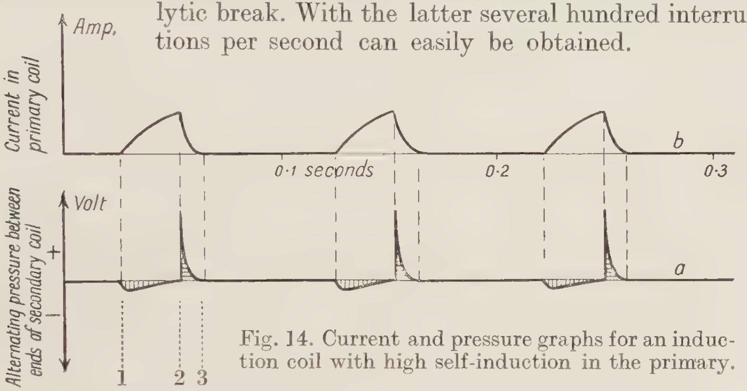


Fig. 14. Current and pressure graphs for an induction coil with high self-induction in the primary.

The secondary of an induction coil supplies a purely *alternating* pressure, and not (as might be thought) an intermittent direct pressure such as exists between the ends of the primary. The variation of the alternating pressure with time differs widely from the simple sine-wave (fig. 2a, p. 149). Fig. 14*a* gives an example for an induction coil of the type used by physiologists. The curve is obtained from a self-registering voltmeter. It is easy to show how its peculiar asymmetry arises. For this purpose we have drawn (at *b*) the graph of the current in the primary circuit, i. e. of the interrupted direct current; this is likewise taken from a photographic record. At the instant 1 the interrupter closes the primary circuit. The primary current rises slowly owing to self-induction or inertia. During the rise the magnetic flux increases and induces the pressures in the secondary circuit shown in the lower curve *a*. At the instant 2 the current is abruptly interrupted and by the instant 3 the magnetic flux has already vanished. The interval of time 2–3 is of course short, but it is finite. The reason is that a conducting arc is formed between the separated contacts every time a switch is opened, and the arc is only broken at the instant 3. All the same, the time 2–3 is much shorter than the time 1–2. Hence the pressures induced during the time 2–3 show sharp peaks. The horizontally and vertically shaded areas of the graph of the alternating pressure are equal. The same number of volt-seconds is induced by the same magnetic flux, first when it appears and then when it disappears.

For numerous purposes, e. g. nerve stimulation and working X-ray tubes, a very high degree of asymmetry in the graph of the secondary pressure is required. We need high peaks of pressure when the primary circuit is broken and small pressures when it is closed. To obtain the first it is important to suppress the arc at the contact of the break. This is done by connecting a practical paper condenser C in parallel, as shown in fig. 15.

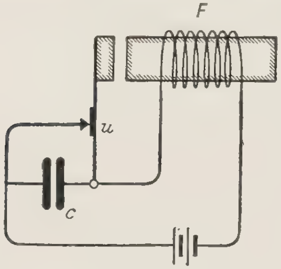


Fig. 15. Blow-out for the arc of a hammer break.

Explanation: the circuit in heavy lines is an oscillatory circuit (see next section), and the alternating current in it overlaps the direct current in the arc. As soon as the resultant current passes through the value zero the arc goes out.

How greatly the shape of the graph of the pressure in the secondary can be altered by self-induction is shown by fig. 16. Here the coefficient of self-induction of the primary is adjusted to be small and the pressure in the secondary is almost symmetrical. These symmetric alternating currents have physiological effects quite different from those of markedly asymmetric alternating currents. Hence results of experiments in which induction coils of differing origin are used cannot be immediately compared. Each observer must give an accurate description of the alternating pressure graph of his own apparatus.

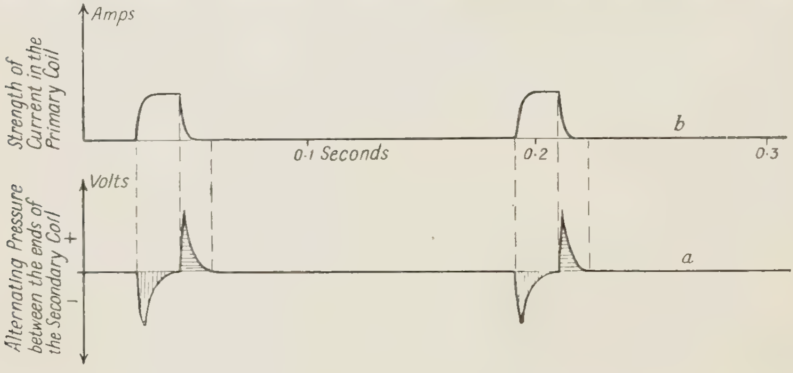


Fig. 16. Current and pressure graphs for an induction coil with low self-induction in the primary.

3. Electrical Oscillations.

The most important sources of alternating current which we have so far come across are generators, i. e. machines with revolving "armatures", usually consisting of rotating coils (fig. 1, p. 148). Some of the common forms in practical use have been illustrated in Chapter VI. In the course of the last few decades another method of producing alternating currents has become of permanent importance, namely the production of alternating currents by means of electrical oscillations.

It is very easy to get a general idea of how electrical oscillations arise, provided we avoid the usual starting-point of the discussion, namely the electric spark, as being an unessential phenomenon of minor importance. The really essential fact connected with the occurrence of electrical oscillations is the inertia of the magnetic field.

Let us briefly recall how mechanical oscillations arise. A very simple mechanical pendulum*, consisting of a sphere in the middle of a spiral spring, is shown in fig. 17a.

In 1 the pendulum is at rest. In 2 the spring is stretched by the hand or otherwise, and possesses potential energy. In 3 the sphere is let go and has just reached its equilibrium position. But there is a great difference between 1 and 3. In 1 the sphere is at rest; in 3 it possesses kinetic energy. It reaches the equilibrium position with the maximum velocity produced by the spring. Now comes the essential point. Owing to its inertia the sphere flies to the right past the equilibrium position. It stretches the spring and reaches the position 4. The process now repeats itself in the reverse direction; the sphere flies to the left past the equilibrium position, and so on. We see that there is a continual exchange going on between the potential energy of the spring and the kinetic energy of the sphere. The inertia of the spherical mass is essential for the production of oscillations. The oscillations are damped and die away; the amplitudes become smaller and smaller at each deflection, because the energy is gradually consumed by friction (cf. fig. 18).

We shall now consider an electrical pendulum or an *oscillatory electrical circuit* (fig. 17b) in a similar way. Instead of the spring we have a condenser, instead of the sphere a coil. In 1 there is equilibrium; the condenser is not charged, there is no current in the coil, and hence no magnetic field either. In 2 the condenser has been charged; its pressure is indicated by the continuous lines of force. In 3 the condenser and the coil are connected to form a circuit. The condenser has just discharged itself and its lines of electric force have disappeared. A current flows in the coil; its lines of magnetic force are shown by continuous lines. Now comes the essential point. The current and the magnetic field possess inertia. The current continues to flow in the direction of the arrow although the condenser in 3 has zero pressure, i. e. no longer acts as a source of current. The current flows on until the state 4 is reached; the condenser again contains an electric field but this time in the direction opposite to that in case 2. In 5 the process has gone on further in the reverse direction; the condenser again possesses no electric energy, but the coil contains a magnetic field, and so on. There is a continual exchange going on between the electrical energy of the condenser $\left(\frac{1}{2} C E^2\right)$ and the magnetic energy of the coil $\left(\frac{1}{2} L i^2\right)$. *An alternating current must flow in the circuit*, and this will be recorded by a moving-coil ammeter if the

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* In this book the word "pendulum" is always used in the sense of a "vibrating mechanical system", and not in the special sense of the "gravity pendulum".

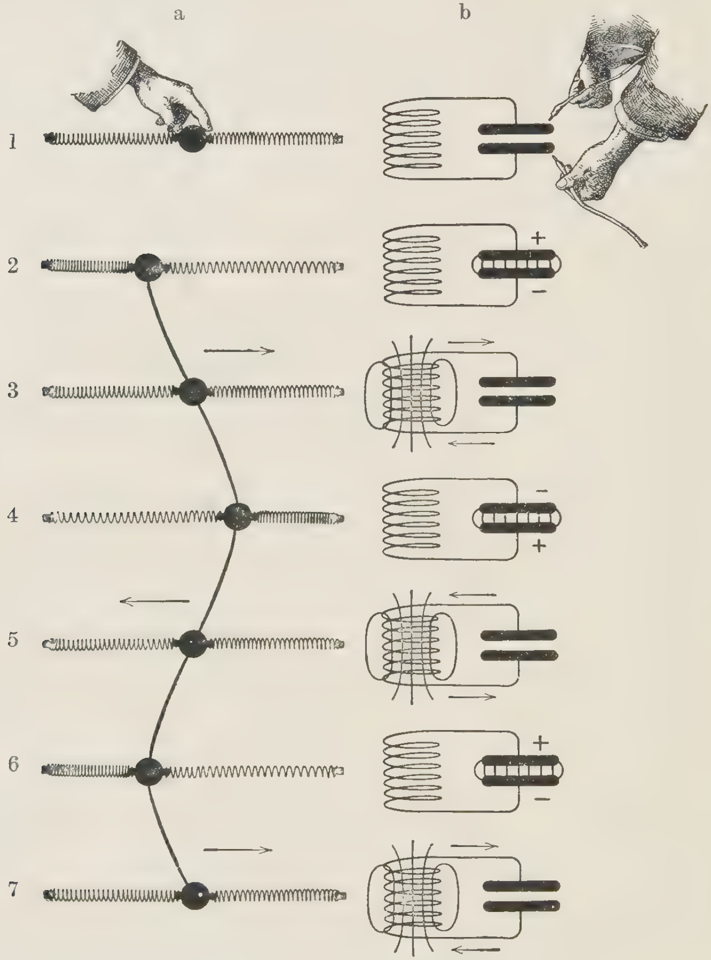


Fig. 17a, b. How mechanical and electrical oscillations arise.

frequency of the alternating current is small enough. Now this can be carried out experimentally even with very modest apparatus.

In fig. 19, *C* is a practical paper condenser whose capacity amounts to 5.10^{-5} coulomb/volt or farad. *L* is a coil with a closed iron core and a fairly large coefficient of self-induction, say 2000 volt-seconds/ampere or henries. *A* is a moving-coil ammeter which takes several tenths of a second to come to rest.

In order to carry out the experiment, we bring the two leads to the condenser momentarily into contact with the 220-volt poles of the town mains, and thus produce the state 2 in fig. 17 b. We then connect the leads with the ends of the large coil; the pointer of the ammeter immediately begins to oscillate backwards and forwards, say once in two seconds. We are observing an alternating current of frequency $n = 0{\cdot}5$ cycles per second. The only difference between it and the alternating currents produced by machinery is that the amplitude decreases fairly rapidly, somewhat as shown in fig. 18. The oscillations are strongly damped. The electrical energy originally stored up in the condenser is rapidly consumed in heating the turns of the coil.

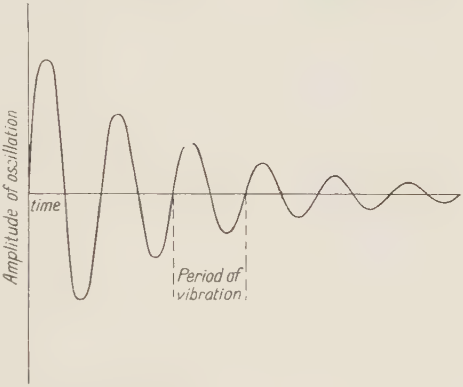


Fig. 18. Graph of a damped oscillation.

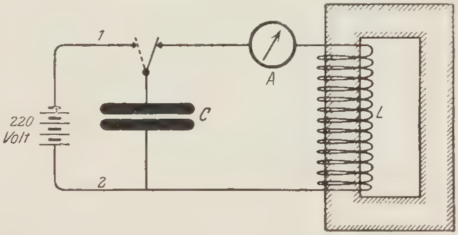


Fig. 19. Observation of the alternating current in very slow electrical oscillations by means of a moving-coil ammeter. The coil is of the type shown in figs. 3, 4, pp. 169, 171. It is convenient to include a 220-volt incandescent lamp in the lead 1, to act as an indicator of the current which serves to charge the condenser *C*.

A mechanical pendulum has the period

$$\tau = 2\pi\sqrt{\frac{M}{D}}, \qquad (2)$$

where *M* is the inert mass (in fig. 17 a the mass of the sphere) and *D* is the tension when the increase of length is 1 cm. In the case of electrical oscillations the place of the mass *M* as a measure of the inertia is taken by *L*, the coefficient of self-induction. Instead of *D* we have the pressure produced in the condenser when the quantity of electricity is 1 coulomb, i. e. E/Q. But by equation (1), p. 37, this quotient is equal to $1/C$. In the electrical pendulum we have to substitute the factor $1/C$ for D. We then obtain

$$\tau = 2\pi\sqrt{LC} \tag{3}$$

for the *period of the oscillating electrical circuit*, or the *period of the alternating current.*

To verify this equation we first reduce the capacity of the condenser from 5.10^{-5} to 5.10^{-6} farad. Experiment actually gives a period about three times as short $(\sqrt{10} \sim 3)$. We then diminish L also. We replace the 2000-henry coil by one of only 30 henries. The moving-coil ammeter is too sluggish to show the frequency once it has been raised in this way. The pointer no longer oscillates. We resort to the bifilar voltmeter as being an instrument of smaller inertia, i. e. we observe the alternating pressure instead of the alternating current for a change (cf. the diagram of the connections in fig. 20).

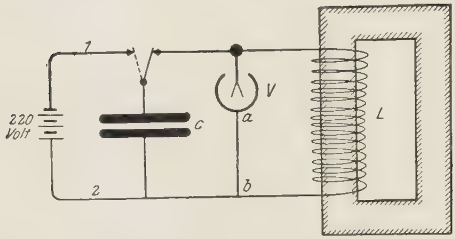


Fig. 20. Observation of the alternating pressure in slow electrical oscillations.

Between a and b there is a battery (not shown in the figure) which maintains a permanent pressure between the threads and the case of the voltmeter. This permanent pressure is alternately raised and lowered by the alternating pressure of the oscillations, i. e. the deflection of the voltmeter depends on the sign of the latter pressure at the instant in question.

When the charged condenser is connected with the coil the threads of the voltmeter vibrate rapidly, but still visibly, backwards and forwards.

When L and C are diminished still further, two things happen: (1) the frequency of the alternating current becomes so great that no instrument whatever with a mechanical pointer can follow the oscillations; (2) the energy $W_e = \frac{1}{2}CE^2$ originally supplied to the condenser becomes very small, unless the condenser is charged to a high pressure E.

The use of high pressures involves a troublesome drawback. We require some kind of switch to connect the charged condenser with the coil and set the oscillations going. But even before the switch contacts touch a spark jumps from one to the other. The circuit is not wholly metallic but is completed by a spark. *This spark sensibly disturbs the oscillations; it damps them by transforming their energy into heat and sound.*

If we are obliged to put up with these disturbing sparks, we can, however, put them to use, and that in two ways:

(1) *The spark may serve as a periodically-working automatic switch.*
(2) *The spark can be used as an inertia-free ammeter.*

The spark acts as an automatic switch e. g. in the arrangement of fig. 21. Instead of a movable contact and a fixed one we have a "spark gap" formed from two metal spheres. The leads 1 and 2 merely serve to charge the condenser from some source of current or other, e. g. an induction machine. When the pressure reaches a certain value a spark jumps, completing the coil circuit. The pressure E at which we wish to work can be adjusted by altering the distance between the spheres.

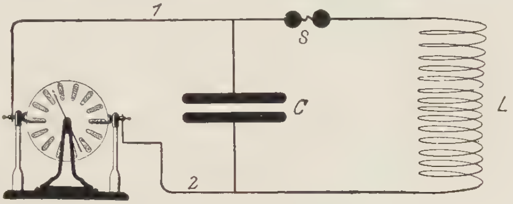


Fig. 21. A spark gap S acting as a switch in an oscillatory electric circuit.

The spark acts as an inertia-free ammeter in that its brightness varies with the strength of the current. There are two maxima in the brightness in every period. To enable the eye to recognize the variations in brightness, we have only to separate in space the images of the sparks which follow each other in time. This can easily be done by means of a rapidly rotating polygonal mirror. Fig. 22 is from a photograph of these spark images. The frequency was 50000 cycles per second. At the beginning the periodic variations in brightness are easily seen. Further on the picture is made indistinct by clouds of glowing metal vapour. At the beginning even the direction of the current at the separate maxima can be recognized. The bright end of the spark always indicates the negative pole.

Fig. 22. Demonstration of electric oscillations by means of a spark ("Feddersen sparks". Photograph by B. Walter).

4. Some Applications of Electric Oscillations.

The alternating current of the town mains usually has a frequency of 50 cycles per second. Using electrical oscillations we can produce alternating currents of a frequency 1000 times as great by very simple means. By reducing the capacity and self-induction still further the frequency may be multiplied by powers of ten. For our present purposes, however, frequencies of several hundreds of thousands per second will suffice. We can once again illustrate three phenomena, whose nature we have already investigated, by means of some very striking experiments with these high frequency alternating currents. The experiments have to do with

(1) electromagnetic induction;
(2) self-induction;
(3) displacement currents.

We shall also go on to use high frequency alternating currents to exhibit two phenomena which we have not previously considered in connection with low frequency alternating current. These are

(4) the self-induction phenomenon known as the "skin effect";
(5) the phenomenon of endless ring-shaped lines of electric force, i. e. an extension of what we described on pp. 113—115 and of fig. 13, p. 113.

(1) *Induction with high frequencies.* In the case of high frequency alternating current the magnetic flux of a coil alters not only very suddenly but also very frequently. Hence very rough apparatus will suffice to demonstrate the inductive effect of this magnetic flux on another coil. In order to do this we have a primary coil F and a secondary coil J (fig. 23). Their dimensions and the numbers of turns in them are given. We first send an ordinary alternating current of a few amperes through the primary, after connecting a small 2-volt lamp to the secondary coil J to act as a sensitive pressure-indicator. The lamp fails to light up, i. e. the induced pressure is less than 2 volts. The lamp still remains dark even if we raise the magnetic flux by inserting a thick iron core into the coils.

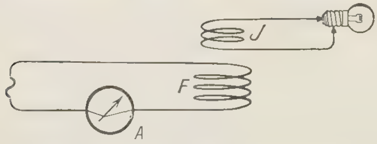


Fig. 23. Induction with low frequency alternating current. F has 3 turns about 30 cm. in diameter. J has 10 turns about 10 cm. in diameter.

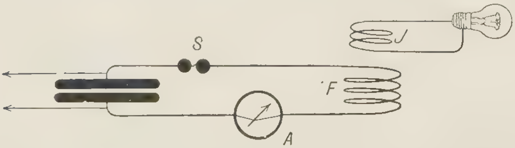


Fig. 24. Induction with high frequency alternating current. The leads marked with arrows serve to charge the condenser. S is a spark gap.

Things are quite different in the case of high frequency alternating current. We make the primary F in fig. 24 into a complete oscillatory circuit. The air gap which acts as a switch is adjusted so that the hot-wire ammeter A again registers a few amperes. Now even a 220-volt lamp can be used to indicate the pressure induced in J. It is very instructive to make oneself thoroughly familiar once more with the phenomena of electromagnetic induction, using the movable secondary coil and this rough indicator. The lamp shines less brightly when the secondary coil is brought into the region of smaller magnetic flux or is tilted relative to its position in fig. 24. If it is tilted through an angle of 90° the lamp goes out, because the coil is no longer penetrated by the magnetic flux; and so on for other positions of the lamp.

If we raise the strength of the current in the primary coil F a still more primitive indicator will suffice. For our secondary coil we take a closed ring-shaped iron wire with a handle which does not conduct heat. In the regions of high magnetic flux the whole wire becomes incandescent. Here we have a "low-pressure transformer for high frequency". Finally we take a simple plate instead of the secondary coil. It is made incandescent by the induced "eddy currents" (p. 158). This is used in practice for heating or melting pieces of metal in vacuum tubes.

If we make the secondary coil J have a large number of turns, say several thousand, the pressure between its ends reaches a correspondingly high value. Long bluish-red sheaves of sparks jump between the ends of the coil (fig. 25). One end of the secondary coil is often connected to earth (by means of a water-pipe or the like). Reddish bundles of sparks, which are often a metre long and much branched, then dart vigorously from the free end.

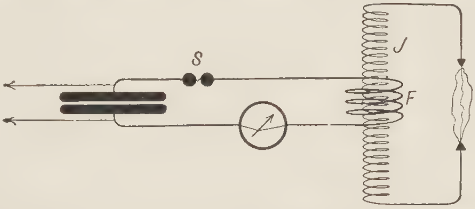


Fig. 25. Induction with high frequency alternating current in a coil. J has a large number of turns ("Tesla transformer").

The magnitude of the pressures which occur in this case does not entirely depend on the transformer ratio n_s/n_p. Here we are tacitly using the artifice of resonance, which we shall not discuss in detail until p. 309.

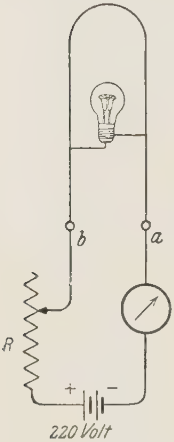
(2) *Self-induction with high frequencies.* On pp. 168—175 we recognized self-induction as the inertia of the magnetic field. This inertia opposes the formation of an alternating current in a coil. The ratio E/i, which we originally defined as the resistance, is much higher for alternating current than for direct current. For alternating current of frequency n we have the equation

$$\frac{E}{i} = \sqrt{r^2 + (2\,n\,\pi\,L)^2}. \tag{1}$$

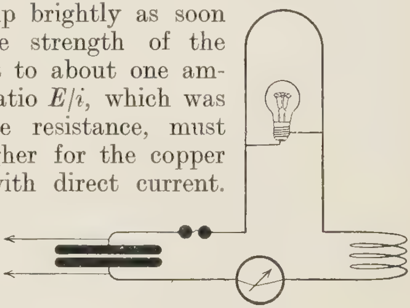
We used a coil of 300 turns and ordinary alternating current with a frequency of 50 cycles per second when we were verifying this equation previously. We also increased the magnetic flux by means of an iron core.

We can proceed in a simpler way with high frequency alternating current. A coil of many turns is no longer required. A single turn or a simple bent piece of wire will suffice. That is, we can clearly show that *not only a coil but any arbitrary conductor possesses a coefficient of self-induction L.*

Fig. 26a shows a bent piece of copper wire, 5 mm. thick, with its ends connected to a 220-volt lamp. We let a direct current of a few amperes flow in at *a* and out at *b*: the lamp remains quite dark, for it is short-circuited by the copper loop. (The ohmic resistance of the latter only amounts to about 10^{-3} ohm, i. e. is vanishingly small compared with that of the lamp, which is about 2000 ohms.) Things are quite different with high frequency alternating current. We include the loop and the lamp in the oscillatory circuit which we have already used several times (fig. 26b). The lamp lights up brightly as soon as we adjust the strength of the alternating current to about one ampere. Hence the ratio E/i, which was defined to be the resistance, must now be much higher for the copper loop than it is with direct current. This simple experiment demonstrates the inertia of the magnetic field in a crude way. There is no new principle involved, but the experiment is important, because the beginner is apt to leave self-induction out of account in conductors which are not coiled up.



a



b

Fig. 26a, b. Apparent resistance with high frequency alternating current.

(3) *The displacement current with high frequencies.* By equation (6a), p. 121, the intensity of the displacement current in amperes is given by

$$i_d = KA \cdot \dot{\mathbf{R}}\,. \tag{6a}$$

In this equation, K, the absolute dielectric constant, is — unfortunately — of the order of 10^{-13}. That is, we can obtain a displacement current of an intensity which is easily measurable only by giving high values either to A, the cross-section of the path of the current, or to $\dot{\mathbf{R}}$, the rate of change of the electric field. The latter can be done in two ways, namely by raising $\mathbf{R}$, the electric intensity, or n, the frequency of the alternating current.

In the case of the intensity $\mathbf{R}$ we have already reached the limits that are convenient in practice, namely 10^5 volts/cm. (fig. 18, p. 117). Even our ordinary alternating current (frequency 50 cycles per second, or number of reversals 100 per second) gives a value of the order of 10^7 for $\dot{\mathbf{R}}$ (the change in the intensity per second). We should then have to make A, the cross-section of the current, of the order of 10^6 sq.cm. (= 100 sq.m.) (and that in a practical paper condenser!) in order to obtain displacement currents of the order of one ampere.

The great disadvantage of the experiment of fig. 18, *p.* 117, *is the minute length of the lines of electric force*, which only comes to $^1/_{50}$ mm. (the thickness of the insulating paper in the condenser). *Now that we have high frequency alternating currents at our disposal, we can manage to work with fields of much lower intensity, and hence we can produce convenient displacement currents even when the lines of electric force are long.* This is actually done in a way to which we shall refer later in connection with electric waves, where we shall have to demonstrate the existence of displacement currents with very long lines of force.

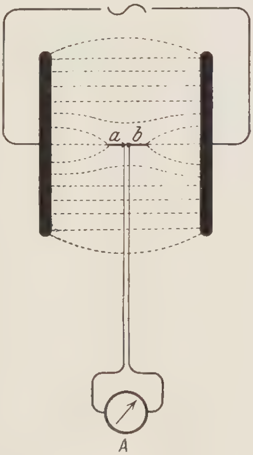


Fig. 27. Displacement current in a high frequency alternating electric field. ~ denotes the source of alternating current (the oscillatory circuit).

Fig. 27 shows our old friend the plate condenser, with the plates about 20 cm. apart. We connect it with a high frequency oscillatory circuit, which acts as a source of alternating current (~) with a frequency of about 10^6 cycles per second. There is a pressure of about 10^5 volts between the plates, and by equation 5, p. 69, the intensity of the field is approximately given by $\mathbf{R} = E/d = 5000$ volts/cm.; i. e. $\dot{\mathbf{R}}$ is of the order 10^{10}. Hence by equation (6a) above we have a displacement current of one milliampere, even with a cross-section of only one square centimetre.

In order to demonstrate the existence of this displacement current we proceed to replace a small part of the lines of force by a metal bar ab 3 cm. long. The field breaks down in this conducting wire and a conduction current flows in it, a high frequency alternating current, in fact. We have to exhibit this alternating current. The best way would be to include a Liliputian alternating current ammeter in the middle of the wire. In default of any such apparatus we divide the wire in the middle and insert a small rectifier (crystal detector) in the gap. Two fine metal wires very close to each other lead from the detector to a direct current ammeter A outside the field. The wires are placed perpendicularly to the lines of force so as not to distort the field. The rectifier and direct current galvanometer merely form a practical substitute for an alternating current ammeter of the required sensitivity. We have used this device before (fig. 16, p. 154).

So — this is the essential point of the experiment — an alternating current flows in the short straight wire with free ends *ab*. The occurrence of this current would puzzle us completely if we were not familiar with the idea of displacement current. A few dotted lines of force, whose variation in time we call the displacement current, at once give us a clear picture of what is happening. It shows that the current does not come to a cul de sac at the free ends of the bar *ab*, but proceeds from there to the boundaries of the field as a displacement current.

It follows that the experiment of fig. 27 is intended to show two things:

(1) A displacement current can bridge even long stretches of lines of force.

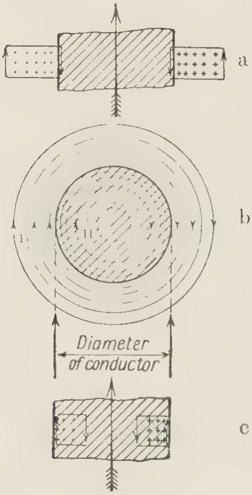


Fig. 28a—c. Magnetic lines of force in the neighbourhood and interior of a wire (shown shaded), and their inductive effect.

(2) Its existence can be proved by destroying its electric field by a metallic wire and producing a conduction current in the wire.

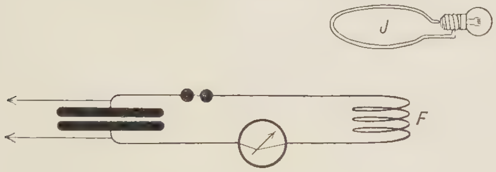
Such a metal wire as e. g. *ab* in fig. 27, will in future be called a *receiver* for short.

(4) *The skin effect.* Fig. 28b shows the cross-section of a straight conductor (shaded) carrying a current. As we know, the conductor is surrounded by closed ring-shaped lines of magnetic force **H**. In fig. 28a a short piece of the same conductor is shown in longitudinal section. The direction of the conduction current, i. e. the direction in which the electrons are moving, is marked by a feathered arrow. The lines of magnetic force encircling the outside of the conductor are visible only at the points (· or +) where they meet the plane of the section. The variation of these magnetic lines of force as time goes on induces closed lines of electric force. Two of these are diagrammatically shown as rectangles, for the case where the conduction current is *increasing*; in the wire the electric lines of force are in the direction opposite to the current. They retard the growth of the current. This is the phenomenon which we know as self-induction. Here we have merely explained the self-induction phenomenon for a straight conductor. We now see why equations (1) or (1a), p. 174, must be used instead of Ohm's law [equation (1), p. 16] even in the case of straight wires conveying alternating current.

Now, however, we come to something new:

The magnetic lines of force do not only encircle a conductor on the outside but they also exist in its interior. For we may imagine that the wire is built up of successive tubes, each surrounding the next. Current flows through the walls of each tube, and no current exists which is not encircled by lines of magnetic force. Some of the latter are drawn in fig. 28b.

Fig. 28c again represents a short piece of the conductor in longitudinal section. The direction of the current is marked by an arrow as before. Dots and crosses indicate the points where the magnetic lines of force in the interior of the conductor meet the plane of the section. When these lines of magnetic force change as time goes on, closed lines of electric force are induced. Two of the latter are drawn diagrammatically as small rectangles. The direction of these induced lines of electric force is opposed to that of the original field along the axis of the coil, and hence the field along the axis of the coil is weakened. In the neighbourhood of the surface of the coil, on the other hand, the induced and original fields are in the same direction, and the current is strengthened there. Hence the self-induction phenomenon in the interior of the conductor has the effect of shifting the current to just under the surface of the conductor.



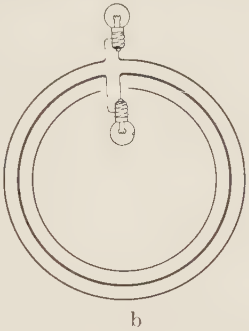


Fig. 29a, b. Skin effect with high frequency alternating current. The leads shown by arrows serve to charge the condenser.

In the case of slow changes, say ordinary alternating current, this shift of the current can only be detected by refined methods of observation. When the frequency is high, however, it makes itself noticeable on a large scale. The current no longer flows anything like uniformly through the cross-section of the conductor; on the contrary, it is concentrated in a thin superficial layer or skin.

In order to demonstrate this shifting of the current, or skin effect, we use the apparatus sketched in fig. 29a. F, the coil of an oscillatory circuit, supplies an alternating field of high frequency. This induces currents in the secondary coil J, which takes the form of a thick ring of copper wire. A lamp included in the secondary circuit serves as a measure of the intensity of the current. We then surround the copper ring with a concentric copper tube having a bore of a few centimetres (cf. fig. 29b). The walls of the tube have the same copper cross-section as the wire. A lamp of the same kind as that in the copper ring is connected between the ends of the tube. We now make the two secondary coils, one inside the other, approach the primary coil F, as in fig. 29a. The lamp between the ends of the tube shines brilliantly, whereas that between the ends of the ring only glows redly or not at all.

In modern high frequency technique this shifting of the current, or skin effect, is extensively taken into account. Instead of solid wires, thin-walled tubes are used, often with a superficial coating of silver, which is a particularly good conductor; or attempts are made to reduce the shifting of the current by dividing the wires into strands composed of many very fine single wires separated from each other by insulating varnish. In thin wires the influence of the shift of the current is proportionally smaller. Further, an even loading of all the single wires with current can be obtained by the device of twisting or plaiting the wire. In this way every single wire is made to pass through the centre of the bundle of wires to just as great an extent as it does through the superficial layers.

(5) *The demonstration of closed lines of electric force.* According to our deeper interpretation of the phenomenon of induction, closed ring-shaped lines of electric force must exist (p. 114). On the previous occasion, however, the electric intensities were too small for the closed lines of force to be indicated by means of gypsum crystals. The high frequency alternating currents derived from electrical oscillations now enable us to make good this deficiency and render closed ring-shaped lines of electric force clearly visible.

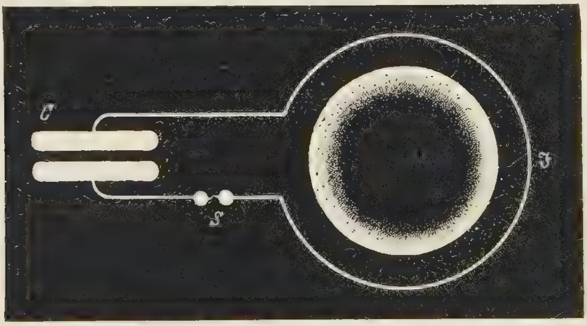


Fig. 30. Demonstration of closed lines of electric force ("Ring current without electrodes").

The intensities which are now within our reach are of course insufficient to arrange crystals of gypsum in order, but they enable us to demonstrate the existence of closed lines of force in almost as clear a way. The method depends on the fact that inert gases, such as neon, light up even with relatively low electric intensities: intensities of about 20 volts per centimetre are sufficient.

The mechanism of this phenomenon does not concern us at present: it will be touched on briefly later on pp. 215—218. A detailed treatment will be found in textbooks on electro-optics. In principle it is the same as the lighting-up of the air of the room shown in the electric spark when the intensity of the field exceeds 30000 volts/cm.

The arrangement of the apparatus is shown in fig. 30. The primary coil F, say of one turn, provides an alternating field of high frequency. Its lines of magnetic force are at right angles to the plane of the paper. These rapidly alternating lines of magnetic force should, according to the hypothesis illustrated in fig. 13, p. 113, be encircled by endless lines of electric force.

We now bring a glass sphere filled with neon at reduced pressure into the region of these closed lines of electric force: a ring-shaped region of the sphere lights up so as to be visible at a distance. We have a picture, though a rough one, of the alternating electric field with its closed lines of electric force without beginning or end. Without the knowledge that closed lines of electric force exist we should later find it quite impossible to understand electric waves, i. e. electromagnetic radiation. The present experiment is therefore intended to supplement our intuitive ideas on the subject.

CHAPTER VIII

The Mechanism of Conduction Currents

1. The Mechanism of Conduction in a Model Experiment.

So far we have always regarded electric currents in conductors as a motion of atoms of electricity. We have purposely refrained from any more detailed statement; it still remains an open question whether negative atoms of electricity alone move in *one* direction, or positive alone in the other, or both at the same time. Data about the velocity of motion are likewise wanting. We shall now give a comprehensive discussion of all these questions. At the same time we shall draw important conclusions about the real nature of atoms of electricity.

We have observed the conduction phenomenon experimentally in two different cases.

(a) *In the decay of the field as in fig.* 1. The plates of a condenser are connected by the conducting body (shown shaded). This is the fundamental experiment described on p. 29. The fall of the pressure is observed by means of a voltmeter.

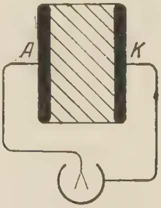


Fig. 1. Demonstration of conduction in the decay of a field.

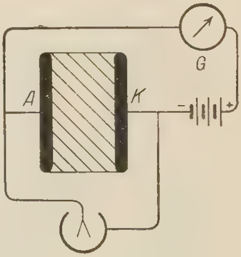


Fig. 2. Demonstration of conduction with permanent currents.

(b) *In permanent currents as in fig.* 2. Here too the plates of the condenser are continually losing atoms of electricity as long as they are connected by the conducting body. But the loss is continually replaced to such an extent by a *source of current* (e. g. a battery, dynamo, or induction machine) that the field and the pressure are *maintained*. The ammeter records the current required to replace the atoms of electricity.

In both cases the condenser plates or boundaries of the field are frequently called the *electrodes*. The positive electrode is often called the anode, the negative one the cathode. These terms have been adopted for conduction phenomena of all kinds.

In both arrangements it is easy to demonstrate the two simple fundamental processes involved in all cases of electrical conduction. They are as follows:

I. Atoms of electricity must come out of one electrode and pass over to the other.

II. In the interior of a conductor there are equal quantities of atoms of electricity of opposite sign and arbitrary origin. The positive atoms reach the negative electrode and the negative atoms reach the positive electrode in some way or other.

Both phenomena are most simply realized in practice by carrying the atoms of electricity on some kind of "electricity carrier". The large-scale way of doing this is to use our old friend the proof-plane (fig. 13, p. 28). With its assistance we shall carry out model experiments on the two phenomena.

For I we move a proof-plane backwards and forwards from one electrode to the other (in the region shaded in figs. 1, 2). In the circuit of fig. 1 the pointer of the voltmeter sinks and the field decays. In the circuit of fig. 2 the voltmeter deflection remains constant. The ammeter (mirror galvanometer) G shows a permanent deflection, provided we do not move the carrier too slowly. The condenser no longer "insulates". The air in it "conducts", because a single large-scale electricity carrier is moving backwards and forwards through the swarm of invisible molecules. In fig. 1 this motion means a current diminishing in time, in fig. 2 a current constant in time.

We can immediately give the intensity of the latter in amperes. Let the carrier contain q ampere-seconds of electricity and let it traverse d, the thickness of the condenser, n times per second. Then

$$i = q n \text{ amperes}. \tag{1}$$

This equation can be slightly changed by introducing u, the velocity of the carrier. The time taken by the carrier to traverse the whole distance d is $t = 1/n$ sec. Moreover, $t = d/u$, so that

$$i = q\, u/d. \tag{1a}$$

If there are many carriers, q denotes their total charge. Or in words: equation (1a) gives the intensity of the current in amperes when the quantity of electricity q moves with velocity u along the lines of force of a plate condenser of thickness d.

Numerical example: $n = 4$ times per second, q for a proof-plane $= 7{\cdot}5.10^{-10}$ coulombs (cf. p. 36). Then $i = 3.10^{-9}$ ampere.

For II we bring two proof-planes touching each other into the air gap, and separate them in the field. Then one proof-plane is positive and the other negative. That is, we have charged the carriers by induction; the origin of the carriers and their charges is of course of no importance.

We move the positive carrier to the negative electrode and the negative carrier to the positive electrode, and so on with other pairs of conductors as often as we like. We again observe current intensities of about 10^{-9} ampere.

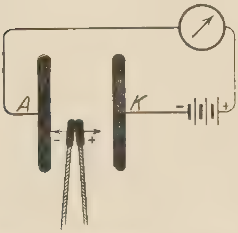


Fig. 3. Model experiment on the conduction current with large-scale electricity carriers worked by hand. Galvanometer as in figs. 16, p. 30, and 68, p. 66.

Equation (1a) holds unchanged: but u now means the sum of the velocities of the positive and negative carriers, i. e.

$$i = q \frac{(u_+ + u_-)}{d}. \tag{1b}$$

Instead of two carriers we can take a great number, say $\mathbf{N}$ per cubic centimetre; then there are $\mathbf{N}Ad$ electricity carriers in the space between the condenser plates or electrodes of area A. Let each one carry $\mathbf{n}$ atoms of electricity, i. e. a charge $\mathbf{n}e$ or $\mathbf{n}.1\cdot6.10^{-19}$ coulombs. Then q will be equal to $\mathbf{N}Ad\,(\mathbf{n}e)$, where A denotes the area of the condenser plates or electrodes. We observe the current

$$i = \mathbf{N}A\,(\mathbf{n}\,e)\,(u_+ + u_-). \tag{1c}$$

We shall soon return to this equation (see the next section).

These two model experiments go straight to the root of the matter. We must go into this more thoroughly in view of apparently important variations from them in practice.

(1) In the model experiments there are no charged conductors in the air originally. They are only brought in by us and charged by us. We may briefly speak of "dependent" conduction, in contradistinction to ordinary "independent" conduction, in which the carriers of electricity exist without our intervention. But this "dependence" (this is a technical expression) is common to our model experiments and numerous cases of electrical conduction in gaseous, liquid, and solid bodies. Many of these bodies do not conduct of themselves. They only become conducting when electricity carriers penetrate into them as a result of our intervention. The independent conduction exhibited by any metal wire is by no means the rule.

(2) In the model experiments the motion of the carriers can be seen with the naked eye. This is certainly rare in conductors, but it does occur.

(3) In conductors the carriers are caused to move *by the field*. The field exerts a force on them (see fig. 74, and equation 10, p. 73). In the model experiment the carriers receive their velocity independently of the field, from our hand in the examples selected.

Herein lies a real but by no means *essential* difference. The motion of the carriers due to the effect of the field can also be imitated in the model experiment. We have only to take carriers of smaller mass and get rid of gravity disturbances. This can easily be carried out in practice by means of soap-bubbles filled with a suitable gas. It is not worth while to do this, however, especially in view of the previous experiments of fig. 83, p. 83. Moreover, the model experiment in which the electricity carriers are moved by hand has a special advantage: it brings in the possibility of a new kind of conduction current, the so-called *convection current*, which so far we have not yet observed. The nature of the convection current is illustrated by means of fig. 4.

Two electrodes are connected by a conducting wire. The wire is connected with an ammeter but not with any source of current. Yet a strong current may flow through the ammeter; all that is required is that carriers of electricity should move between the plates in the direction of the arrow. *The origin of their velocity is quite arbitrary.* In the model experiment it is sufficient to move one or two proof-planes, *which have previously been charged in some way*, by hand (fig. 4). Other possibilities suggest themselves. The carriers may be shot into the space between the plates through a side hole *a*. We shall meet with this case later on in connection with cathode rays. Or we may place the long axis of the condenser vertically and use gravity to move the carriers. Such a case will occur on p. 265. *A convection current, then, is a conduction current between two electrodes in which the electricity carriers do not obtain their velocity from an electric field between those electrodes.*

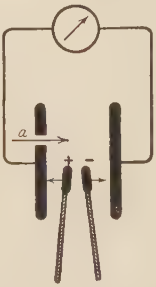


Fig. 4. Convection current. Galvanometer as in figs. 16, p. 30, and 68, p. 66.

The following erroneous idea often makes it more difficult for us to understand conduction phenomena: we expect a current to be recorded on the ammeter only when electricity carriers *reach* the electrodes. In actual fact the ammeter reacts to any motion that alters the distance of the electricity carriers from the electrodes. It reacts to every change of the electric field or every *displacement current*. This can be particularly easily shown by the model experiment of fig. 3 (by charging the carriers beforehand by means of an induction machine and then bringing them into the field without letting them touch each other). — Equations (1a—c) still hold.

So much for the model experiments. We now proceed to deal with individual cases of conduction phenomena in various bodies in detail. Our observations will enable us to draw conclusions about the nature, number, origin, and velocity of the electricity carriers.

2. Dependent Conduction in the Air of the Room with Visible Electricity Carriers. The Meaning of Ohm's Law.

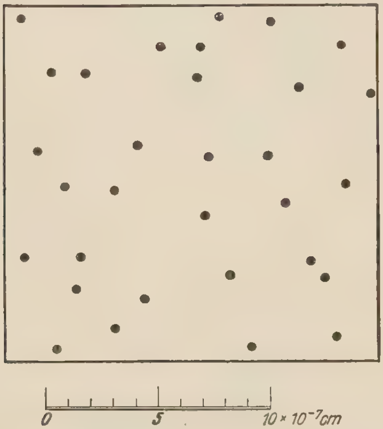


Fig. 5. Diagrammatic snapshot of the air of the room, magnified $3 \cdot 10^6$ times.

We begin with gases because we are particularly familiar with their atomic structure. Fig. 5 will serve to recall the most important facts briefly; it represents a snapshot of the air of the room, linearly magnified about 3000000 times. The molecules are shown by black dots. Their spherical shape is arbitrary and makes no difference to the discussion. The diameter of the molecules is about $3 \cdot 10^{-8}$ cm., and the average distance between them is about ten times as great. Between the molecules there is empty space. Fig. 6 similarly gives — again figuratively speaking — a time exposure lasting about 10^{-8} second. The trajectories of three molecules are drawn, this time, however, magnified only 100000 times. The straight pieces are the "mean free paths" between two collisions (about 10^{-5} cm.). Every break in the line corresponds to a collision with one of the molecules which are not shown in the figure. The velocity amounts on the average to about 500 metres per second at room temperature. 1 c. c. of the air of the room contains about $3 \cdot 10^{19}$ molecules. This struggling mob is penetrated by the electric field, as in fig. 59, p. 59. The electricity carriers have to work their way through this chaos; not exactly a simple task for carriers of small kinetic energy.

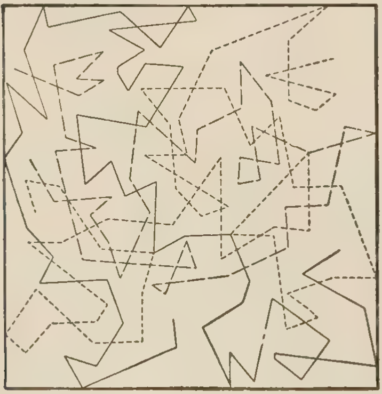


Fig. 6. Free paths of three gas molecules in the air of the room, magnified 10^5 times.

For electric fields of low intensity the air of the room is an excellent insulator. The field of a condenser in air lasts for hours. Air can conduct well dependently, however. We have only to see to the required stock of electricity carriers.

Our model experiments in last section, in which proof-planes passing between the molecules in the air were used as electricity carriers, were perhaps on too large a scale for the taste of many readers. We shall therefore repeat one of the two experiments in a more refined form, for it is essential that we should get a thoroughly clear conception of the real nature of dependent conduction. The experimental arrangement sketched in fig. 7 serves for this purpose. It is closely related to the apparatus of fig. 3, but a cylindrical condenser is used instead of a plate condenser. In practice it is convenient to let air stream through this condenser. Note the tapering funnel-shaped ends and the rubber-tube connections.

The essential feature of the experiment is the way in which the electricity carriers are supplied. They consist of electrically charged particles of dust; for example, the positive carriers may consist of wheat flour*, and the negative of flowers of sulphur. The electrical charges result from "frictional" electrification (p. 61). The two powders are driven through the metal nozzle (*D*) of an atomizer by means of a current of air. The particles become charged by contact with the walls and edge of the nozzle. The air streams through the condenser laden with particles of dust carrying electricity. The galvanometer indicates a surprisingly constant current of about 10^{-8} ampere. Reversing the direction of the field in the condenser reverses the direction of the current.

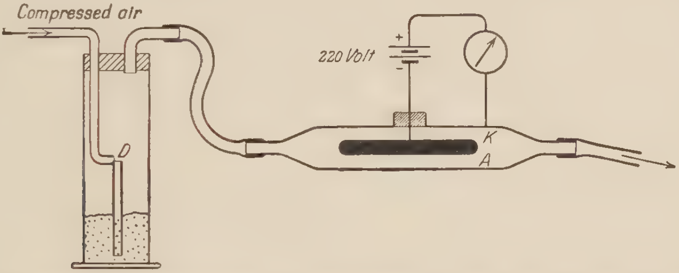


Fig. 7. Conduction of electricity by electrically charged dust particles. Galvanometer as in figs. 16, p. 30, and 68, p. 66.

We let the current flow for some minutes in the same direction, e. g. as in fig. 7. We then open the condenser. We find that the inner cylinder, the cathode, is quite uniformly covered with a thin coat of flour. Similarly, the inner surface of the outer cylinder, the anode, is coated with sulphur. The electricity carriers have given up their burdens, the atoms of electricity, and have remained sticking to the boundaries of the field or electrodes. Playfully speaking, we have an "electroplated" coat of flour or sulphur, as in nickel-plating etc. by electrolysis (cf. p. 242).

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* Red lead is another suitable substance.

By the giving up of the charge we do not mean quite the same thing in the case of negative carriers as in the case of positive carriers. The negative carrier gives up its superfluous electrons. The positive carrier completes its stock of electrons by robbing the metallic electrode of electrons (cf. pp. 246—250).

This is no model experiment. It is a genuine case of dependent conduction of electricity through air, and yet the carriers are still easily visible.

The experiment enables us to explain the mechanism of dependent conduction still further, with the help of an important experimental result. We repeat the experiment using lower pressures instead of 220 volts. We find that current and pressure are proportional to each other. *Ohm's law holds under these particular conditions. So far we know of Ohm's law only for metallic conductors at constant temperature* (p. 16). *Here we have found another particular case in which it holds.* This is important, for it leads us to form a very plausible idea of the *mechanism* which lies at the bottom of this simple law.

In order to make the geometrical relationships very simple, we imagine the experiment with dust particles acting as electricity carriers performed with the plate condenser of fig. 3. Of course the only object of using the cylindrical condenser in fig. 7 is to make it easier to introduce the dust-laden air without contamination.

The current produced by the moving dust carriers is

$$i = \mathbf{N} A (\mathbf{n}\, e)(u_+ + u_-) \tag{1c}$$

by equation (1c). (A = area of the condenser plates or electrodes, $\mathbf{N}$ = number of carriers per cubic centimetre, $(\mathbf{n} e)$ = charge on a single carrier, $(u_+ + u_-)$ = the sum of the velocities of the positive and negative carriers.)

Further, E/i is in general defined to be the resistance r. Hence we can write

$$\frac{E}{r} = \mathbf{N} A (\mathbf{n}\, e)(u_+ + u_-),$$

or, if we introduce the electric intensity $\mathbf{R} = E/d$,

$$r = \frac{d}{A} \frac{1}{\mathbf{N}(\mathbf{n}\, e)} \frac{\mathbf{R}}{u_+ + u_-}. \tag{2}$$

Now we are given the fact that Ohm's law holds, i. e. r turns out to be constant for the conduction of current by dust carriers. How can this be derived from equation (2)?

The simplest way is by making the variables satisfy the two following conditions:

(A) $\mathbf{N}(\mathbf{n} e)$, the total charge of the carriers in a cubic centimetre, is to remain constant; it is not to be altered by a noticeable fraction by the field or the motion of the carriers.

(B) $\left(\frac{u_+ + u_-}{\mathbf{R}}\right)$ is to remain constant; the velocity of the carriers is to be adjusted to a definite value u proportional to $\mathbf{R}$, the intensity of the field. We have $u = \mathbf{R}v$, where we shall call the factor of proportionality v the "mobility". *The mobility v is the velocity of the carriers in the field $\mathbf{R} = 1$ volt/cm.*

Satisfying condition (A) is a purely practical matter: we have only to blow the air containing carriers through the condenser in sufficient quantity and with a suitable velocity.

How about condition (B)? Can the carriers really move with constant velocity in spite of the fact that the electric field acts on them continually with the force $F = (\mathbf{n}e)\mathbf{R}$, by equation (10), p. 73? Further, can this constant velocity u be proportional to the field intensity $\mathbf{R}$? The answer to this double question is familiar to us from mechanics.

The carriers must move in a viscous medium with friction; the force called the friction must increase in proportion to the velocity. Imagine spheres of equal sizes but of different weights falling in glycerine. The velocity of fall is constant (after a short initial acceleration) and proportional to the weight of the sphere. There is internal friction even in air, just as in glycerine: think of the constant rate of fall in air of small bodies such as down feathers and, in particular, particles of dust.

What is true of bodies falling in a gravitational field is also true of the motion of our dust particles which carry electricity in the air-filled condenser. Here, however, the constant force is not the weight (which we may neglect) but the force $F = (\mathbf{n}e)\mathbf{R}$. The velocity of the dust carriers is *constant* in spite of the existence of the force F, and is proportional to F and hence also to $\mathbf{R}$. Condition (B) is fulfilled. The carriers possess a characteristic "mobility" v.

In view of future applications we shall immediately transform equation (1c), which forms the basis of conduction phenomena in general, for the particular case where the carriers have a definite mobility v [see under (B) above]. We obtain

$$i = \frac{A}{d}\mathbf{N}(\mathbf{n}\,e)(v_+ + v_-)E, \tag{3}$$

or for the ratio E/i, which is in general defined as the resistance,

$$r = \frac{d}{A}\frac{1}{\mathbf{N}(\mathbf{n}\,e)(v_+ + v_-)}. \tag{3a}$$

To summarize: the experiments of this section have led us to two important results:

(1) We have succeeded in producing a typical example of dependent conduction of electricity in the air of the room, even with electricity carriers which can easily be seen under the microscope.

(2) We observe that Ohm's law, the proportionality of current and pressure, holds in this case, provided the two conditions mentioned in (A) and (B) are fulfilled.

The experiments are supplemented in a very satisfactory way by direct microscopical observation. We see the carriers moving through the field in opposite directions. The reader should visualize the corresponding lines of force diagram in fig. 74, p. 73. The carriers have differing velocities, as the latter depend on the sizes and charges ($\mathbf{n}e$) of the carriers. u_+ and u_- in equation (1c) are of course only meant to be average values. The velocities of the carriers can be measured with a stop-watch and the fact that they are proportional to the intensity **R** established. The remarkably fascinating sight is best observed by each for himself; in projection, disturbances readily arise from the heating of the air by the lamp.

The case of dependent conduction discussed here is used on a very large scale in practice. Large masses of air are freed from injurious dust in this way. The condensers used are of gigantic dimensions.

In addition, the reader should not overlook the fact that a single dust carrier has been observed under the microscope. This is the classical experiment which enabled the charge e of the single atom of electricity to be determined as being equal to $1{\cdot}6 . 10^{-19}$ coulombs (see pp. 83—85).

3. Dependent Conduction in Air. Ions as Electricity Carriers.

In the preceding sections the charged particles of powder were electricity carriers of a size visible under the microscope. According to the conception of dependent conduction with which we are already becoming more familiar, the size of the carriers is a matter of very subsidiary importance. We shall not be surprised to learn that there also exist carriers far below the limits of the microscope; as the smallest electricity carriers in air we shall now have to consider the molecules of the air itself.

We are already familiar with molecules and atoms acting as carriers of electricity. We call them ions; negative ions when they have too many electrons, positive ions when they have too few electrons. We have only to complete our previous rough description by a note on "large" or "complex ions": these are molecules which have attracted a covering layer of neutral molecules, just as an electrical conductor becomes covered with dust, or, in general, they are forms which are intermediate in size between molecules and visible particles of dust.

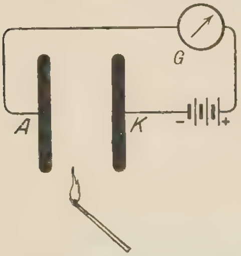

Fig. 8. A burning match acting as an ionizer. Galvanometer as in figs. 16, p. 30 and 68, p. 66.

How do we bring ions into a gas, or how do we ionize the molecules of a gas? There are many methods; we give three examples:

(1) Chemical reactions at high temperature. Fig. 8 shows a burning match acting as an ionizer.

(2) Röntgen- or X-rays.

(3) The rays given off by radioactive substances.

These ionizers enable us to gain a more thorough-going knowledge of dependent conduction in gases. The actual mechanism of the formation of ions is discussed on pp. 218—221.

To begin with, a simple experiment shows that the ions have a considerable length of life. In fig. 9 AK is an ordinary plate condenser connected with the ammeter G. To the right of it is placed a bunsen flame to act as ionizer. The ions formed rise straight up with the warm air, i. e. they do not reach the condenser; but they can, of course, be conducted to the place where they are to be measured by a lateral stream of air. This can easily be done by means of a small fan V worked by hand. It turns out that some ions still remain after traversing the distance between the flame and the condenser. Hence the life of an ion must amount to several tenths of a second.

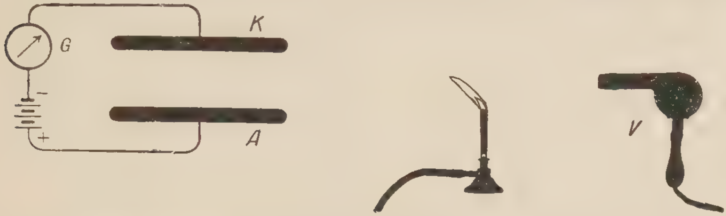

Fig. 9. The carrying along of the ions formed by a bunsen flame by means of the current of air from a fan. Galvanometer as in figs. 16, p. 30, and 68, p. 66.

In view of the mutual attraction between ions of opposite signs this result is very surprising. Yet the lines of force diagram of fig. 7, p. 24, and fig. 79, p. 78 is true for every pair of ions, and this must result in the combination of the opposite charges. Or, as it is somewhat inappropriately called, the "recombination": for it is by no means necessary that the partners which were originally separated should recombine.

The "recombination" can be followed out by measurement, for which the apparatus represented in fig. 10 may be used. Three small cylindrical condensers have their hollow outer cylinder in common. The inner cylinders are three separate wires insulated by amber. Each of them can be connected to the bifilar voltmeter; in fig. 10 it is the lowest that is so connected. Under the hollow cylinder, which is placed vertically, there is a small gas flame acting as an ionizer. The hot gases rise up through the "chimney".

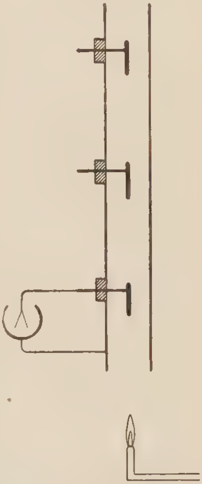

Fig. 10. Length of life and recombination of ions.

We produce a field in the lowest of the three condensers by connecting it momentarily with the town mains. The field breaks down within a second. In the middle condenser a field lasts for several seconds and in the upper one for about half a minute. From the upward velocity of the warm air and the distance between the separate condensers it can be calculated that in less than $^1/_{10}$ second the majority of the ions are lost byre combination.

Those that reach the upper end of the chimney in fig. 10 are practically nothing but bulky complex ions. Without troubling ourselves further about them, we may regard them as products of chemical reaction. Owing to their large size, they are less mobile than the others. Hence a residue composed of them can only arrange itself in pairs very slowly when the stock of small nimble ions is exhausted.

The slow-moving complex ions from flame gases may occasionally be a nuisance. A single bunsen burning in a room may disturb fine measurements. There is a simple means of getting rid of them, however, namely an electric field. This is shown by the experiment represented in fig. 11. An electric field is produced in the lower condenser, say by connecting it with the town mains. The middle and upper condensers now no longer receive any ions from the flame.

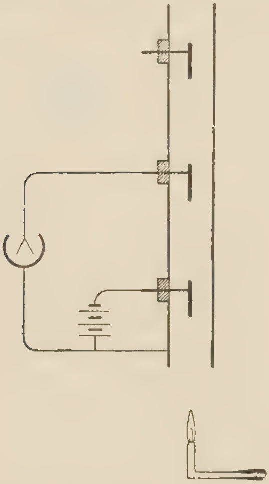

Fig. 11. Removal of ions by an electric field.

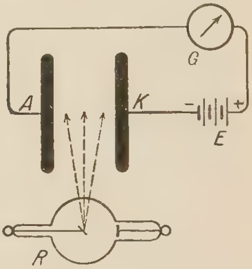

Fig. 12. Ionization of the air of the room by X-rays. Galvanometer of the type in fig. 16, p. 30, and fig. 68, p. 66.

4. Dependent Ionic Conduction in the Air of the Room. X-rays as an Ionizing Agent. Ionic Mobility.

X-rays form a very convenient and effective ionizing agent. In fig. 12 AK is the usual plate condenser. The X-rays are incident in the direction of the arrows. G is a moving-coil galvanometer, E the town mains. We can easily obtain currents of the order of 10^{-8} ampere.

The ions produced by X-rays are practically all of the same molecular type. This we infer from the velocity which they attain in the electric field.

The measurement of the velocity of the ions offers no difficulty. We compare their velocity in the electric field with that of a current of air. Fig. 13 is a diagram of one of the numerous ways in which the experiment may be carried out in practice. A narrow strip (ab) of a tube is penetrated by X-rays. A current of air of known velocity blows the ions obliquely through the field of a cylindrical condenser. The inner electrode of the cylinder is subdivided, and an ammeter is connected to the right-hand half. When the air is at rest or moving slowly the paths of all negative ions end on the left-hand half of the inner electrode. When the velocity of the air exceeds a definite value, the ions formed at *a*, i. e. at the inner wall of the outer cylinder, reach the right-hand half of the inner electrode. Their path is shown by a dotted line. The ammeter *A* indicates their arrival by a deflection.

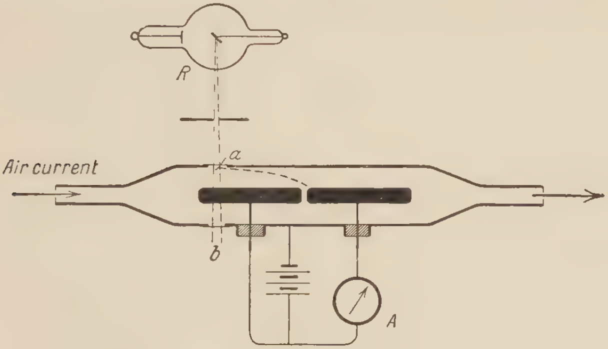

Fig. 13. Measurement of ionic velocity.

By measurements of this kind we find that u, the velocity of the ions, is proportional to $\mathbf{R}$, the intensity of the field. We have $u = \mathbf{R}v$. By what we said on p. 201, v, the factor of proportionality, is the *mobility* of the ions. v is equal to $u/\mathbf{R}$, i. e. to the *ionic velocity in a field of* 1 *volt/cm.* In the case of the air of the room we find that the negative ions have a mobility of $1{\cdot}89\ \frac{\text{cm.}}{\text{sec.}}\Big/\frac{\text{volt}}{\text{cm.}}$, and the positive ions a mobility of $1{\cdot}37\ \frac{\text{cm.}}{\text{sec.}}\Big/\frac{\text{volt}}{\text{cm.}}$. Similar values are obtained for other gases at the same pressure.

In the plate condenser of fig. 12 we experimented with a field intensity $\mathbf{R}$ of about 50 volts/cm., i. e. the ions moved steadily through the crowd of air molecules with a velocity of almost 1 metre per second.

The mobility of an ion is determined by its diameter and by the internal friction of the air. A simple calculation leads to the conclusion that the ions formed by X-rays are of the order of magnitude of molecules.

5. Dependent Ionic Conduction in Air. The Saturation Current.

As yet we know of only *one* kind of relationship between current and pressure, namely the proportionality relationship expressed by Ohm's law. As we have frequently emphasized, however, Ohm's law is valid only in particular cases. Often it is very far from the truth. A typical example of this will now be brought forward.

We again use the apparatus of fig. 12 but with a battery of variable pressure as source of current. The relationship between the current i and the pressure E which we obtain in this way is shown in fig. 14. Initially the current rises in proportion to the pressure. Ohm's law, $E/i=$const., is true. When the pressure is raised further the current rises by a less than proportional amount. It finally becomes constant, independently of the pressure between the electrodes. This constant maximum value is called the *saturation current.*

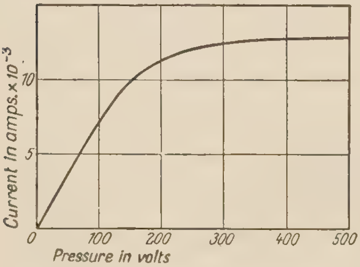

Fig. 14. The saturation current.

In order to find out the meaning of this current-pressure curve, we refer back to the fundamental equation of the conduction current, namely equation (1c), p. 196:

$$i = A\,\mathbf{N}(\mathbf{n}\,e)\,(u_+ + u_-).$$

u, the velocity of the electricity carriers, is proportional to the field strength $\mathbf{R}$ for ions formed by X-rays also, and therefore proportional to the pressure $E = \mathbf{R}\,d$ (d is the distance between the electrodes or the thickness of the condenser). We know this from last section. The ions have a definite mobility v. Hence we can use equation (3), which is

$$i = \frac{A}{d}\,\mathbf{N}(\mathbf{n}\,e)\,(v_+ + v_-)\,E\,, \tag{3}$$

where A, d, and v are constant. Hence the experimental relationship between the current i and the pressure E, i. e. the "saturation curve" of fig. 14, can only be interpreted in one way. *The quantity* $\mathbf{N}\,(\mathbf{n}\,e)$, *the total ionic charge in a cubic centimetre of air, must depend on the pressure E, and must, in fact, decrease for high pressures.*

It is not difficult to make the reason for this dependence clear. The X-rays produce the same number of ions inside the condenser in every second. Hence in a condenser with no field the stock of ions would necessarily increase continually if recombination did not produce a stationary equilibrium value of $\mathbf{N}$.

With low pressures in the field only a small fraction of the ions formed in every second moves to the electrodes. The losses which thereby arise, however, are to a great extent made good owing to the losses due to recombination being reduced by the motion of the ions. $\mathbf{N}$, the number of ions in a cubic centimetre, may therefore remain practically constant in spite of the motion of the ions in the field. Hence, by equation (3), i and E are proportional to each other; Ohm's law is a good approximation for low pressures in fig. 14 also.

When the pressure is raised the loss due to the ions moving to the electrodes soon predominates. $\mathbf{N}$, the number of ions in a cubic centimetre, becomes smaller and smaller. The current i approaches its maximum or saturation value asymptotically.

Hence it is possible to have saturation currents only when the motion of the carriers in the field uses up the stock of carriers. The fact that Ohm's law holds, on the contrary, means that the stock of carriers is inexhaustible in comparison with the amount of ions moving.

Saturation currents in ionized gases play a great part in connection with scientific measurements. We often have to measure the total amount of ions formed per second e. g. by X-rays. This is a relative measure of the intensity of the X-rays concerned. Such measurements have no meaning unless the current has reached its saturation value. *It is only with the saturation current that we catch all the ions formed per second without losing any owing to recombination on the way.*

In conclusion, we add a word or two about *spontaneous conduction of electricity* in air. Hitherto we have regarded air as an almost perfect insulator; and in this we are perfectly justified, for in the absence of ionizing radiations etc. air contains only a vanishingly small number of carriers, unless they are introduced from outside. Among the molecules in a cubic centimetre of the air of the room, numbering about 3.10^{19}, there are only a few thousand ions. This stock of ions is maintained by "spontaneous" ionization. About five pairs of ions are formed per second in a cubic centimetre of the air of the room. In spite of their continual formation, the stock is kept limited by recombination, diffusion to the walls, and the electric fields which are always present.

In order to measure the value of $\mathbf{N}$, the *number of ions* in a cubic centimetre, at any moment, we have to use pressures so small that Ohm's law still holds. The motion of the ions in the field must not appreciably diminish the stock. $\mathbf{N}$ may then be calculated from equation (3). ($\mathbf{n} = 1$ for ions formed from air, A is the area of the plates of the condenser, E is the pressure used, and the ionic mobilities are known from p. 205.)

The *saturation current* i_s serves to determine the number of ions formed per second. All the ions formed per second in the volume of the condenser Ad are caught without any losses. i_s/Ade gives the number of ions formed per second in a cubic centimetre.

The spontaneous ionization of the air is caused by the radiation from radioactive substances, which are never absent from any part of the earth. Part of the radiation is of cosmic origin; say two of the five ions formed in a cubic centimetre per second are due to this. (It takes the form of penetrating X-rays, which possibly originate in the system of fixed stars.)

What we have said in this and the two preceding sections about dependent conduction in the air of the room will suffice. Our statements are qualitatively true for other gases also. The inert gases, however, must be subjected to a pressure of several atmospheres before they exhibit the same phenomena.

6. Dependent Conduction of Electricity in a High Vacuum.

The relationships involved in dependent conduction of electricity through gases at comparatively high pressures, e. g. through the air of the room, are very simple; the electricity carriers follow the pull of the electric field and move through the crowd of gas molecules, overcoming the internal friction of the gas. Their velocity u is proportional to $\mathbf{R}$, the intensity of the field.

We now turn to another case distinguished by great simplicity, namely, dependent conduction of electricity through a high vacuum. We shall thereby be led in a very simple way to a fundamental conclusion about the nature of the electron or negative atom of electricity.

The sketch in fig. 15 will serve to make it clear what is meant by the technical term "high vacuum". Again speaking figuratively, this diagram is a time exposure of about 10^{-3} second. The paths of two molecules only are shown. Here the free path has become large compared with the dimensions of the containing vessel. Hardly any collisions now take place between the molecules and their companions, most of the encounters being between molecules and the walls. By means of modern mercury-vapour pumps we can easily lower the density of the gas to 10^{-9} of its normal value. For all that, there are still about 10^{10} molecules flying about in every cubic centimetre even then. The velocity of the molecules is still determined by the temperature only, that is, it is about 500 m./sec. for air at room temperature.

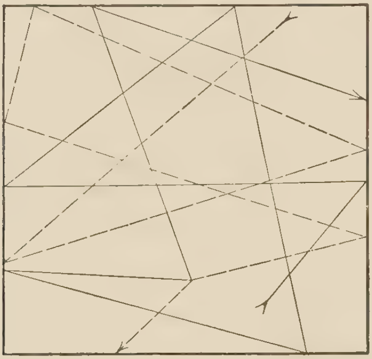

Fig. 15. Paths of two molecules in a high vacuum.

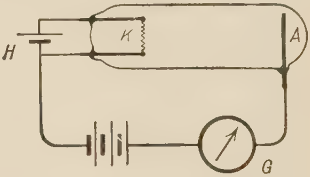

Fig. 16. Current of electrons from an incandescent cathode.

A high vacuum contains no electricity carriers of its own at all. From this point of view a high vacuum is the very best insulator that we have available. The electricity carriers must therefore be brought into the high vacuum from outside. Among many and varied methods of doing this the simplest is to make the negative plate of the condenser, the "negative electrode" or "cathode", incandescent. To anticipate the result of this experiment, *electrons* are emitted by the incandescent cathode. This is shown by means of the apparatus sketched in fig. 16. K is an electrically-heated tungsten wire, H the battery for heating it. A is the other boundary of the electric field, the "positive electrode" or "anode". The ammeter registers a current of e. g. several milliamperes, but only when the hot wire is the negative electrode or cathode. In the reverse case no current flows. This agrees with our hypothesis that negative atoms of electricity or electrons are emitted by the glowing wire.

The electrons are caught by the electric field when they emerge from the incandescent electrode, i. e. the hot wire. The ammeter shows that even when the electric pressure is low all the electrons reach the anode. The current does not rise any further when the pressure is raised; the "saturation" value has been reached (cf. fig. 14).

By equation 10, p. 73, the electric field exerts a force $F = e\mathbf{R}$ on the electrons, if, as always, we denote the charge on the electron by e. When the gas is at high pressure, the electricity carriers are retarded by the internal friction of the gas. They have a definite "mobility" v. Their velocity u turns out to be proportional to $\mathbf{R}$, the intensity at the moment under consideration. We have $u = \mathbf{R}v$. It is quite different in a high vacuum, where there is no friction. Hence by one of the fundamental laws of mechanics the electrons are continually *accelerated* and their velocity increases continually. They "fall" (a technical expression used in a transferred sense) through the electric field, and thereby receive a large amount of kinetic energy $\frac{1}{2}mu^2$, with which they impinge on the opposite electrode, the anode (fig. 16). In a short time the anode glows brightly as a result of the impact of the invisible carriers (E is about 800 volts).

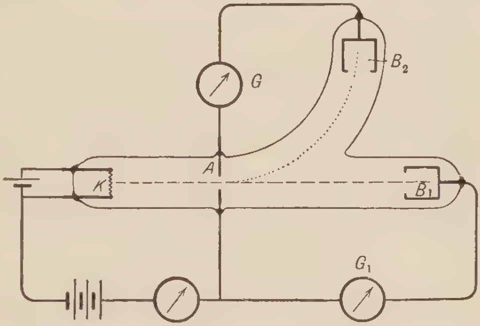

Fig. 17. Rectilinear path of the invisible cathode rays, and their circular path in a magnetic field. The magnetic field **H** is to the right of A, perpendicular to the plane of the paper and directed towards the reader.

If, as in fig. 17, the positive electrode A has a hole in it, the electrons fly through the hole. They fly in a straight line — still with constant velocity — till they reach the nearest obstacle, e. g. the hollow box B_1. *Such a swarm of electrons flying like shot we call cathode rays*[*]. Cathode rays are quite invisible. The deflection of the ammeter G_1 connected between A and B_1 serves to indicate their presence. Note that an electric field is not produced between A and B_1 by means of a source of current (as might be thought); but the electrons continue to fly along with the velocity previously imparted to them. Here we have a *typical convection current* like that in fig. 4, p. 197.

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[*] [Cathode rays were first observed by Plücker in 1859 and were subsequently investigated by Hittorf and Crookes. Crookes was the first to suggest that the cathode rays are negatively charged particles moving at a high speed. The truth of this was confirmed about 1897 by the brilliant researches of several physicists, notably Perrin, Kaufmann, Lenard, and J. J. Thomson. The method described in the next section is due to J. J. Thomson.]

A hollow box is used instead of a plate for the following reason: cathode rays ionize the top layers of molecules when they impinge on solid bodies. During this process electrons are split off; these are scattered in all directions and travel on as slow cathode rays or so-called "secondary" radiation. The latter may be so abundant as to reverse the sign of the charge on B_1. This is largely avoided by transferring the point of impact of the primary rays to the bottom of the hollow box.

This convection current, i. e. the swarm of electrons flying in a definite direction which we call cathode rays, must, like every current, possess a magnetic field. This magnetic field must be affected by other magnetic fields, or, to speak more precisely, its direction must be altered by other magnetic fields. The experimental demonstration of this is immediately successful; it is likewise shown in fig. 17. Like any conductor traversed by a current, the cathode rays are deviated in a direction at right angles to their own motion and to the lines of magnetic force (figs. 17 and 18, pp. 143—144). They no longer strike on B_1, but upon the box B_2 placed at the side, whence they are led to the ammeter G.

So much for the qualitative observation of these invisible cathode rays. We have still to consider the hypothesis that the convection currents or cathode rays which we have observed here really consist of electrons and not of negative ions. We have to show that atoms of negative electricity or of pure "electron gas" are flying freely and are not being carried by chemical molecules or atoms. In order to do this we must ascertain m, the mass of the single cathode ray particle. This is done in the next section.

7. The Atomic Weight of the Electron Deduced from Observations on Cathode Rays.

In fig. 17 the electrons between the electrodes K and A are accelerated by the electric field which supplies each separate electron with kinetic energy $\frac{1}{2}mu^2$. The work supplied to each single electron during the acceleration is known to be eE, by equation (12), p. 74, where e again denotes the charge on the electron ($1{\cdot}6 . 10^{-19}$ coulombs, by p. 85) and E the pressure between K and A in volts. Hence

$$\frac{1}{2} m u^2 = eE \cdot 10^7 \text{ ergs.} \tag{4}$$

This equation involves two unknowns, m, the mass of a single electron, and u, its velocity. In order to determine them separately a second equation is required. This is easily obtained from the magnetic deflection of the cathode rays in the space between A and B_1, where there is no electric field.

Let $\mathbf{N}$ be the number of electrons in a cubic centimetre of the radiation and A the cross-section of the radiation.

Then by equation (1c), p. 196, the strength of the current producing the cathode rays is

$$i = \mathbf{N} A e u \text{ amperes.}$$

Now let a uniform magnetic field of intensity $\mathbf{H}$ perpendicular to the plane of the paper be produced in the space to the right of A (fig. 17). The magnetic field exerts a force on the cathode rays; this force is situated in the plane of the paper, at right angles to the direction of the rays at the point in question. The force is constant in a uniform field, so that the electrons move in a circular path. If $\mathbf{H}$, the intensity of the field, is suitably chosen they come to the bottom of the curved tube at B_2, where they are detected and measured by means of the hollow box B_2 and the ammeter G.

If we consider a current of length l, i. e. in this case a length l of cathode rays, the force exerted on it is

$$F = 10^7 i \mathbf{B} l = 10^7 \cdot \mu \mathbf{H} \mathbf{N} A e u l \quad \text{(dynes)}$$

by equation (9), p. 145. This is the centripetal force which makes the electrons move in the circular path of radius r. We have the equation for the centripetal force

$$F = \frac{M u^2}{r},$$

which is familiar to us from mechanics. Here M is the total mass of the electrons contained in l cm. length of the radiation, i. e. $M = \mathbf{N} A l m$. Combining the above expressions we get

$$\mathbf{H} r = \frac{m}{e} \cdot u \cdot \frac{1}{10^7 \mu}. \tag{5}$$

This is the other equation containing m and u. Hence, in order to determine m and u from equations (4) and (5) we only require to find

(1) the acceleration of the electrons produced by a known pressure E;

(2) r, the radius of curvature of the path of the electrons in the magnetic field $\mathbf{H}$. Measurements of this kind lead to the results given below (see Table III, page 212).

According to the kinetic theory of gases, m_{H}, the mass of the hydrogen atom, is $1 \cdot 65.10^{-24}$ gm.; that is, the mass of the electron is about $^1/_{1800}$ of that of the hydrogen atom.

The masses of the various chemical atoms are as a rule counted not in grams but in multiples of the mass of the hydrogen atom. These relative atomic masses referred to H = 1* are in accordance

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* More accurately, O = 16 and H = 1·008.

Table III.

Accelerating pressure E in volts	H, the magnetic intensity required to produce a circular path of radius 1 cm., in ampere-turns per cm., or Hr, the product in equation (5), in ampere-turns	u, the velocity of the electrons,		e/m, the ratio of e, the charge of the electrons, to m, its mass, in coulombs/gram	Mass of an electron in grams
		in km./sec.	in fractions of the velocity of light, $c = 3.10^{10}$ cm./sec.		
10^2	27	6.10^3	2%	$1{\cdot}77.10^8$	9.10^{-28}
10^3	88	2.10^4	6·5%	$1{\cdot}77.10^8$	9.10^{-28}
10^4	276	6.10^4	20%	$1{\cdot}73.10^8$	$9{\cdot}2.10^{-28}$
4.10^4	590	12.10^4	40%	$1{\cdot}62.10^8$	$9{\cdot}8.10^{-28}$

with long-standing usage, called atomic weights, although weight is really a force, not a mass. Using this phraseology, therefore, the electron has an atomic weight of $^1/_{1800}$ and stands at the head of the periodic table as being by far the lightest of the elements.

The result given in this section which is of most immediate importance for us is as follows. *In conduction phenomena negative atoms of electricity, i. e. electrons, can traverse the electric field freely, without being carried by any electricity carrier whatever.*

8. Applications of the Dependent Conduction of Electrons through a High Vacuum.

The phenomena discussed in last section are of fundamental importance for our knowledge of atoms of electricity: besides, practical applications of these phenomena are becoming more and more important. We shall consider only three examples: the high vacuum X-ray tube, the three-electrode valve or triode, and the two-electrode valve.

(1) *The high vacuum X-ray tube**. X-rays are produced when fast electrons impinge on obstacles, especially when these consist of metals of high atomic weight. Fig. 18 is a diagrammatic longitudinal section through an X-ray tube of this kind. The source of electrons, a tungsten wire heated by a battery E, is at K. A is the positive electrode, which in this case is usually called the anticathode; it consists of a block of tungsten or tantalum. A and K are connected with a source of current at high pressure (at the present day pressures up to 250000 volts are used). The X-rays are produced at the bright spot where the electrons strike the anticathode. Visible light of rather long wave-length is produced at the bright spot besides the invisible X-rays, but the former is usually masked by the brilliant glow of the anticathode due to its bombardment by the electrons.

Screens with a coating which fluoresces under the action of X-rays are used to show their presence. The coating usually consists of salts of platinum or zinc.

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* [Usually called the *incandescent cathode* tube, the best-known type being known as the *Coolidge tube*, after its constructor.]

Altering the temperature of the hot wire alters the number of electrons and consequently the intensity of the X-rays. The velocity of the electrons, and hence the penetrating capacity of the X-rays, is varied by varying E, the pressure between K and A. In order to make the bones of the hand visible on the fluorescent screen, E must amount to about 40000 volts. Further details are to be found in textbooks on optics.

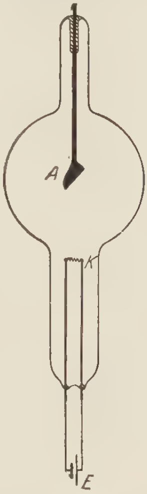

Fig. 18. X-ray tube with an incandescent cathode in a high vacuum.

(2) *The three-electrode valve* *. The alteration of the strength of electric currents is a problem involved in many important questions in physics and engineering alike. As a rule some form of switch with movable contacts is used. All practical forms of switchgear, however, possess more or less mechanical inertia. The apparatus with the least inertia that we have so far met with is the "microphone", in which contacts between plates and grains of carbon have their resistances greatly altered by minute alterations in the closeness of contact (p. 282). Even the mechanical vibrations of the plate caused by the human voice are sufficient. On this fact depends the well-known use of the microphone as a current regulator in telephony.

But even the microphone has sharp limitations. Like all other mechanical forms of switch, it fails whenever we require it to work at high frequencies, i. e. above a few thousands per second.

In this respect the dependent conduction of electricity through high vacua has enabled us to make a real advance. Nowadays we can construct many kinds of instruments for regulating electric currents which are entirely free from mechanical inertia. The simplest practical form is the so-called three-electrode valve.

This is shown in Fig. 19. K is the hot cathode, A the anode. Between them is the regulating "grid" N. In practice N and A usually take the form of hollow cylinders surrounding K. In addition, the source of current E_1 and the ammeter G form a circuit.

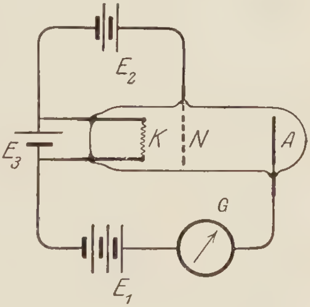

Fig. 19. A three-electrode valve.

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* [Invented by Lee de Forest in 1907 as an improvement on the two-electrode valve (p. 214) invented by J. A. Fleming in 1904.]

The electrons emitted by K can only pass through the grid N if N has no appreciable negative charge.

Otherwise the electrons "fall" back to the hot wire. Fig. 20 shows the grid-plate characteristic of a three-electrode valve. The abscissa is the pressure between the grid and the hot wire, which may be produced e. g. by the battery E_2 in fig. 19. i is the strength of the current shown by the ammeter G. The pressure e between the grid and the cathode "regulates" i, the strength of the current in the circuit; a negative charge on the grid diminishes i, a positive charge increases it.

The numerical values in fig. 20 merely refer to a particular practical form. The pressure between cathode and anode is taken to be 60 volts. If the pressure is smaller, or if the meshes of the grid are closer, the curve is displaced towards the right. At the present day three-electrode valves are manufactured with characteristics which quantitatively differ very widely but which are similar from a theoretical point of view.

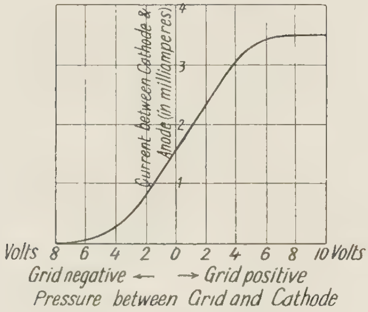

Fig. 20. Grid-plate characteristic of a three-electrode valve.

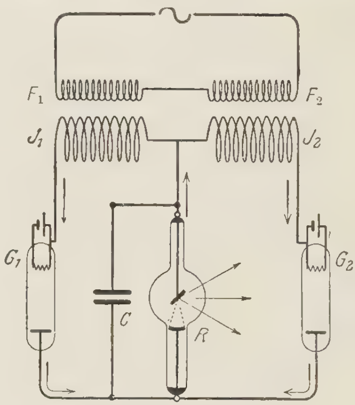

Fig. 21. Rectifying apparatus with hot-cathode valves.

In our case about 46% of the electrons pass through the grid when the latter is uncharged. The current i attains its maximum value when the grid has a positive charge and the pressure is 8 volts. When the grid has a high negative charge (greater than 6·5 volts) the circuit is broken. Hence the three-electrode valve may be regarded as an inertia-less switch, although this title does not do justice to the efficiency of the valve as a current regulator capable of fine adjustment.

The valve is an indispensable auxiliary to modern telegraphy and telephony. Among other things it has at last made the construction of an efficient current amplifier for telephony possible. The small variations of pressure arriving from the line are made to regulate powerful currents at the receiving station (imagine that G is a telephone receiver like that in fig. 15, p. 154). Apart from this, the importance of the valve in connection with scientific measurements is continually increasing.

(3) *The two-electrode valve.* Except in particular cases, incandescent

metal wires emit electrons only, and not positive electricity carriers. Hence a high vacuum in which dependent electronic conduction is going on acts as a valve. For example, it lets through only every second half-period of an alternating current. Instead of the original alternating current we obtain an intermittent direct current. By including large condensers in the circuit these variations of the current with time can be greatly reduced. Rectifiers of this kind are used in the laboratory, especially in conjunction with transformers, when direct current at high pressure is required e. g. for X-ray tubes (fig. 21).

9. Independent Conduction in Gases and Vapours at Low Pressure.

Dependent conduction in gases and vapours at high pressure and dependent conduction in a high vacuum are distinguished by their great simplicity. We therefore began by giving an account of them in the last six sections. Historically, however, they did not by any means form the starting-point of the explanation of the mechanism of conduction. The attention of observers was first attracted by the many coloured luminous forms of conduction in gases, e. g. the spark (lightning), the brush (St. Elmo's fire) and the elaborate tubes with rarefied gases which form well-known playthings.

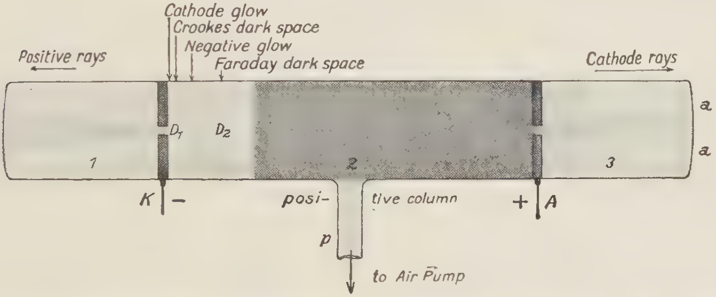

Fig. 22. The characteristic parts of the luminous discharge, semi-diagrammatically.

In all these cases the conduction begins as independent conduction. The electricity carriers appear in the electric field without our intervention. Nearly always the electricity carriers are very small; in fact they are ions. Besides these, numerous free electrons take part in the motion. In the first instance this is mere hypothesis; an account of the experimental foundations for it follows. The details of the phenomena are very involved and in parts only the main outlines have been elucidated. In spite of this, however, we cannot dispense with a short survey of the phenomena we have mentioned, in view of their practical importance. *In this section we begin by considering the conduction of electricity through rarefied gases*, and in fact that form of the phenomenon which is known as the luminous discharge.

Fig. 22 shows a cylindrical glass tube; A and K are two condenser plates or electrodes, connected to a source of current at a pressure of several hundred volts. The tube *p* leads to an ordinary air-pump. Let the pressure of the air be first lowered to about 0·2 mm. of mercury. The tube between *K* and *A* is then filled with a characteristic glow of light.

The feeble pink cathode glow is situated on the actual surface of the cathode *K*. It is followed by the Crookes dark space D_1 and the violet negative glow, which is sharply bounded by the broad Faraday dark space D_2. The rest of the tube as far as the anode is filled with the red positive column, which often exhibits conspicuous bright and dark striations.

Both electrodes are perforated. Feebly glowing bundles of rays extend from the holes into the spaces 1 and 2 where there is no field. These are the luminous traces of the cathode rays and canal or positive rays (see below) traversing the gas.

The effect of lowering the pressure of the gas further is to make the Crookes dark space and the negative glow wider. The colours grow paler. At *a*, where the cathode rays impinge on the glass, the latter gleams with a vivid green fluorescence. In the gas the bluish traces of the cathode rays can easily be followed into the space 2 and we can see quite clearly that they originate from the cathode. With progressive lowering of the pressure the whole tube between *A* and *K* eventually glows with a greenish light, but much more feebly than before. Finally the light begins to flicker and conduction ceases entirely.

So much for external appearances. We have to show in the first instance that in the region 3 we have really to do with the *traces of the cathode rays*. This is demonstrated qualitatively by the facts that these traces are readily deflected by a magnet and that the deflection is in the same direction as for cathode rays. They can also be identified quantitatively by determining the atomic weight of the flying particles from their deviation by a magnet (pp. 210—212).

Now for the *positive rays**. These chiefly consist of swarms of positive ions. In order to ascertain their charge, we collect them in a hollow Faraday box. In order to find the atomic or molecular weight of the ions we use a method which is the same in principle as that which we used for the cathode rays; but much higher magnetic intensities **H** are needed in order to produce a sufficiently curved path. The reader should remember that *m*, the mass of the ion, occurs in the numerator of equation (5), p. 211, and that even the lightest ion, that of hydrogen, has a mass about 1800 times that of the electron.

Besides positive ions, negative and neutral atoms and molecules are always found among the positive rays. These arise as a result of the positive ions exchanging their charges with the gas as they fly through it. Frequent collisions with the molecules of the gas take place, and these are the cause of the luminous traces which make the positive rays visible.

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* [These have been investigated by Goldstein, Wien, J. J. Thomson, Aston, and others.]

So much for the facts obtained by visual observation. Important additions may be made to them by ascertaining the distribution of pressure between K and A. This is carried out in the usual way by means of proof-wires (p. 64); we may use thin wires, as there are so many ions present that a flame collector is unnecessary. The results of such measurements are shown diagrammatically in fig. 23. The greater part of the total pressure is located on the short piece of the lines of force in the Crookes dark space; this pressure is called the *cathode fall of potential.* It follows that there is a very strong field in the Crookes dark space. Then there follows a region of low intensity extending nearly all the way along the tube. The pressure all along this part only amounts to a fraction of what we obtain for the cathode fall of potential. There is a small "anode fall of potential" just before the anode is reached.

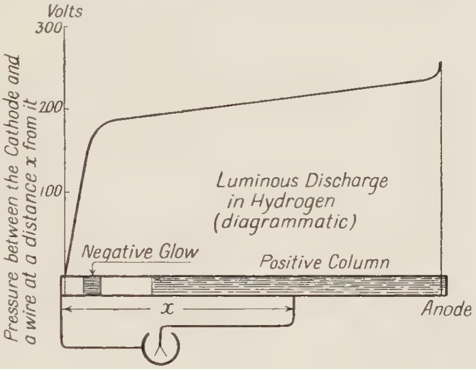

Fig. 23. Distribution of pressure in the luminous discharge.

The whole mechanism of the phenomenon of independent conduction in the luminous discharge depends on "ionization by collision". The diffusion of gas containing ions also plays an important part. By "ionization by collision" we mean the splitting off of electrons when electrons or ions collide with molecules. The details are somewhat as follows: positive ions move up to the cathode and attain a high velocity owing to the cathode fall of potential. Some of them fly through the holes in the cathode as positive rays. The others strike the cathode with a velocity sufficient to ionize the molecules of the surface (causing the cathode glow). The electrons split off in this way traverse the cathode fall of potential in the opposite direction. They leave the dark space with a high velocity as cathode rays. Their rectilinear paths are perpendicular to the violet negative glow. The electrons give rise to ions and more electrons by ionizing the molecules with which they collide. The electrons are accelerated towards the anode by the weak field in the middle of the tube. By colliding with the gas molecules they produce more electrons. The number of electrons increases like the progress of an avalanche, and in this way the positive column is formed. This supplies the positive ions which finally strike the cathode.

The cathode rays gain practically all their velocity within the region of the cathode fall of potential. In the subsequent parts of their paths the electric intensities are insufficient to influence the magnitude or direction of the cathode rays to a noticeable extent. Hence the position of the anode does not affect the paths of flight of the cathode rays. The size and shape of the positive column are of quite subordinate importance. The anode *A* may be placed in any tube added at the side (cf. fig. 24); but the anode must not be too near the cathode. If there is not enough room for the Crookes dark space to be formed, independent conduction does not take place at all: the most important part, the cathode fall of potential, must be present.

This fact is utilized in the tube shown in fig. 24. The side tube on the left contains a simple piece of apparatus for ascertaining the charge on the cathode rays. A hollow cylinder 1 with a sieve-like cover serves as the anode. The cathode rays coming from *K* pass through the sieve and arrive in the second hollow cylinder 2, where they come to a standstill and charge the cylinder. A pressure of several thousand volts is produced between 1 and 2, and small sparks appear at the spark gap *F* included in the circuit. The smallness of the space between the cylinders prevents any dark space being formed and thereby prevents a current from passing in the highly rarefied gas.

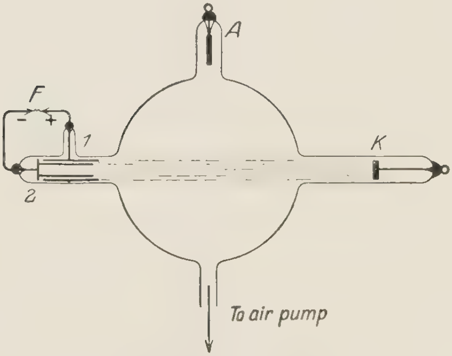

Fig. 24. Finding the charge on the cathode rays.

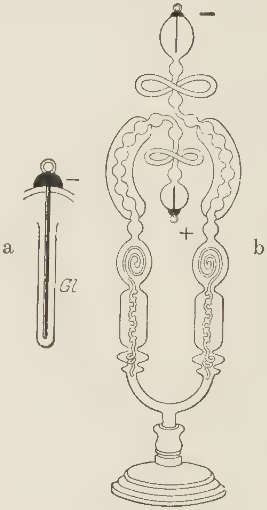

Fig. 25 a, b. Old-fashioned vacuum tubes (Geissler tubes). On the left the negative glow is shown covering the cathode like a skin.

10. Some Applications of Independent Conduction in Gases at Low Pressure.

(1) *Geissler tubes.* Any oldish collection of apparatus contains playthings of the kind shown in fig. 25 b, namely, fantastically ornate glass tubes with wire electrodes in each of their blown-out ends. An induction coil or induction machine is used to provide current. The negative glow covers the cathode like a skin floating freely, closed in

front and with a somewhat raised edge behind. It is indicated by *Gl* in fig. 25a. All the rest of the tube is filled by the positive column. The play of colours is often heightened by the use of tubes made of fluorescent glass.

A simple form for practical use is shown in fig. 26. This is what is called a spectrum tube. It is used for investigating the spectra of gases under electrical excitation. The middle of the tube is usually a small capillary. The high current density is meant to increase the surface brilliancy of the positive column.

Geissler tubes are being used to an increasing degree in practice for lighting, particularly for advertising purposes. The tubes are often many yards long and not infrequently take the form of letters. They are usually filled with the inert gas neon, which gives a positive column of a beautiful orange-red colour. The cathode, which is the most essential part of the apparatus, is usually inserted where it cannot be seen.

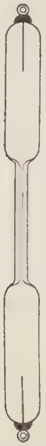

Fig. 26. Vacuum tube for investigating spectra.

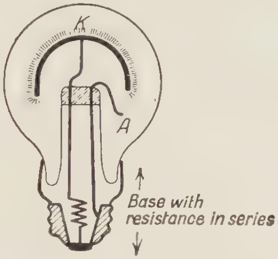

Fig. 27. Neon lamp.

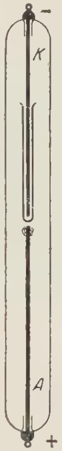

Fig. 28. The glow-light oscillograph.

(2) *The neon lamp.* The cathode is a rather large sheet of metal, which may be e. g. hemispherical as in fig. 27, the anode a smaller wire. The cathode is covered with the cathode glow, which forms a freely floating layer. The positive glow is quite atrophied, no visible sign of it being left except for a small glowing cloud touching the anode.

The cathode fall of potential only amounts to about 68 volts. This is obtained by special choice of the gas (neon) and of the metal of which the electrodes are made (iron with a coating of potassium). These lamps can be worked directly off the town mains.

(g) *The glow-light oscillograph.* Oscillograph is the name given to measuring instruments of low inertia which enable us to record rapidly alternating currents.

A glow-light oscillograph is shown in fig. 28. The electrodes take the form of two long wires placed in a straight line. The negative glow forms a freely floating skin covering the cathode. It has been found by experiment that l, the length of the negative glow, is a measure of the strength of the current.

In making measurements with the glow-light oscillograph we observe or photograph the negative glow by means of a rotating mirror, which, as we are already aware, transforms phenomena occurring one after the other in time into phenomena occurring beside each other in space. Fig. 29 shows such a photograph of the alternating current in a damped oscillatory circuit. The reader should imagine the glow-light oscillograph inserted instead of the spark gap in fig. 21, p. 185. In the oscillograph the positive column is completely atrophied. Traces of it can be seen in fig. 29 as bright horizontal strips in the direction of the arrow, but they are scarcely sufficient to cause inconvenience.

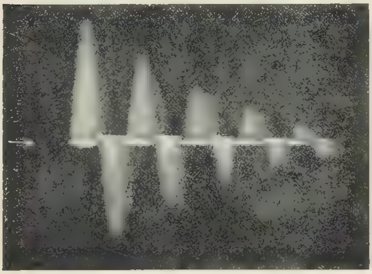

Fig. 29. A damped electrical vibration (from a retouched photograph).

(4) *The luminous discharge with a cathode of glowing oxide.* As a rule the cathode fall of potential in the luminous discharge amounts to several hundred volts, but this can be reduced to about 60 volts by using electrodes of the alkali metals in inert gases. A luminous discharge may thus be produced even at the pressure supplied by the town mains (as in the neon lamp).

Even more effective than the alkali metals are the oxides of the alkaline earth metals; with these the cathode fall of potential falls to only a few volts. A luminous discharge can then be easily produced with only about 20 volts between cathode and anode.

Fig. 30 shows a practical form of one of these "Wehnelt's cathodes". An electrically heated plate of iridium or tungsten, B, carries a speck of oxide of about one square millimetre in area. This speck forms the starting point of a very sharply defined bundle of cathode rays, which can easily be followed up by the bluish traces left in the gas.

These oxide cathodes enable us to produce very "soft" cathode rays, i. e. rays of very low velocity. These can be very easily deviated by magnetic and electric fields. Using fields with an intensity **H** of only a few ampere-turns per centimetre the rays may be made to traverse closed circular paths in small glass vessels (cf. fig. 30 and the second column of Table III, p. 212).

(5) *The cathode ray oscillograph.* The apparatus is sketched in fig. 31. K is the cathode, A the anode, and B two stops which allow only a narrow bundle of cathode rays to pass. The rays fall on a screen at a. The point where they impinge on the screen shines with a bright fluorescent light.

The oscillograph may equally well be used to measure either current or pressure. In order to measure a current with it, the magnetic field of the current is made to deflect the bundle of rays; this is done by means of coils, as e. g. the coil F.

In order to measure pressure, the cathode rays are deflected by an electric field; this is done by the condenser plates A′ and K'.

The amount of the "deviation" or "distance fallen through" (x) is easily calculated. Let the electron traverse a length y of the condenser with velocity u in the time $t = y/u$ seconds (fig. 32). In this time the electron "falls" through the distance

$$x = \frac{1}{2} b t^2, \tag{6}$$

where b denotes the acceleration given to the electron in the direction of the lines of force by $\mathbf{R}$, the intensity of the electric field. By one of the fundamental laws of mechanics,

$$\text{force} = mb,$$

or by equation (10), p. 73,

$$10^7 e\mathbf{R} = mb,$$

if the force is measured in dynes. By substituting the values of b and t in (6) we obtain

$$x = \frac{1}{2} 10^7 \frac{e}{m} \cdot \mathbf{R} \cdot \frac{y^2}{u^2}. \tag{7}$$

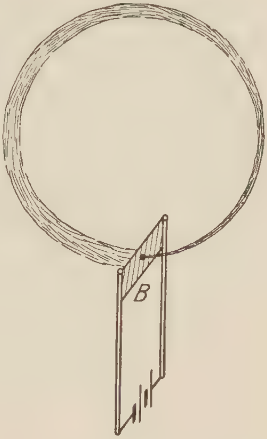

Fig. 30. A bundle of cathode rays which start from a Wehnelt cathode and are bent into a circle by a uniform magnetic field. The lines of magnetic force are directed towards the reader. The evacuated glass vessel and the anode are not shown in the figure.

Many practical variants of the apparatus exist. Occasionally it is constructed with glowing cathodes covered with oxide, which give a low cathode fall of potential and hence slow rays which are easily deviated. The apparatus is also made with a glowing cathode for producing dependent discharges in a high vacuum. This form might have been described in connection with the last section but one.

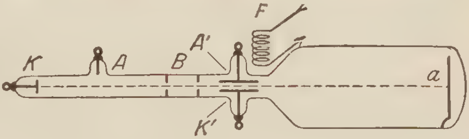

Fig. 31. Braun's cathode ray oscillograph.

(6) *The gas-filled X-ray tube* (fig. 33). K is the cathode, shaped like a concave mirror, A the anode, AK the anticathode. (The presence of an auxiliary anode A makes it easier to exhaust the tube in process of manufacture.) The pressure of the air is about 0·001 mm. of mercury. At this pressure the negative glow and the positive column are no longer visible. Feeble traces of the cathode rays in the gas left in the tube are occasionally to

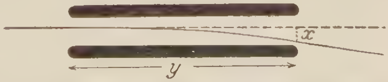

Fig. 32. Deviation of electrically charged rays in the uniform electric field of a flat plate condenser.

be seen between K and the bright spot where the rays impinge on AK. The near half of the sphere shines with a green light. This fluorescence is not due to the X-rays but to cathode rays diffusely reflected from the anticathode.

If the penetrating power of the X-rays is raised by increasing the electric pressure at which the tube is worked, the intensity of the X-rays is increased at the same time. This is a disadvantage of this form of X-ray tube as compared with the high vacuum type.

The amount of gas in the X-ray tube diminishes while the tube is working. Some of the gaseous ions produced gradually combine with the metal and glass parts of the tube. Many devices are used for replacing the lost gas. As an example, a palladium tube is shown at Pd in fig. 33. This tube is heated red-hot by a flame; hydrogen from the flame gases can then diffuse through it into the X-ray tube.

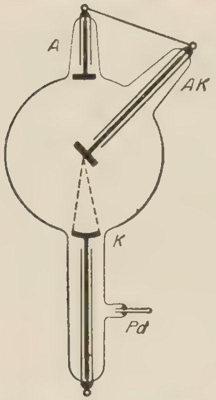

Fig. 33. Gas-filled X-ray tube with a concave cathode.

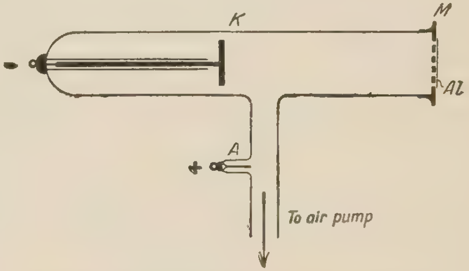

Fig. 34. Cathode ray tube with aluminium window, after Lenard.

(7) *Lenard's window.* We are given a glass tube evacuated to a pressure of about 0·001 mm. of mercury, with a cathode plate K and an anode A, the latter being placed anywhere we like (fig. 34). The cathode rays fall on a metal plate M, which is perforated like a sieve. The holes are covered with thin aluminium foil or thin blown glass Al (made air-tight with grease). Although these windows have no holes at all and are gas-tight, the cathode rays fly through them and emerge into the free air of the room. The molecules of the air shine with a reddish-violet shimmering light owing to their becoming ionized by collision. Suitable screens (e. g. of zinc silicate) fluoresce visibly at a distance.

Lenard's window enables the cathode rays to be investigated outside the space in which they arise. By means of it Lenard himself has laid the firm foundations of modern atomic theory and of its first bold attempts to picture model atoms.

(8) *The mass spectrograph and the nature of the atom of positive electricity.* The methods of the chemist only enable us to determine

atomic and molecular weights when relatively large quantities of the substance involved are available. With the help of positive rays these numbers can be ascertained even with very minute quantities of substance. In this case we do not find the mass of *atoms* or *molecules* but of positive atomic or molecular *ions*. The two masses differ only by the minute and known mass of the electrons whose loss transforms the neutral atoms or molecules into positive ions.

The principle of the method of measurement is as follows. The substance to be investigated is introduced in the form of a gas or vapour into the space between the electrodes AK of an evacuated vessel (fig. 35). Dependent conduction is brought about by a strong electric field between A and K (producing a luminous discharge as in fig. 22, p. 215). The perforated cathode has two slits 1 and 2. Some of the positive ions in the form of sharply bounded positive radiation fly through this opening into the actual space for experiment. A special pump with very rapid suction maintains a high vacuum in the vessel where the experiment is to be conducted, although some gas or vapour is always streaming through the opening. In the vessel the positive rays are deviated by an electric field and a magnetic field. In the electric field the path is a parabola. For x, the deviation, we have

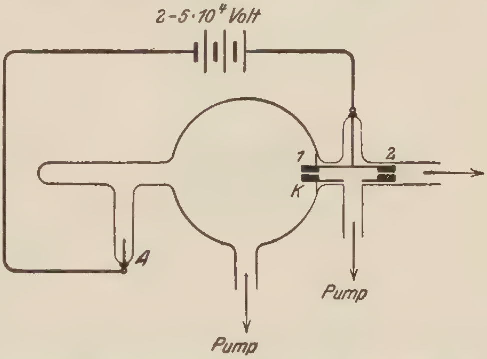

Fig. 35. Discharge tube for the production of positive rays.

$$x = \frac{1}{2} 10^7 \frac{\mathbf{e}}{m} \cdot \mathbf{R} \cdot \frac{y^2}{u^2}, \tag{7}$$

from fig. 32, p. 221.

In the magnetic field the path is a circle. For the radius of curvature of the path equation (5), p. 211, again holds:

$$r = \frac{1}{\mathbf{H}} \frac{m}{\mathbf{e}} \cdot u \cdot \frac{1}{10^7 \mu} \cdot \tag{5}$$

Here in both equations $\mathbf{e}$ means $\mathbf{n}$ elementary quantities of electricity or $\mathbf{n} . 1 \cdot 6 . 10^{-19}$ coulombs, if the ion is $\mathbf{n}$-valent.

By combining (5) and (7), we can eliminate u, the velocity of the ions, and find the ratio e/m. Substituting the ionic charge $\mathbf{e} = \mathbf{n}e$ ($\mathbf{n} = 1, 2, 3, \ldots$) gives us m, the required mass in grams of the single atomic or molecular ion. The chemist does not use the gram as his unit of mass for atomic or molecular weights, but the mass of a hydrogen atom*, i. e. $1 \cdot 65 . 10^{-24}$ gm. On dividing by this number we obtain the ordinary relative values of the atomic or molecular weights from the measured values of m.

The most perfect of the many practical forms of the experimental vessel is certainly the *mass spectrograph*, due to Aston, which is sketched in fig. 36. In the figure the electric field **R** bends the rays downwards. They then travel in a straight line to the magnetic field **H**, which is of circular cross-section, with its lines of force at right angles to the plane of the paper. The rays end at B on a photographic plate after traversing another straight path. This arrangement offers a special advantage: the production of the positive rays in the field AK involves unavoidable fluctuations in the velocity of the ions, and this would lead to a haziness of the image at B. If we adjust the mass spectrograph of fig. 36 properly, we can ensure that the rays with the same value of e/m but with different velocities u reach the same point B. Fast rays go by the path 1, slow rays by the path 2. The blackening of the plate at B is sharply defined and its position can be accurately measured.

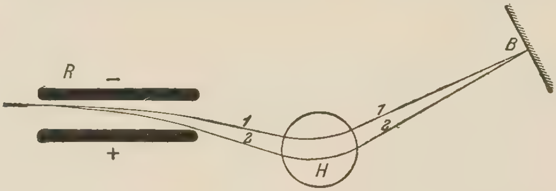

Fig. 36. Path of the rays in Aston's mass spectrograph.

The mass spectrograph gives us much more information than chemical methods do. For example, the chemist obtains the atomic weight 35·46 for chlorine. The mass spectrograph shows that chlorine is a mixture of two kinds of atoms ("isotopes") of atomic weights 35 and 37. Details of these matters, which are of fundamental importance for the understanding of the periodic table of the elements, would lead us too far out of our way here.

Only one point still remains to be mentioned. In none of the numerous quantitative investigations of positive rays has *an elementary quantity of positive electricity with a mass less than* $1 \cdot 65 . 10^{-24}$ *grams, i. e. smaller than the mass of a hydrogen atom, ever been found.* All endeavours to discover an atom of positive electricity of minute mass corresponding to the electron have so far been fruitless. *The hydrogen ion probably represents the atom of positive electricity. It has recently been called the "proton" or the "hydrogen nucleus"*. Further experimental developments, however, have still to be awaited.

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* More accurately, $^1/_{16}$ of the mass of an oxygen atom.

11. Independent Conduction of Electricity in Gases at High Pressure. (Discharge from Points, Brush Discharge, Spark, Arc.)

These well-known phenomena have formed the subject of numerous investigations, and yet we are still very far from understanding their details quantitatively. We shall confine our account of them to a few characteristic features.

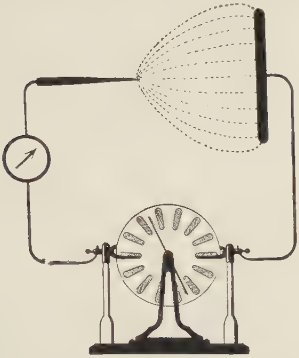

Fig. 37. Lines of electric force between a point and a plate, diagrammatically.

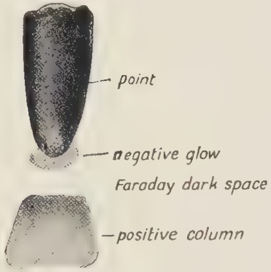

Fig. 38. The discharge from a point in air as seen with the microscope.

(1) *The discharge from points.* This occurs in non-uniform electric fields in the regions where the electric intensity **R** is high. As an example, fig. 37 shows a condenser formed of a point and a plate connected to an induction machine. Let the pressure be about 2000 volts. Where the lines of force crowd together at the point there is a brush shining with a bluish-red light visible to the naked eye. Further details are shown by microscopical observation. We find the same phenomena as we observed in the luminous discharge at low pressure. In fig. 38 we see the negative glow immediately in front of the point acting as *cathode*, then the Faraday dark space and adjacent to the latter a distinct positive column. These names are the same as those of fig. 22, p. 215.

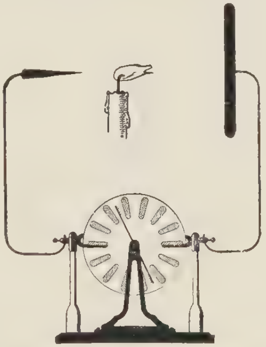

Fig. 39. The electric wind.

The formation of electricity carriers as a result of ionization by collision is confined to the region of the small luminous brush. In the luminous discharge at low pressure the region of ionization by collision fills the whole interval AK between the electrodes.

The carriers of electricity of the same sign as the point stream out from the luminous brush into the main dark portion of the path of the current, and drag the molecules of air along with them owing to internal friction. This causes the "electric wind", which is shown blowing a flame to one side in fig. 39.

Of course the phenomenon of the electric wind is not confined to discharges from points; but it is particularly easy to demonstrate in this case. An electric wind, i. e. a dragging of the gas along by the ions traversing it, occurs in the most varied forms of dependent and independent conduction of electricity through gases. All that is required is that an excess of electricity carriers of one sign should be moving in the part of the conducting path concerned; otherwise the effects of the ions moving in opposite directions cancel. In the case of the discharge from a point the above condition is particularly well fulfilled. In fig. 39 there are no ions moving in the whole dark path of the current at all except those of *one* sign, namely that of the point.

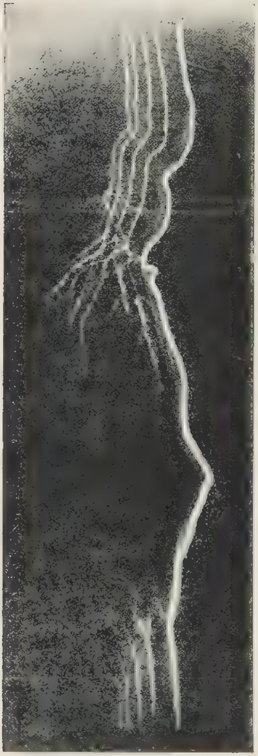

Fig. 40. Formation of an electric spark. The anode is at the top. (Photograph on a moved plate, by B. Walter).

(2) *The brush discharge.* In the case of the discharge from a point a galvanometer inserted in the circuit shows that the current remains very steady. If the intensity of the field is raised further a new form of independent conduction appears, namely the brush discharge. Single luminous lambent and branching threads are seen darting far out into the field of the condenser, and the galvanometer shows large fluctuations in the strength of the current.

A non-uniform electric field is again essential for the production of the brush discharge. The brushes emerging from the anode are longer than those from the cathode. (This is true for the air of the room, in which the brushes shine with a reddish light.) The brushes are sufficiently long to enable us to find out without difficulty which is the positive pole of an induction machine, induction coil, etc.

(3) *The spark.* The formation of sparks is associated with extremely great fluctuations in the strength of the current. Sparks are produced in non-uniform electric fields whenever the brush discharge bridges over the whole electric field between the electrodes AK. Fig. 40, which is from a photograph on a moved plate, shows the gradual formation of a spark between two points which are the boundaries of the field. The brushes in successive impulsive currents become longer and longer. If the brushes on either side become united the electric field breaks down, giving rise to a brief impulsive current of high intensity. The light due to vigorous ionization by collision is often blinding and a loud explosive noise may be produced by strong local heating along the path of the current.

The largest sparks known to us, namely lightning flashes, are formed in the same gradual way.

Table IV gives some numerical values showing the connection between the pressure and the length of the sparks obtained. The

Table IV.

Lengths of Sparks in Ordinary Air.

At a pressure of	Between points	Between spheres of		Between planes
		5 cm. diameter	30 cm. diameter	
20000 volts	15·5 mm.	5·8 mm.	6·0 mm.	6·0 mm.
40000 ,,	45·5 ,,	13 ,,	13·3 ,,	13·7 ,,
100000 ,,	220 ,,	45 ,,	35·7 ,,	36·7 ,,
200000 ,,	410 ,,	262 ,,	75·3 ,,	75·3 ,,
300000 ,,	600 ,,	530 ,,	126 ,,	114 ,,

table refers to electrodes with various radii of curvature. The corresponding lines of force diagram is given in fig. 7a, p. 24. From the table we see how favourable a small radius of curvature is for the passage of sparks. A small radius of curvature means that the field is very far from being uniform, and hence that high intensities which favour ionization by collision occur near the surface of the electrodes.

The phenomenon known as the "lag" of the spark also provides evidence for the part played by ionization by collision in the formation of sparks. If the electrodes are connected with the source of current for a short time only, sparks will scarcely begin to pass at the pressures given in the table. On the contrary, we can exceed these pressures by several times their value without sparks being formed, provided we leave the electrodes in connection with the source of current for only about $^1/_{100}$ of a second.

The air of the room in itself contains only a few ions, usually of the order of a thousand per cubic centimetre. Hence an interval of time which can be conveniently measured must obviously elapse in order that the number of ions *along the whole of the interval between the electrodes* may be raised by many powers of ten by ionization by collision. The fact that the lag can easily be prevented is evidence in favour of this interpretation of the phenomena. In order to bring about the initial conditions required to make ionization by collision rapidly effective we have only to ionize the air between the electrodes strongly by means of X-rays or the rays from radioactive substances.

(4) *The arc.* The form of independent conduction in gases and vapours which is known as the arc is produced by apparatus arranged as in fig. 41. E is a source of current (a battery or the town mains) of at least 40 volts. R is a practical form of resistance (fig. 30, p. 18). A and K are the electrodes, which are usually rods of metal or carbon. The arc is produced by separating the rods after making them touch

for a moment. An ammeter in the circuit shows that the current is very steady.

In essentials the arc resembles the luminous discharge much more closely than the discontinuous brushes or sparks do. Its use as a source of light, especially in projection lanterns, is well known. In this case a current of 5 amperes usually suffices. For technical purposes, e. g. for welding, arcs of hundreds of amperes are used. Hence the source of electricity must supply very considerable quantities of electricity per second in order to maintain the field between the electrodes.

The chief source of the electrons which carry the arc is the cathode, which becomes white-hot. The importance of these electrons is best shown by the vain attempt to maintain an arc with a cold cathode. An experiment of this kind is shown in fig. 42. One electrode is a metal plate, the other a rod of carbon or metal. As long as the rod is the cathode, the arc can be moved about over the plate in any way, following every movement of the cathode. But if the rod is the anode the arc is invariably broken. Its cathode end cannot be pulled along to a neighbouring cold part of the plate.

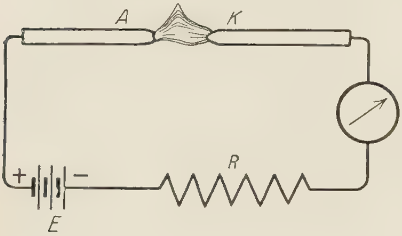

Fig. 41. The electric arc.

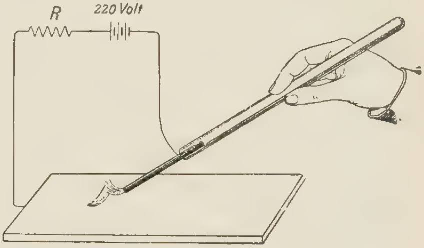

Fig. 42. Arc with moving cathode.

Very high temperatures occur in the vapour of the arc. In the crater which is hollowed out in the anode by evaporation the temperature amounts to about 3700° C. The production of this high temperature is the chief reason for the importance of the arc in the physical laboratory. The arc is indispensable for many welding processes and numberless optical investigations.

As a single one out of numerous examples we shall mention the mercury arc lamps found in every laboratory. This is the most convenient source of ultra-violet light down to about $\lambda = 200\,\mu\mu$*. The electrodes consist of liquid mercury in the two limbs of a tube of quartz glass (fig. 43). The arc "burns" in an atmosphere of mercury vapour, often at a pressure greater than atmospheric. In medical advertisements the mercury arc lamp is described as a source of "artificial sunlight".

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* [μ is the symbol for a *micron*, i. e. 10^{-3} millimetre; $\mu\mu$ denotes a thousandth of a micron.]

In electrical engineering the arc is continually gaining in importance owing to the construction of "mercury vapour rectifiers". These rectifiers enable alternating current to be transformed into direct current without mechanical apparatus and practically without any attention. They depend on the fact that an arc can only exist between a mercury electrode and an iron electrode if the mercury is the cathode.

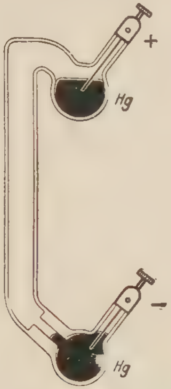

Fig. 43. Mercury lamp made of quartz glass.

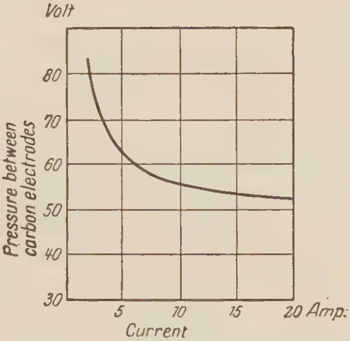

Fig. 44. Characteristic of an arc.

Rectifiers of this kind are made even for powers exceeding 1000 kilowatts.

So much for our summary survey of independent conduction of electricity at high pressures. One other point remains to be discussed, namely the connection between current and pressure in independent conduction through gases. This relationship is important in connection with experimental technique.

Hitherto we have met with two characteristic forms of the relationship between the current and pressure in conduction phenomena:

(1) Current and pressure are proportional to each other, i. e. Ohm's law is true.

(2) The current reaches a saturation value when the pressure is raised, e. g. as in fig. 14, p. 206.

In the luminous discharge and the arc — we shall confine ourselves to these cases — a third form of relationship occurs, the so-called *falling characteristic* shown in fig. 44. The pressure graph is obtained from measurements with the apparatus shown in fig. 45. The strength of the current is varied by means of the sliding resistance R and the pressures between the ends of the arc which correspond to various strengths of current are measured. *Raising the current causes the pressure between the ends of the arc to fall; this is the meaning of* *the falling characteristic.* The curve in fig. 44 is not prolonged to meet the vertical axis: neither the arc nor the luminous discharge persists until the pressure rises above a definite maximum value which depends on the conditions of experiment.

The falling characteristic arises from the combined effects of a series of phenomena which are not independent of each other. In connection with electrolytic conduction we shall have an opportunity of explaining the production of a falling characteristic in a comparatively simple particular case (p. 245). The phenomena in the luminous discharge and the arc are too complicated.

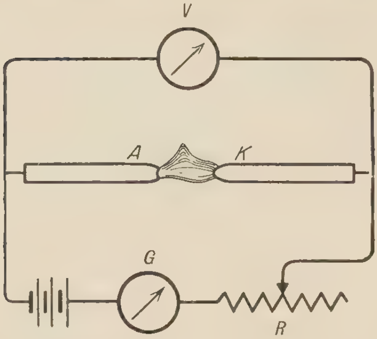

Fig. 45. Apparatus for obtaining the characteristic of an arc. The voltmeter V must have such a high resistance that the current through it is negligible in comparison with the current in the arc.

12. Conduction in Liquids. General Considerations.

Liquids are distinguished from gases and vapours by the fact that their molecules are packed ever so much more closely together. The distance between the molecules only amounts to a fraction of the diameter of a molecule, and, moreover, the molecules are swarming about with vigorous but haphazard thermal motions. A box full of ants is a childish but very apt picture of what is going on.

According to our fundamental experiments (pp. 194—197) conduction of electricity always consists of a motion of atoms of electricity in the electric field. If a liquid is to conduct electricity, it is necessary that atoms of electricity should be able to get through the tangled mass of closely packed molecules.

In the case of gases we have learned to distinguish between two cases:

(1) The atoms of electricity are carried by material carriers: *conduction by carriers.*

(2) Electrons move about quite freely: *conduction by electrons.*

We should expect the same two cases to occur in conduction in liquids, and both are actually observed.

Conduction by carriers is met with in all kinds of liquids*, conduction by electrons so far in liquid metals only. Electronic conduction in liquid metals resembles that in solid metals to a very great extent. Hence liquid and solid metals are conveniently treated together, and this is done on pp. 246—254. At first, therefore, we shall confine ourselves to *conduction by carriers* in liquids.

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* [In this type of conduction the liquid is called an *electrolyte.*]

13. Electrolytic or Ionic Conduction in Aqueous Solutions.

In the case of conduction through gases, we began with a case in which we could be perfectly clear about the origin and nature of the carriers of electricity. We shall now proceed in the very same way with conduction in liquids; we therefore begin with conduction in aqueous solutions.

Pure water is quite a poor conductor. Fig. 46 shows a tube about 10 cm. long with a cross-section of about 1 sq.cm. *A* and *K* are two metal electrodes. The tube is filled with ordinary distilled water. With 220 volts between the electrodes the ammeter registers only about 2.10^{-4} of an ampere. If we proceed to add to the water about 5% by weight of common salt (NaCl), the water conducts well; the ammeter registers about 1·5 amperes.

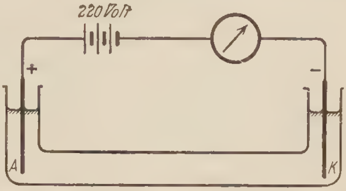

Fig. 46. Ionic conduction in water.

It follows that electricity carriers must have been introduced into the water by the salt. Of what nature are they?

We remember the air which was made conducting by means of charged dust (p. 199). On that occasion positive flour carriers and negative sulphur carriers moved in the electric field. Subsequently we found the carriers at the electrodes after they had given up their charges: sulphur at the anode, flour at the cathode.

What is the meaning of the corresponding results in the case of water made conducting by salt? At the cathode we find metallic sodium (Na), at the anode chlorine gas (Cl). Hence the conclusion: the positive carriers in water are sodium ions, the negative carriers chlorine ions.

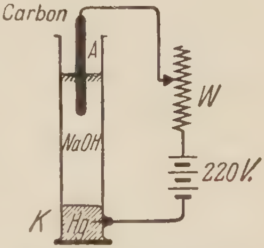

Fig. 47. To show the presence of sodium ions at the cathode.

In order to show the presence of the sodium and the chlorine at the electrodes the experiment must be arranged with this end in view. For example, the anode should consist of a carbon rod. The chlorine would immediately react chemically with metallic electrodes, instead of rising in visible bubbles. The discharged sodium ions, i. e. sodium atoms, immediately react with the water at the cathode, forming NaOH with vigorous evolution of hydrogen bubbles. This might be taken to mean that the positive carrier is hydrogen. In order to avoid this reaction, the cathode is made of mercury (fig. 47). The sodium atoms can immediately diffuse into this liquid metal and are thus protected from the molecules of water. The sodium can subsequently be recovered from the mercury,

e. g. by distilling off the latter. A qualitative test for the presence of sodium in the mercury is even more simple. After the current has been stopped, hot water is poured over the mercury. A vigorous evolution of hydrogen immediately begins on the surface of the mercury and the water gives the reactions of NaOH, e. g. colours litmus paper blue.

In the apparatus used here the current in course of time exhausts the stock of ions available, so that the current decreases as time goes on. The chlorine atoms escape in the form of gas and the sodium atoms disappear in the mercury. To this extent we may call the conduction dependent, but it is not usual to do so.

By suitable choice of the conditions of experiment we can keep the stock of ions unaltered. As an example, we shall take sulphuric acid (H_2SO_4) in water. We first use lead electrodes. Hydrogen ions appear at the cathode as positively charged carriers. The negative carriers turn out to be the molecular ions SO_4. They immediately react with the lead electrode and form a coating of lead sulphate ($PbSO_4$), so that the stock of ions is diminished in this case also. The circumstances are altered, however, if we replace the lead electrodes by platinum electrodes. We used the latter e. g. in fig. 13, p. 10. The SO_4 ions cannot react with the platinum electrodes, so they react with the water. Sulphuric acid is re-formed and bubbles of oxygen come off. It is not the supply of sulphuric acid that is diminished, but the supply of water. Hence we often speak briefly of the "decomposition of water". (We should again be far wrong if we took the oxygen to be an electricity carrier; it is merely a product of a secondary reaction.)

These examples can be multiplied indefinitely. *In all cases of electrolytic conduction care must be taken to recognize the actual ions appearing at the electrodes and not to confuse them with products of secondary reactions.* The results of all these experiments may be briefly summed up as follows. *In all aqueous solutions of salts and acids the atoms of metal or hydrogen move to the cathode as positive carriers: so do radicals like ammonium* (NH_4) *which can replace metals or hydrogen chemically. The acidic radicals, on the other hand, move to the anode as negative ions.*

So much for the nature of the carriers. Where do they come from? Are they produced as soon as the solution is made? Or is the electric field required to split up the salts and acids into their ionic constituents? The answer was given by chemistry long ago: the ions exist even before conduction begins. The splitting up of the substances, their "electrolytic dissociation", takes place as soon as they are dissolved.

The proof of this depends on investigations of osmotic pressure. This is found to be e. g. twice as large for solutions of common salt as one would expect from the number of molecules of NaCl dissolved. From this we infer that every molecule is split into two individuals, namely the positive sodium ion and the negative chlorine ion.

This splitting up of neutral molecules into ions is not by any means a new fact which refuses to fit into the framework of other experiments. There is no doubt that chemical forces are of an electrical nature. Chemical affinity is a problem of the electric fields of the atoms. To a very rough degree of approximation a molecule of NaCl can be thought of as being like the lines of force diagram of fig. 7a, p. 24 (attraction between opposite poles). The force of attraction between the two oppositely-charged ions in this rough conception of the molecule is

$$F = \frac{1}{4\pi K}\,\frac{qq'}{x^2},$$

by equation 16, p. 79. Thus here the absolute dielectric constant K occurs in the denominator. K is 81 times as great for water as for air or for a vacuum (i. e. $K' = 81$). Hence surrounding the NaCl molecule with water means reducing F, the force holding it together, to $^1/_{81}$ of its value for air. The thermal motions of the molecules may then be sufficient to separate the two ionic constituents of the NaCl molecules from each other.

14. The Ionic Charge. Faraday's Law of Equivalents. Avogadro's Number N.

Once more we briefly recall the phenomena of conduction through gases with visible electricity carriers in the form of dust. The separate particles of flour or sulphur bear charges of differing magnitude. The defect of electrons in the positive carriers and the excess of electrons in the negative carriers vary from one carrier to another. The defect or excess is determined by the help of microscopic observation of single particles and is always found to be $\mathbf{n}.1{\cdot}6.10^{-19}$ coulombs, where **n** means a whole number which is usually small. $1{\cdot}6.10^{-19}$ coulombs is the elementary quantity of electricity, the smallest number of coulombs ever observed in nature.

How about the charge on the carriers in ionic conduction in aqueous solutions?

The observation of individual particles which was so successful in the case of dust carriers in air cannot be employed here. Owing to their minute size, single ions are beyond the reach of microscopic observation. In spite of this the question of the ionic charge can be answered with great certainty for electrolytic conduction also. We owe the possibility of doing this to the "law of electrochemical equivalents" discovered by Faraday in 1833.

In order to formulate this law and make its significance in this connection clear we must first recall some of the fundamental ideas of chemistry.

The chemist measures the quantities of substances reacting in *gram-atoms* and in *gram-molecules* or *mols*. By these terms he means the quantities of substance which contain a number of grams equal to the numerical value of the atomic or molecular weight of the substance in question.

Examples:

1 gram-atom of hydrogen means 1 gm. of hydrogen.
1 gram-atom of oxygen means 16 gm. of oxygen.
1 gram-atom of sodium means 23 gm. of sodium.
1 mol of water (H_2O) means 18 gm. of water.
1 mol of oxygen means 32 gm. of oxygen in the form of the diatomic gas found in the atmosphere (O_2).
1 mol of ozone means 48 gm. of the gas ozone or "triatomic" oxygen, each molecule of which consists of three atoms of oxygen.

Moreover, according to the fundamental notions of chemistry, atomic weights and molecular weights are themselves relative values for the weights, or more accurately for the masses, of the various kinds of atoms and molecules. The mass of the single sodium atom is to be 23 times as great as the mass of the single hydrogen atom, and so on. But this is only possible if all the "gram-atoms" and all the "mols" contain one and the same number N of atoms or molecules. This universal constant is called *Avogadro's number**. Its numerical value has been determined to be 6.10^{23} from the kinetic theory of gases and in several other independent ways. So much for the chemical definitions.

Faraday investigated a whole series of electrolytes quantitatively. He compared the quantities of substances liberated with the quantities of electricity which had flowed through the solutions. In this way he discovered a result of remarkable simplicity, which in modern terms is expressed as follows:

A gram-atom of an element or a mol of a compound is liberated by n·96500 *coulombs, where* n *denotes the chemical valency of the element or of the compound.*

For example, we require for the liberation of each of these:

1 gram-atom or 1 gm. monovalent hydrogen†	96500 coulombs
1 gram-atom or 35·5 gm. monovalent chlorine	
1 gram-atom or 108 gm. monovalent silver	
1 mol or 62 gm. monovalent NO_3	
1 gram-atom or 16 gm. divalent oxygen	2·96500 coulombs
1 mol or 96 gm. divalent SO_4	
1 gram-atom or 55·8 gm. divalent iron (from ferrous salts)	
1 gram-atom or 55·8 gm. trivalent iron (from ferric salts) and so on.	3·96500 coulombs

The chemist calls the quotient gram-atom/valency or mol/valency a "gram-equivalent". Thus Faraday's discovery may be stated in this way. 1 gram-equivalent of a substance is electrolytically liberated by 96500 coulombs. Hence the name "law of equivalents".

So much for the empirical facts. What do they prove about the question at issue, the charge on the single ion in electrolytic conduction? The answer is easily given:

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* [Sometimes *Loschmidt's number*.]

† Another way of putting this: e/m, the ratio of the charge to the mass for the hydrogen ion, is equal to 96500 coulombs/gram.

The N ions in a gram-atom or mol carry the charge $\mathrm{n}.96\,500$ coulombs through the electrolyte. Hence each of the N ions carries the charge $\frac{\mathrm{n}.96\,500}{N}$ coulombs, where n denotes the chemical valency of the ion.

As we mentioned above, Avogadro's number N has been found to be 6.10^{23}. Hence for the charge on a single n-valent ion we obtain

$$\frac{\mathrm{n}.96\,500}{6.10^{23}} = \mathrm{n}.1{\cdot}6.10^{-19} \text{ coulombs}.$$

Or in words:

The charge on every ion, no matter whether it is an atomic ion or a molecular ion, is n *times the elementary quantity of electricity* $e = 1{\cdot}6.10^{-19}$ *coulombs, where* n *is the chemical valency of the ion.*

Thus the question about the ionic charge has been accurately answered in a very simple way, which reveals a close connection between electrical and chemical quantities.

Faraday's law of equivalents contains a discovery of great importance. Its atomic interpretation leads us to the equation

$$\underbrace{N}_{\substack{\text{number of the}\\ \text{carriers or ions}\\ \text{in the gram-}\\ \text{atom or mol}}} \cdot \underbrace{\mathrm{n}\,e}_{\substack{\text{charge of the single}\\ \text{carrier or ion = n}\\ \text{elementary quantities}\\ \text{of electricity}}} = \underbrace{\mathrm{n}}_{\substack{\text{chemical}\\ \text{valency}}} \cdot 96\,500 \text{ coulombs}$$

or

$$N e = 96\,500 \text{ coulombs}. \tag{8}$$

This equation must be considered as a fundamental equation of modern atomic theory. It gives us the relationship between the two universal constants N, Avogadro's number, and e, the elementary quantity of electricity. Using it, every determination of e gives us a value for N, and vice versa. It will be used on p. 294 to test a particularly simple way of determining N.

15. Ohm's Law for Electrolytic Conduction. The Velocity of the Ions.

We are now acquainted with the nature and charge of the electricity carriers in aqueous solutions. The carriers are ions which arise as a result of electrolytic dissociation, i. e. the splitting up of neutral molecules when dissolved. Each ion carries a number of elementary quantities of electricity equal to its chemical valency n.

The electric field drags these ions through the more or less viscous fluid. In this case Ohm's law is in general true to a very close approximation: current and pressure are proportional to one another. In order to show this, we may use the apparatus shown in fig. 48. The shaded area represents the electrolyte. For the latter we take, for the sake of convenience, a strip of filter paper saturated with conducting water and placed on a piece of plate glass. A and K are two metal electrodes placed on top of the paper. G is a moving-coil ammeter and V a bifilar voltmeter.

On measuring corresponding values of E and i we obtain e. g. the diagram shown in fig. 49. The relationship between pressure and current is represented by a straight line pointing towards the origin.

With accurate measurements we find that the straight line does not pass exactly through the origin but cuts the pressure axis at a small finite pressure (of the order of 1 volt). Hence the observations do not give $E/i =$ const. but $(E - \varepsilon)/i =$ const. This deviation of the straight line, however, is merely due to a practical detail in the experimental arrangements. The voltmeter measures not only the pressure between the ends of the column of liquid but the so-called polarization ε in addition. The polarization is located between the electrode and the column of liquid. We shall come back to this point (which is quite unimportant here) in a more important connection in the next chapter (p. 274).

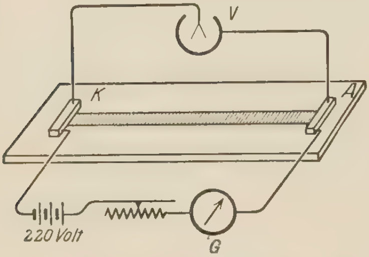

Fig. 48. Apparatus for obtaining the characteristic of an electrolyte.

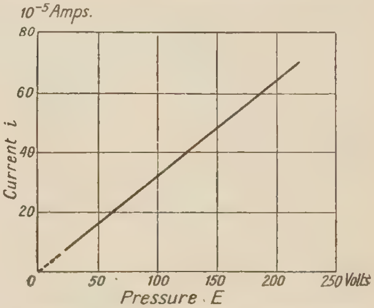

Fig. 49. Ohm's law for electrolytic conduction.

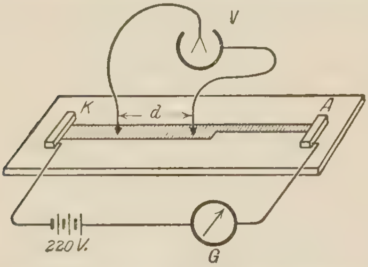

Fig. 50. To show how the resistance depends on the length and cross-section of the electrolyte.

The ratio E/i, as usual, is defined to be the resistance r. r turns out to be proportional to d, the length of the column of liquid, and inversely proportional to its cross-section A. We have

$$r = \frac{d}{A} \cdot \sigma, \tag{9}$$

where σ is a constant. The effects of d and A on the resistance are very simply exhibited by means of the apparatus shown in fig. 50. Equal currents are made to flow successively along paths with two or three differing cross-sections. Two "proof-wires", small metal wires with insulating handles, enable us to determine the pressure between the ends of current paths of differing length and cross-section.

The factor of proportionality σ is called the specific resistance.

Its numerical value has a simple physical meaning; it is equal to the resistance of a cube with an edge of length 1 cm. ($A = 1$ sq.cm., $d = 1$ cm.). Table V gives some numerical values for σ; all these are for a temperature of 18° C. ("room temperature"). σ has a large temperature coefficient; it decreases by about 2% per degree of temperature rise. Hence exact measurements of current and pressure in electrolytes require the temperature to be kept scrupulously constant by means of water-baths. Besides values of σ the table also gives the reciprocals $1/\sigma$. $1/\sigma$ is called the *conductivity* (or *specific conductance*) $\varkappa$; the numerical value of $\varkappa$ gives the length of a conductor of cross-section 1 sq.cm. which has a resistance of 1 ohm.

Table V.

Specific Resistance and Conductivity of Aqueous Solutions (at 18° C.).

Substance	Concentration η		Specific resistance σ in ohm.cm.	Conductivity $\varkappa$ in ohm^{-1}.cm.$^{-1}$		Remarks
Common salt (NaCl)	10^{-7}	mols per cubic centimetre	$9{\cdot}3.10^{4}$	108.10^{-7}		$\varkappa/\eta$ approaches a constant limiting value (108) as the concentration falls
	10^{-6}		$9{\cdot}4.10^{3}$	107.10^{-6}		
	10^{-5}		$9{\cdot}8.10^{2}$	102.10^{-5}		
	10^{-4}		$10{\cdot}9.10^{1}$	92.10^{-4}		
	10^{-3}		$13{\cdot}5$	74.10^{-3}		
	5	weight per cent	15	67	10^{-3}	
	20		5·1	196		
Zinc sulphate ($ZnSO_4$)	5		53	19		
	20		21	47		
Hydrochloric acid (HCl)	5		2·5	395		
	20		1·3	762		
	40		1·9	515		
Sulphuric acid (H_2SO_4)	5		4·8	209		
	20		1·5	653		
	30		1·4	740		
	40		1·5	680		
For comparison:	Copper		$1{\cdot}8.10^{-6}$	$5{\cdot}7.10^{5}$		

So much for the empirical facts. The essential features of Ohm's law have already been made clear on pp. 200—201, so that we can refer back to equation (3) (p. 201), which we deduced on that occasion, namely

$$i = \frac{A}{d}\,\mathbf{N}\,(\mathbf{n}\,e)\,(v_+ + v_-)\,E\,.$$

Comparing this with equation (9), we obtain

$$\frac{1}{\sigma} = \varkappa = \mathbf{N}\,(\mathbf{n}\,e)\,(v_+ + v_-)\,. \tag{9a}$$

The product $\mathbf{N}(\mathbf{n}e)$ is the total charge of the carriers of *one* sign in one cubic centimetre. For an electrolyte it can be immediately obtained from Faraday's law of equivalents. We only require to know the concentration of the solution, i. e. the number of mols of dissolved substance in 1 cubic centimetre of water. The sum of the *mobilities of the carriers*, i. e. $v_+ + v_-$, may then be calculated from the measured values of $\varkappa$.

We shall illustrate this by an example. By Table V, $\varkappa = 108.10^{-7}$ ohm^{-1}. cm.$^{-1}$ if 10^{-7} mol NaCl is dissolved in 1 c.c. water. Both sodium ions and chlorine ions are monovalent, so that $\mathfrak{n} = 1$. Hence the charge on the carriers of *one* sign contained in *one* c.c. of solution is $10^{-7}.96500$ coulombs. Accordingly we have

$$v_+ + v_- = \frac{108.10^{-7}}{10^{-7}.96500} = 11{\cdot}2.10^{-4} \frac{\text{cm.}}{\text{sec.}} \Big/ \frac{\text{volt}}{\text{cm.}}.$$

According to the present calculation the mobility of the ions in water, i. e. their velocity in a field of $\mathbf{R} = 1$ volt/cm., turns out a thousand times smaller than that of the ions in the air of the room or in other gases at atmospheric pressure. This is really due to the much greater internal friction in liquids. Besides, we ought never to think of a sodium ion in aqueous solution as being merely a simple atom of sodium which has lost one electron. Neutral molecules of water always form a layer over the sodium atoms, just like dust on a larger charged body.

Direct measurements of the ionic mobilities in aqueous solution confirm the order of magnitude of the calculated results. The method is easy to follow, as it involves the use of coloured ions whose motions can be directly traced by the eye. *A* and *K* are ordinary platinum electrodes in a glass U-tube (fig. 51). Two different aqueous solutions of equal density are arranged in layers in the tube, $KMnO_4$ above, KNO_3 above.

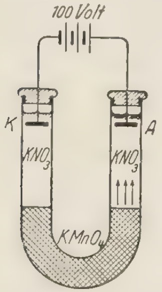

Fig. 51.
Measurement of the velocity of coloured ions in an electric field.

The K and NO_3 ions do not absorb any visible light and hence do not colour the water. The MnO_4 ions, on the contrary, do; they form a distinct purplish-red colouring-matter.

A pressure of say 100 volts is produced between *K* and *A*. The length of the tube is about 20 cm., so that the intensity of the field is about 5 volts/cm. We can watch the continuous column of coloured ions moving quite slowly towards the anode, at about the rate of 1·8 mm. per minute. If the pressure is doubled we see that the velocity is doubled. The velocity of the ions turns out to be actually proportional to $\mathbf{R}$, the strength of the field. The factor of proportionality v, the ionic mobility of the MnO_4 ions, is found to be $6.10^{-4} \frac{\text{cm.}}{\text{sec.}} \Big/ \frac{\text{volt}}{\text{cm.}}$, which is of the order of magnitude anticipated.

The proportionality between u, the velocity of the carriers, and $\mathbf{R}$, the strength of the electric field, is conditioned by resistances of a frictional nature (p. 201). Hence the internal friction of the liquid must brake the ions in this case. *The existence of this friction between the ions and the liquid can be proved experimentally.*

In fig. 52 we see a ring-shaped trough with an insulating bottom. The two cylindrical side walls form the electrodes A, K. The liquid is a solution of any salt. The ions move radially through it. The trough is set on the north pole of a bar magnet, i. e. the magnetic lines of force penetrate the ring of liquid almost vertically. The moving ions are now deviated by the magnetic field, like any other current. This deviation is in a direction at right angles to the direction of motion of the ions and at right angles to the lines of magnetic force (fig. 17, p. 143). Now if friction occurs between the ions and the liquid, the lateral motion of the ions must be transferred to the liquid. This is actually the case: the liquid begins to circulate vigorously in the direction of the arrow. The motion can be made visible at a distance by means of a floating cork with a small flag attached to it. The direction of rotation is reversed if the current is reversed. Thus the existence of friction between the ions and the water is confirmed.

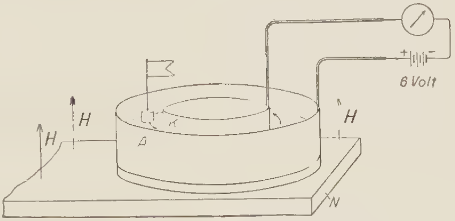

Fig. 52. Friction between ions and liquid.

The brief expression "friction between the ions and the water" may be misleading. External friction in the boundary layer between the carrier and the liquid is certainly not what is meant. The layer of liquid immediately bounded by the carrier adheres to the carrier. What is meant is the internal friction of the liquid: the friction of the "tubes of flow" of the liquid which slide along each other where they spread out round the carrier. See fig. 53, where the carrier or the ion is diagrammatically represented as a simple sphere. In this model experiment single tubes of flow have been made visible by dissolved colouring-matter.

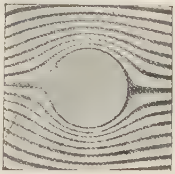
Fig. 53. Tubes of flow where there is internal friction.

Highly concentrated electrolytes exhibit important pecularities. The validity of Ohm's law is merely simulated owing to the smallness of the electric intensities ordinarily used. When the range of electric intensities is extended there is no question of the quotient E/i being constant, as the deviations may exceed 50%. The latter are particularly large in the case of polyvalent ions. The conductivities ($\varkappa$) given in Table V for some high concentrations are of practical importance, it is true. In reality, however, the numbers merely represent *lower* limiting values. The upper limiting values are reached when the electric intensity is of the order of 10^5 volts/cm. If we divide these *upper* limiting values by the high concentrations used (η), we find that the quotient $\frac{\varkappa}{\eta}$, the molecular conductivity, is equal to the value $\left(\frac{\varkappa}{\eta}\right)_\infty$ obtained in the limiting case of indefinite dilution, e. g. 108 for NaCl (Table V on the right). Thus with high intensities the factor which slows down the ions in concentrated electrolytes is absent (Max Wien).

Tentative explanation: The motions due to the mutual electric attractions between the ions are superposed on the thermal motions of the ions in the solution. In the long run negative ions must predominate in the neighbourhood of a positive ion and conversely. The excess charge of the opposite sign limits the motion of the ion. Time is required, however, for the formation of this disturbing excess charge, and when the ion attains a high velocity (of the order of a metre per second) owing to the electric intensity being high, sufficient time is not available. Hence in the case of high electric intensities the disturbing factor is absent altogether.

In the case of low electric intensities and small ionic velocities there is sufficient time for the excess charge to be formed, but even then its restrictive effect on the motion of the ions may at least be greatly reduced by the following artifice. The motion of the ions is confined to minute paths traversed first in one direction and then in the opposite, i. e. high frequency alternating fields are used (H. Zahn).

16. How the Heating Effect of the Current Arises.

The friction on the ions causes the well-known heating of the path of the current which is usually referred to as the "heating effect of the current" or the "Joule effect". Except for the vanishingly small remainder $\frac{1}{2} m u^2$, friction consumes the whole kinetic energy $\frac{1}{2} m U^2$ which the single ion would receive from the electric field *if there were no friction.* It is easy to find the total amount of energy, $\sum \frac{1}{2} m U^2$, which all the ions along the path of the current receive from the field and dissipate as friction per second.

Let q again be the charge of all the ions in the path of the current, $u_+ + u_-$ their velocity, i. e. the total length of path traversed by them per second, and d the distance between the condenser plates or electrodes.

Between the ends of the path $(u_+ + u_-)$ there is a pressure ε, which is only a small fraction of E, the whole pressure between the electrodes. In fact we have

$$\varepsilon = E \frac{u_+ + u_-}{d}.$$

In moving the charge q along the path $u_+ + u_-$, the electric field does work

$$q\,\varepsilon = q\,E \frac{u_+ + u_-}{d},$$

by equation (12), p. 74. On the other hand, this ionic motion means a current

$$i = q\frac{u_+ + u_-}{d} \text{ amperes},$$

by equation (1b), p. 196. Hence we obtain

$$q\varepsilon = iE \qquad (10)$$

as the energy which the electric field gives up to the ions per second and which is immediately lost again by the ions in the form of heat developed by friction.

Hence the heat developed by the current along the conducting path per second amounts to iE *volt-amperes or watts.* This result, which has here been deduced for the special case of an electrolyte, is true in general, for we have merely made use of two equations which hold for *any* conduction current.

If in a particular case, e. g. electrolytes or metal wires, *Ohm's law holds*, E may be replaced by ir. We then obtain the expression ri^2 watts for the *heat developed by the current per second.*

We have again reduced the transformation of electrical energy into another form of energy to the fundamental experiement of fig. 73, p. 72, and equation (12), p. 74, namely $W = qE$.

The heating effect of a current in an electrolyte is shown in a very striking way in our own bodies. For example, if we send a conduction current of a few amperes through our arms and shoulders (fig. 54) we feel a strong heating effect, especially in the joints of the hand.

In the joints of the hand the path of the current has the smallest cross-section A, i. e. the greatest resistance r per unit length,. by equation (9), p. 236. Hence ri^2, the heat developed by the current per second, also reaches its greatest value there.

It is quite out of the question to carry out this experiment with direct current. The ions in the contents of the cells move in the electric field up to the cell boundaries. Hence the concentrations in the cell-contents are changed. With currents of low intensity [$i < 10^{-2}$ ampere (fig. 31, p. 18)] the changes of concentration cannot take place to any extent worth mentioning, for they are continually being equalized again by thermal diffusion. With currents of several amperes, however, the compensating effect of diffusion is nothing like sufficient to do this. Even within a fraction of a second, such currents cause chemical changes in the cell-contents which are fatal to the organism.

Instead of direct current, which at higher strengths is dangerous to life, we must use high frequency alternating currents ($n > 10^5$ cycles per second). Then since the motion of the ions is known to be slow no conceivable alteration of concentration can arise within the short time of a half-period (say a wave-crest). In any case it would be immediately reversed when the current flowed in the opposite direction in the next half-period (wave-trough). This reversal is most completely attained when successive amplitudes of the alternating current are exactly equal. If we use an oscillating circuit (fig. 54) as a source of alternating current, its oscillations should be as undamped as possible, i. e. should die away slowly. The simplest way of attaining this in practice is to use a special form of spark gap in the oscillatory circuit. Instead of the well-known spark gap formed by spheres we use a "quenching gap", which consists of a series of silvered copper plates separated by thin rings of mica.

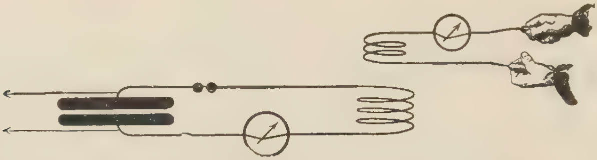

Fig. 54. Lack of physiological effect of high frequency alternating current.

17. Practical Applications of the Electrolysis of Aqueous Solutions.

On pp. 231—240 we dealt mainly with the mechanism of electrolytic conduction in aqueous solutions; but we must not overlook the practical importance of electrolytic conduction either.

In the first place we mention *electroplating*, which is of great practical importance in many industries. The metallic ions coming to the cathode are used to produce metal coatings. The electrodeposition of nickel and gold on common metals, e. g. iron, is now well known to all. Non-conducting bodies, such as wax models, wooden blocks for illustrations, etc., are first rubbed over with powdered graphite and thereby receive a superficial conducting layer. They can then be used as cathodes and can be coated with copper or some other metal by means of the current. The production of good electroplated coatings requires considerable experience. It is essential that suitable metallic salts should be selected (complex salts are generally used) and that the current density should be kept at a definite value.

Besides this we shall briefly mention the production of insulating surface layers by means of electrolytic conduction. The case represented in fig. 55, with aluminium and lead electrodes in an aqueous solution of an alkali borate, is particularly well known. E is a source of current at a pressure of about 40 volts. As long as the aluminium plate is the cathode, several amperes flow through the electrolytic cell with a vigorous evolution of gas. Everything is quite different if the current is reversed. Within a few seconds the current falls practically to zero. When the aluminium acts as the anode it becomes covered with an invisible insulating layer, which holds out against a pressure of say 40 volts. The coating probably consists of an insoluble compound of aluminium and a skin of hydrogen.

This so-called "aluminium cell" is used in two ways:

(1) *As a rectifier or valve.* The battery E in fig. 55 is replaced by a source of alternating current at a pressure of less than 40 volts. The assumption that the cell lets through only the one set of half-waves of an alternating current is verified, for the *moving-coil* voltmeter shows a permanent and constant deflection. This is impossible for alternating current, so that the rectifying effect of the cell is proved.

In order to rectify alternating pressures above 40 volts we connect several valve cells in series.

(2) *As a condenser of high capacity.* The sheet of aluminium and the electrolyte form the plates of a condenser, the dielectric being the thin invisible layer. As the distance between the plates is very small, the capacity is very high, by equation 6, p. 69; it may reach several microfarads per square decimetre of plate surface.

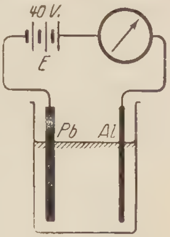

Fig. 55. The aluminium cell.

18. Ionic Conduction in Fused Salts and in Supercooled Liquids (Glasses).

In all the previous examples of ionic conduction in liquids we used aqueous solutions of salts and acids throughout. The splitting up of the dissolved molecules into ions or in short their *electrolytic dissociation* is the essential feature of the phenomenon.

Other solvents besides water may be considered, e. g. alcohol and ether; but in them the amount of dissociation is much smaller.

By what we have said on p. 233, the special behaviour of water is connected with its remarkably high dielectric constant ($K' = 81$). Ether has a dielectric constant of only 4·1 and alcohol one of 25. Formic acid, on the other hand, has a dielectric constant comparable with that of water, namely $K' = 62$. Accordingly the dissociation of e. g. NaCl in formic acid is scarcely less than it is in water. A high value of K', however, is not the only decisive factor, as other chemical factors are involved; thus e. g. a solution of HCl in formic acid does not conduct electrolytically at all.

Molecules in *fused salts and bases show considerable electrolytic dissociation*, as well as those in solutions. As examples we may mention fused NaCl (carbon electrodes being used) or fused NaOH. The electrolysis of such fused substances plays an important part in modern metallurgy.

We must further mention *electrolytic conduction in glasses.* Regarded from the physical point of view, a glass is not a solid body, but a supercooled liquid of very great internal friction. Every solid body has a well-defined *melting-point.* Glass, on the contrary, has no melting-point. All that happens when it is heated is that the internal friction steadily decreases. The glass first becomes viscous like pitch, then mobile like oil.

The apparatus sketched in fig. 56 may be used to demonstrate electrolytic conduction in ordinary glass. The anode consists of a liquid alloy of sodium and mercury in a porcelain crucible heated to about 300° C. The cathode K is an aluminium wire. G is a glass tube containing rarefied helium. The source of current has a pressure of about 200 volts. The current passes from K to the hot glass wall as a typical *luminous discharge* — the essential features of the positive column are well developed (cf. fig. 22, p. 215). The current then passes through the glass wall as an electrolytic current. The positively charged sodium ions move from A through the glass to its inner wall, where they are discharged by becoming united with electrons from the "positive column". *The metallic sodium is separated as a shining mirror on the inner wall* (shown dotted in the figure). *Yet the glass is still solid enough to resist the external atmospheric pressure.* Thus the ions are capable of moving through exceedingly viscous liquids if the strength of the field is sufficiently great.

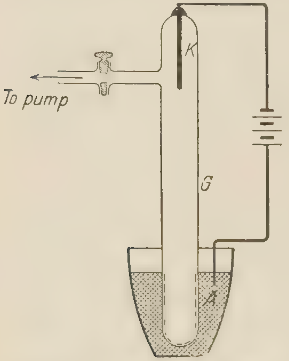

Fig. 56. Motion of ions through solid glass.

The specific resistance of electrolytic conductors falls as the temperature rises (p. 237). *The temperature coefficient of σ must be negative.* This can be shown in a very striking way with a glass rod which conducts electrolytically. In fig. 57 a glass rod as thick as a pencil is imagined to be clamped between two metal springs A and K, which act as electrodes. The town mains provide the current (220 volts). The ammeter G has a range up to about 50 amperes. R is a protective resistance of about 5 ohms, to prevent the current from increasing too much and burning out the fuses.

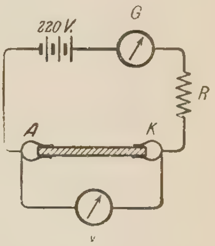

Fig. 57. The effect of temperature on ionic conduction in glass.

At room temperature the current is too small to be measured and the voltmeter V shows a pressure E of 220 volts between the two ends of the glass rod. Glass is of course a very good insulator at room temperature, but does not remain so when heated by a bunsen burner. A measurable current is registered even before the glass becomes red-hot. The bunsen is immediately removed, yet the current i continues to rise further. The greater the current, the greater is its heating effect. In a short time the glass becomes white-hot and a few moments later it melts right through and drops down.

This experiment is also instructive in another connection. In the case of the independent discharge through a gas, especially in the electric arc, we met with the *falling characteristic* (fig. 44, p. 229). *As the strength of the current increases, E, the pressure between the ends* *of the path of the current, falls.* Here, by means of the voltmeter V, we observe the same thing for the glass rod.

For the electrolytic conduction in the glass rod Ohm's law is true *to a first approximation;* but here the temperature is presupposed to be constant. The conductor must be shielded from a rise of temperature resulting from the heating effect of the current, either by making it give up the heat or by cooling it artificially. Otherwise the original conduction phenomenon is masked by the secondary influence of the rise in temperature. The temperature coefficient of specific resistance is negative for electrolytes, so that we get the graph of the falling characteristic.

Thus we can easily see how the falling characteristic arises in the case of electrolytes, and similarly in the case of the discharge through a gas. Even in the case of the arc we cannot alter the strength of the current alone. The temperature of the path of conduction and of the electrodes are always altered simultaneously, and thereby other factors as well. The complicated current-pressure curve of the falling characteristic then arises from incomplete separation of the single variables.

19. Conduction in Liquids of High Specific Resistance.

According to p. 29, we can pass continuously from conductors to insulators. An insulator is a conductor of extremely high specific resistance. This is no less true for liquids than for solids. Table VI gives some examples of these liquids which conduct badly (or insulate well or very well).

Table VI.

Substance (at room temperature)	Specific resistance σ in ohm · cm.
Acetone	$1{\cdot}4.10^3$
Ethyl alcohol	5.10^6
Distilled water	10^5 to 10^6
Very pure water in a vacuum	$2{\cdot}5.10^7$
Oil for transformers	10^{13} to 10^{14}
Petroleum	10^{16}
Liquid air	10^{18}

The numerical values give at the same time the resistance in ohms of a cube with an edge of 1 cm.

Very pure water is produced by vacuum distillation. Its conductivity has been investigated particularly thoroughly. Here we have a case of true ionic conduction. A very small fraction of the water (about 1·4 mgm. per metric ton) is split up into positive H ions and negative OH ions. Among the 3.10^{22} odd molecules in a cubic centimetre of water ($= {}^1/_{18}$ mol) there are about 4.10^{13} pairs of ions. This value for the electrolytic dissociation of water has been established in various ways which agree quantitatively.

In highly insulating liquids it is predominantly a case of *dependent* conduction. The carriers are usually foreign molecules present as impurities; they can be partly removed by repeated vacuum distillation. A further amount can be extracted by the application of electric fields for days at a time. The carriers which finally remain even in the purest liquids are ions which originate from the molecules of the liquid itself. Electrolytic dissociation may also exist to an extremely small extent. Essentially, however, it is a question of the formation of ions by the ever-present radiations from radioactive substances, just as in the case of spontaneous conduction in air. These radiations form e. g. in hexane about 200 ions per second per cubic centimetre, i. e. about 20 times as many as in air. One of the reasons for this difference is obvious: hexane absorbs the ionizing rays more strongly than air with its small density.

The conduction of highly insulating liquids can be raised by many powers of ten by ionization by X-rays, radioactive radiations, etc. The phenomena are to a great extent similar to those observed in ionized air. We readily observe saturation currents, we find that the ionic velocity is diminished in accordance with the higher internal friction, and so on.

20. **Conduction in Metals.**

The nature of electrical conduction in metals places them in a special position as compared with all other substances. *The motion of the atoms of electricity through metals takes place practically entirely without material carriers.* Even after the passage of large amounts of current we do not find at the electrodes any discharged carriers such as we are familiar with in discharges through gases and above all in electrolytic conduction. In the present state of our knowledge this negative finding permits of only one explanation. *In metals it is only the electrons that move; the atoms of positive electricity keep their positions even in an electric field.* We are forced to this conclusion as long as we know of atoms of positive electricity only in connection with atoms of matter, i. e. only in the form of *ions. For short we call metallic conduction electronic conduction and use the two terms as meaning the same thing.*

The attempt has been made by all manner of means to prove the existence of a certain number (even if a very small one) of material carriers besides the electrons. This has been successful only in isolated exceptional cases. For example, when lithium is dissolved in liquid mercury it is possible to demonstrate that a motion of Li ions takes place to a measurable extent in the electric field. It has also been found possible to make positive H ions or protons move along palladium wires. The difficulty of such experiments lies in this, that high electric intensities cannot be produced in metals; cf. the numerical calculation at the end of the present section. The intensities available, which are at most only a few thousandths of a volt per centimetre, are insufficient to drag ions in easily measurable quantities through liquid, much less solid, metals.

Ohm's law is rigorously true for metallic conduction. Current and pressure are always proportional to each other as long as all the other conditions, and above all the temperature, are kept constant.

The apparatus sketched in fig. 58 suffices to demonstrate the truth of Ohm's law. R is a practical resistance*, a wire of constantan or the like (cf. fig. 30, p. 18) usually wound in a coil. We use sources of current (E) of different pressures, measure corresponding values of E and i, and find that the quotient E/i is constant, no matter what E and i are. As always, the resistance r is defined to be the quotient E/i. Its numerical value measured in volts/ampere (or, for short, "ohms") is directly proportional to d, the length of the wire, and inversely proportional to its cross-section A. We again have equation (9),

$$w = \frac{d}{A} \cdot \sigma ,$$

in which σ is defined to be the specific resistance.

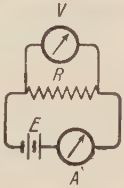

Fig. 58. Demonstration of the truth of Ohm's law for a metallic conductor R. The resistance of the voltmeter V must be large compared to R in order that the current flowing through the voltmeter may be neglected.

An immense number of methods of measuring the quotient E/i, i. e. r, the resistance of a conductor, are available in practice. All of them depend at bottom on the rigorous validity of Ohm's law. As in the case of all measurements, the details are of importance to the specialist only.

Table VII gives some values of the specific resistance for various metals and alloys at room temperature. The values of σ are considerably smaller throughout than for the best conductors among the electrolytes mentioned in Table V. Some alloys are distinguished by particularly high specific resistances. The coils of wire in practical resistances such as those in fig. 30, p. 18 are made of these alloys.

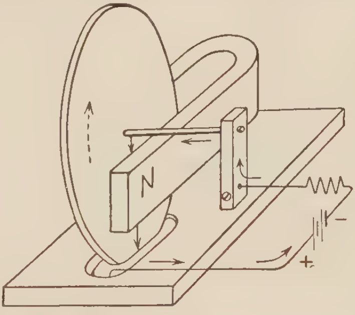

Fig. 59a. Friction on the electrons in a metallic conductor ("Barlow's wheel").

According to our previous discussion of the general principles of conduction currents, the fact of the validity of Ohm's law means that there must be "friction" between the carriers of electricity and the material of the conductor. This "friction" between the atoms of electricity and the atoms of the conductor can be shown to exist in metals just as simply as in the case of electrolytes. A suitable form of apparatus is shown in fig. 59a. A disc of copper which is free to revolve is traversed radially by the current. The shaft serves as one electrode and a bowl filled with mercury as the other. The copper disc is penetrated at right angles by the lines of magnetic force of a horse-shoe magnet. When the current passes the disc begins to revolve. The moving electrons are laterally deviated in the same direction as cathode rays are, and carry the copper disc with them by friction.

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* Cf. the note on p. 15.

Table VII.

Substance at 18° C.	Specific resistance σ in ohm.cm.	Specific Conductance $\varkappa = \frac{1}{\sigma}$ in $\text{ohm}^{-1}.\text{cm}^{-1}$.
Pure metals		
Silver	$0\cdot016.10^{-4}$	62.10^{4}
Copper	$0\cdot017_5.10^{-4}$	57.10^{4}
Sodium	$0\cdot045.10^{-4}$	22.10^{4}
Tungsten	about $0\cdot05.10^{-4}$	20.10^{4}
Platinum	$0\cdot108.10^{-4}$	$9\cdot3.10^{4}$
Lead	$0\cdot208.10^{-4}$	$4\cdot8.10^{4}$
Graphite — single crystal*	about $0\cdot4.10^{-4}$	$2\cdot5.10^{4}$
Mercury	$0\cdot96.10^{-4}$	$1\cdot0_4.10^{4}$
Alloys		
Brass (66% copper + 34% zinc)	$0\cdot063.10^{-4}$	$15\cdot8.10^{4}$
Manganin (84% copper + 4% nickel + 12% manganese)	$0\cdot42.10^{-4}$	$2\cdot4.10^{4}$
Constantan (60% copper + 40% nickel)	$0\cdot49.10^{-4}$	2.10^{4}
Powders pressed together and heated so as to form a coherent mass		
Metallized carbon filaments in incandescent lamps	3.10^{-4}	$0\cdot33.10^{4}$
Ordinary carbon filaments in incandescent lamps and carbons in arc lamps	40.10^{-4}	$0\cdot02_5.10^{4}$
	If the factor 10^{-4} is left out, the numbers give the resistance in ohms of a wire 1 m. long and of cross-section 1 sq. mm.	The numbers give $10^{4}\times$ length in metres of a wire of section 1 sq. mm. and resistance 1 ohm.

A variant of this experiment is very instructive. If we increase the current in a water-pipe, i. e. the quantity of water flowing through it per second, the pipe *bursts*. On the other hand, if we increase the current in an electrical conductor, the reverse takes place: *when the current load is heavy the conductor becomes constricted.* This is conveniently shown with a liquid metal conductor, say mercury in a shallow pipe P with a cross-section of several square centimetres (fig. 59b). The copper bars K and A serve as leads for the current (of the order of 1000 amperes).

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* It is worth noting that pure single crystals of graphite exhibit a rise of specific resistance with rise of temperature, just like pure metals. The causes of the usual negative temperature coefficient of commercial preparations of graphite and of the same phenomenon in many metallic alloys (cf. manganin in fig. 62, p. 252) are probably similar.

In order to understand the phenomenon the reader should refer to fig. 28b, p. 190, in which a conductor is placed perpendicular to the plane of the paper. Its cross-section, which is taken to be circular, is shaded. The electrons are taken to be approaching the reader from below. This settles the direction of the circular lines of magnetic force: they run clockwise. The lines of magnetic force do not merely surround the conductor on the outside, but also exist in the *interior* of the conductor. Hence the electrons *in* the conductor must pass through a magnetic field, in which they are displaced (*radially* towards the centre of the conductor in the plane of the paper; cf. fig. 17, p. 143). The metal of the conductor is dragged along with them as a result of "friction", and this motion towards the centre leads to the constriction of the conductor ("pinch phenomenon").

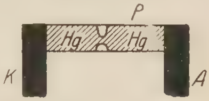


Fig. 59 b. Demonstration of electronic friction: constriction of a conductor owing to a high current load.

The energy of the electrons which is destroyed by friction appears as the heating effect of the current in metals also. The considerations brought forward in the case of electrolytes (p. 237) apply equally well to metals. We have merely to equate v_+, the mobility of the atoms of positive electricity, to zero.

Very erroneous ideas of the mobility and velocity of the atoms of negative electricity or electrons in metals are often met with. People usually associate electrical phenomena with the idea of something timeless and practically instantaneous. This, however, is true only within very narrow limits. The condition which we call an electric field is certainly propagated with a very high velocity, which in the limiting case may be equal to that of light (Chap. XI). But in this field the motion of the electrons takes place with a velocity so small that it takes one aback.

Equation (9a) (p. 237) holds in this case also, just as it does for any conduction current which satisfies Ohm's law; we have

$$\frac{1}{\sigma} = \mathbf{N}\,(\mathbf{n}\,e)\,v_-\,,$$

where v denotes the mobility of the electrons, i. e. their velocity in a field of 1 volt/cm. $\mathbf{N}$, the number of electrons in a cubic centimetre of metal, is unknown, but as a first approximation we may take it as being equal to the number of atoms of the metal in a cubic centimetre. 1 c.c. of copper (= 8·7 gm. = 0·137 mol) contains about 8.10^{22} atoms. $\mathbf{n}e$, which in this case is the charge on an electron, is equal to $1{\cdot}6.10^{-19}$ coulombs. $\sigma_{\mathrm{Cu}} = 1{\cdot}7.10^{-6}$ ohms (Table VII). Hence we obtain $v_- = 46 \frac{\text{cm.}}{\text{sec.}} \Big/ \frac{\text{volt}}{\text{cm.}}$. Now normally it is quite impossible to maintain an intensity **R** of 1 volt/cm. in copper. A pressure $E = 1$ volt between the end surfaces of a copper cube with an edge of 1 cm. would give a current density of 6.10^5 amperes per sq.cm. In actual fact we can scarcely go beyond a current density of 600 amperes per sq.cm. in practice, owing to the heating effect of the current. This corresponds to a field strength **R** of 10^{-3} volt/cm. In this case the electrons merely crawl through the copper conducting wires with a velocity u of not more than about $^1/_2$ mm./sec.

The details of the mechanism of electronic motion in metallic conducting wires are still very obscure. For many purposes the metal may be regarded as a porous lattice, through the meshes of which the electron "gas" streams, being forced along by the electric field. It is certain, however, that this is only a very rough model.

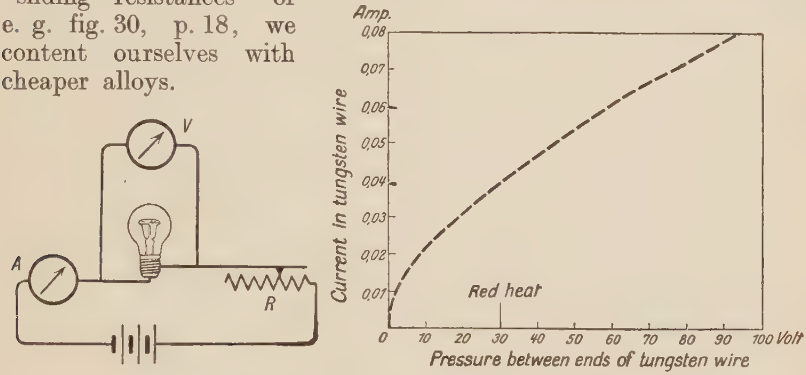
21. The Effect of Temperature on Metallic Conduction. Superconduction.

The specific resistance of metals depends on the temperature. In contradistinction to electrolytes, the temperature coefficient is positive, i. e. *the resistance of metals increases with rising temperature.* In order to show this it is convenient to use an ordinary metallic filament lamp of 25 candle-power (fig. 60a). The lamp contains a tungsten wire which is coiled up in the familiar way. This is heated by the passage of the current (equation 10, p. 241). E, the pressure between the ends of the tungsten wire, is measured by the voltmeter V, and the current flowing through the wire by the ammeter A. The town mains serve as the source of current. A practical resistance R with a sliding contact enables us to vary the strength of the current within wide limits. Corresponding values of E and i are observed. An example is shown in fig. 60b. With current strengths above about 0·03 amperes the wire already glows visibly; white heat is reached at 0·13 amperes. There is no question of the quotient E/i being constant; it obviously rises with increasing temperature.

In quantitative experiments the temperature of the wire is measured instead of merely being estimated from the colour with which it glows. In addition to this the increases in the length d and in the cross-section A caused by the heating are taken into account. Results like those represented in fig. 61 are obtained in this way. The specific resistance is taken as ordinate. As examples we have selected copper, which is the most important conductor in practice, and also lead, platinum, and iron. Similar curves hold up to high temperatures for all other *pure* metals, even for those which are only melted with difficulty, like tungsten (W).

The graph is quite different for alloys. Their specific resistance depends to a far smaller extent on the temperature. Fig. 62 shows this for 18-carat gold. By systematic trial alloys have even been discovered whose specific resistance may be taken as constant over a wide range of temperature. Manganin, which is composed of copper, nickel, and manganese, is an alloy of this kind.

Alloys like manganin are invaluable in connection with the science of measurement. The coils of practical "accurate resistances" and of "standard resistances" are made of such alloys. For the simpler "sliding resistances" of e. g. fig. 30, p. 18, we content ourselves with cheaper alloys.



a b

Fig. 60a, b. Increase of the resistance of a metallic conductor (the spiral of tungsten wire in an incandescent lamp) with temperature. The resistance of the voltmeter *V* in the diagram must be large compared with that of the incandescent lamp, in order that only a vanishingly small fraction of the current indicated by *A* may pass through the voltmeter.

On the other hand, the marked temperature effects in pure metals have also come to be of considerable use in making measurements. Many varieties of "resistance thermometer" or "bolometer" have been constructed. In the simplest case we use a source of current at constant pressure, say an accumulator, an ammeter, and a coil of pure copper wire. The coil is the actual body of the thermometer. If the coil is dipped into liquid air, the current is tripled, and so on. The scale of the ammeter may be directly re-graduated in degrees centigrade. So much for practical applications.

We now return to fig. 61. The resistance of *pure* metals decreases continually with fall of temperature. In the case of copper and lead, measurements have been carried down to — 253° C., the temperature of liquid hydrogen. One would expect that in the gap between — 253 ° C. and the absolute zero — 273 ° C. the curves would continue as indicated by dots, i. e. that the specific resistance would gradually approach zero asymptotically, the thermal motion of the metal atoms being completely brought to rest. Experiment, however, which is always the

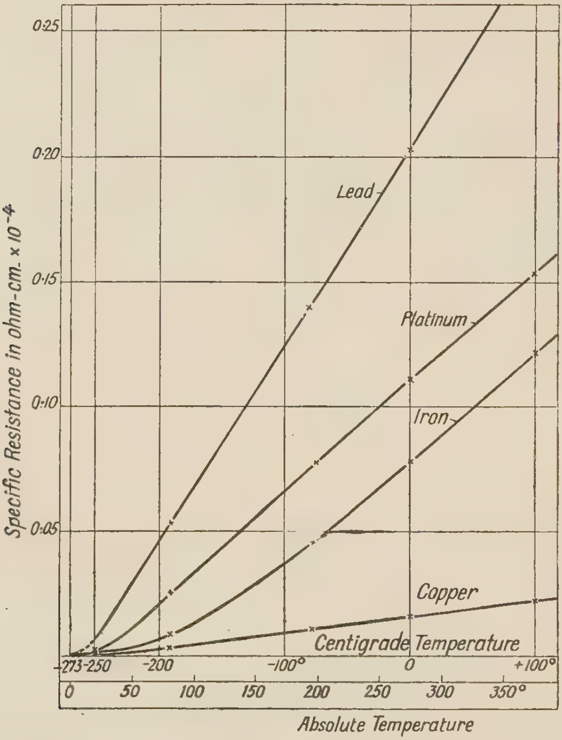


Fig. 61. The effect of temperature on the specific resistance of pure metals.

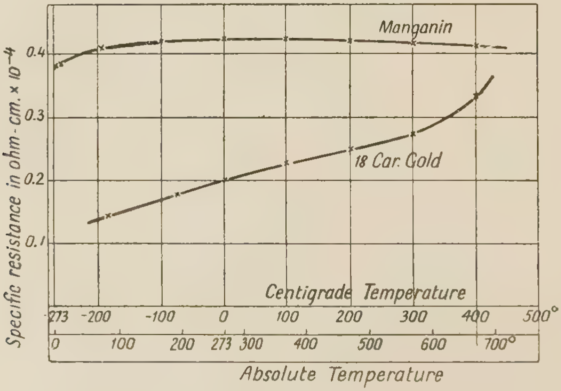
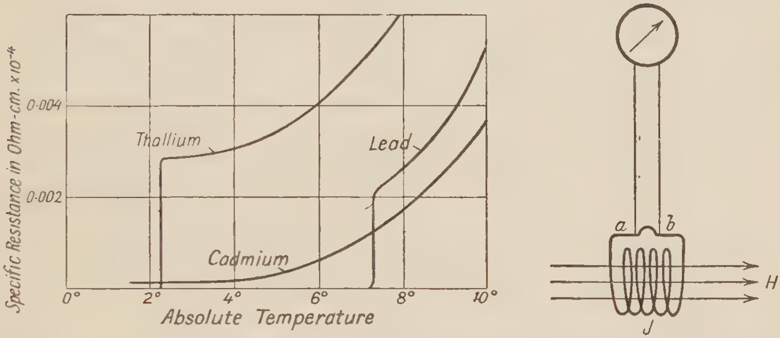


Fig. 62. The effect of temperature on the specific resistance of alloys.

final court of appeal, has decided otherwise. At very low temperatures something quite new and unexpected turns up, namely the phenomenon which its discoverer Kamerlingh Onnes calls *superconduction.*

Fig. 63a consists of the left-hand bottom corner of fig. 61 on a larger scale. Here the values of the specific resistance of lead are magnified for the last ten degrees above the absolute zero. These temperatures are obtained by using liquid helium. Down to 7·3° abs. the specific resistance of lead continues to fall in the way one would expect from the dotted continuation of the curve for lead in fig. 61. Then, however, the specific resistance suddenly drops within a hundredth of a degree to a quantity too small to be measured, amounting to no more than 10^{-12} of its value at room temperature. Superconduction has been shown to occur under similar circumstances in some other metals, and recently in certain alloys also. As a further example, results of measurements on thallium (Tl) are given in Fig. 63. The critical temperature at which its resistance drops is only 2·3° above the absolute zero. Whether superconduction finally appears in all metals if they are sufficiently pure and the temperature is sufficiently low, is still uncertain. The majority of metals exhibit a constant finite limiting value of the resistance even at the lowest temperatures. The curve for cadmium given in fig. 63a is an example of this. However this may be, superconduction is in any case a phenomenon of fundamental importance.



a b

Fig. 63a, b. The effect of temperature on the specific resistance in the neighbourhood of the absolute zero. Superconduction of lead and thallium.

Superconduction has at last made it possible to realize in practice an experiment of which we made important use above in imagination. A current which has once been introduced into a superconducting coil may be made to continue almost indefinitely even in the absence of a source of current (pp. 140, 159).

Let *J* be a coil of lead wire and **H** a magnetic field originating in any way we please. The wire is cooled down to about — 270° C. by means of liquid helium. The magnetic field is then made to disappear. While it is disappearing it produces an electric field with closed ring-shaped lines of force. This field accelerates the electrons in the lead wire and an induced current flows in the coil. When the magnetic field has completely disappeared the electric field ceases to exist also. The electrons are no longer accelerated, but continue on their way with constant velocity. Kamerlingh Onnes has demonstrated the existence of this current in two ways, firstly by the deviation of a neighbouring magnetic needle, and secondly by means of the apparatus sketched in fig. 63b. Two neighbouring points of the lead coil, *a* and *b*, are connected to a galvanometer. When the current has flowed long enough, the superconducting wire is cut through between *a* and *b*. The current henceforth flows through the galvanometer. The coil of the latter is not superconducting. The electrons are rapidly retarded, so that the galvanometer only exhibits a ballistic deflection. In the superconducting metal the currents are maintained practically indefinitely; the decay per hour does not even amount to ne per cent. Yet the current strengths involved need not be small, for currents of over 100 amperes have been observed in these experiments.

In order to explain the magnetic properties of bodies we assumed the existence of currents circulating without diminution in the interior of molecules; we spoke of them as *molecular currents.* We remind the reader of our explanation of permanent magnetism and of the characteristic permeabilities of different substances (pp. 158 to 160). Superconduction has provided us with powerful experimental support for this assumption. It enables us to imitate the molecular currents circulating without diminution, whether in coils or as eddy currents, on a scale large enough to be perceptible to the senses. The only regrettable fact is that present-day experimental methods do not as yet enable us to exhibit the currents in superconducting metals in simple lecture experiments. Meantime this is one of the few experiments which are frustrated by considerations of expense.

22. The Connection between Electrical Conductivity and Thermal Conductivity in Metals.

Bad conductors of electricity are generally bad conductors of heat also, e. g. glass. The glass-blower can hold one end of a glass rod in his hand without any inconvenience, although the other may be heated white-hot in a flame. The same is true of metals also. By Table VII silver has a large electrical conductivity, German silver a small one, and it is easier to burn one's fingers with the handle of a silver teapot than with the handle of a German silver one.

The connection between electrical conductivity and thermal conductivity has been investigated experimentally very thoroughly. The thermal conductivity λ is defined to be the length of a bar of 1 sq.cm. cross-section which lets one watt-second* of heat energy flow through it per second when the difference of temperature between its end surfaces is 1° C.

Fig. 64 gives the ratio $\lambda/\varkappa$, i. e. of the thermal conductivity to the electrical conductivity, for various metals and temperatures. These measurements show two things:

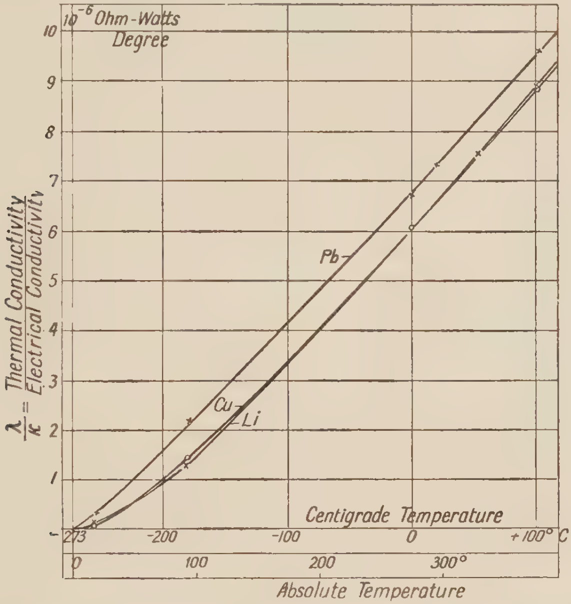


Fig. 64. The ratio of the thermal conductivity to the electrical conductivity at various temperatures.

(1) The values of the quotient $\lambda/\varkappa$ for all metals at high temperatures are very similar. The curves for the different metals do not deviate greatly from each other. To a good approximation λ and $\varkappa$ are proportional to each other (Wiedemann-Franz rule).

(2) As a rough first approximation the quotient $\lambda/\varkappa$ is proportional to the absolute temperature, i. e. the temperature reckoned from — 273° C. as zero.

In spite of the utmost efforts all attempts to explain this state of affairs have so far failed completely. We still know far too little about metallic conduction. The electrons move in the electric field through the metal, while the latter remains fixed; but we are still very much in the dark as to how this happens, which is a curious state of affairs in view of the extreme importance of metallic conduction in practice.

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* [I. e. one joule.]

23. Independent Conduction of Electricity in Solid Non-metals.

What we have said about the conduction of metals is just as true for solid metals composed of crystals as for liquid metals. The change of state of aggregation of course causes a discontinuity in the conductivity. According to an empirical rule, the specific resistance is halved when the metal freezes. In both cases, however, it is only electrons which move. The idea that the minute electrons should be able to get through the fixed scaffolding of a crystal lattice is not a strange one. Thus far conduction in the solid metal is no more difficult to understand than that in the liquid metal.

The same is also true for solid bodies which illustrate the passage from metals to non-metals. A large number of metallic sulphides and oxides, which are mostly opaque, belong to this class. All these bodies conduct metallically; only electrons move in them. We never find charged carriers at the electrodes.

We come up against a difficulty, however, when we pass on to typical non-metallic bodies which conduct *electrolytically* in the liquid state when fused. Even in the solid condition many of them undoubtedly exhibit ionic conduction to a considerable extent. We shall mention a few cases where this certainly occurs.

(1) *Rock salt* (NaCl). Crystals of rock salt form the stock elementary example of a lattice built up of ions. The arrangement of the positive sodium ions and negative chlorine ions is illustrated in fig. 65. Rock salt melts at + 780° C.; but even at temperatures considerably below its melting-point, between 600° and 700°, the existence of ionic conduction in it can be demonstrated by projection to a large audience. A crystal about a cubic centimetre in volume is placed between two metal electrodes mounted between springs in an electric furnace. The electrodes are connected through an ammeter to the town mains. The sodium is liberated at the cathode in the form of rapidly growing violet filaments. The phenomenon resembles the "lead tree" (shown in fig. 14, p. 10) in an aqueous electrolyte. If the current is reversed the filaments of sodium are absorbed and grow out again from the new cathode.

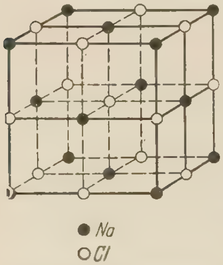

Fig. 65. Arrangement of the sodium ions and chlorine ions in rock salt.

The experimental difficulty which has long hindered the investigation of ionic conduction is also strikingly shown by this experiment. The metal filaments soon form a bridge right across the crystal; thereafter the current flows through the metal bridge and ionic conduction through the crystal ceases.

This difficulty can be successfully avoided only in particular cases, namely those where it is possible to insert a layer of silver iodide (AgI), crystallized in the regular system, between the cathode and the crystal. In this case the metal ions caused to approach the cathode by the electric field are not discharged at the boundary of the silver iodide; on the contrary they remain as *ions* owing to the formation of iodide. Instead of them, silver ions move through the last part of the path of the current (consisting of silver iodide) to the cathode, where the silver is deposited in a compact layer. Regularly-crystallized silver iodide is so far the only substance known in which this inconvenient formation of filaments does not take place. The investigations described in (2) and (3) below only became possible as a result of this device.

(2) *Silver chloride, bromide, and iodide.* All three crystals exhibit ionic conduction only; for in their case Faraday's law of equivalents is rigorously true.

The verification of this is made easier by the fact that the ions will pass through two crystal surfaces pressed together, without cementing them together. Several pieces of crystal are pressed together to form a column, which is placed between the electrodes. After a measured quantity of electricity has passed, the changes of weight of the two pieces of crystal next the electrodes are determined.

At the cathode we merely find discharged ions, i. e. deposited *metal.* At the anode there is not a trace of *halogen. Practically it is only the atoms of metal that move. The halogen atoms of the lattice remain "fixed".*

(3) *Lead chloride* ($PbCl_2$). Faraday's law of equivalents holds rigorously. It is a case of pure ionic conduction; *but practically it is only the chlorine ions that move, the metallic atoms of the lattice remaining "fixed".*

(4) *The motion of ions extraneous to the lattice.* In crystals of sodium nitrate ($NaNO_3$) the positively-charged sodium ions normally move. Instead of them, lithium ions can be drawn through the crystal by the electric field. We have to take mercury containing lithium as the anode, as lithium ions from aqueous solutions will not enter the crystal, probably because they are too heavily laden with a layer of neutral water molecules (cf. p. 238). The motion of the lithium ions does not destroy the lattice, although that of other ions, e. g. copper, does so. The advance of the copper ions can be traced by a green coloration of the crystal, which is previously clear and colourless.

So much for the facts. *It is certain that in the electric field ions pass through the crystal.* The meaning of this is hotly disputed. There are two sharply-opposed points of view. According to the first, the ions really exchange places even in the crystal lattice. It is thought that the bonds between neighbouring ions can be loosened to such an extent, now here, now there, by thermal motions, that the electric field can pull one ion into its neighbour's place. The ion actually moves through the lattice, always slipping through at the places which are weak or "loosened" at the moment. Or, in other words, the thermal diffusion of the constituents of the lattice (ions), which takes place even in crystals, is directed in a particular way by the field. According to the second interpretation, it is absurd to think of the ions which are bound together to form a lattice exchanging places. It is certain that no actual crystal lattice is like the ideal picture sketched by the theorist. Any actual crystal contains a vast number of ultra-microscopic flaws, channels, cracks, and pores. These and not, as might be thought, the solid regions of the lattice, form the path for the ions, according to the second interpretation. Both interpretations may cite in their favour the great decrease of the resistance with rise of temperature. A crucial test is still wanting.

The temperature coefficient of the specific resistance is negative in all cases of electrolytic conduction through non-metallic crystals. The resistance shows a large rise as the temperature falls. *Many crystals with typical ionic lattices are very good insulators at room temperature,* e. g. rock salt (NaCl) and almost all other clear transparent crystals of salts.

Strangely enough, the numerical value of the specific resistance at room temperature has only been determined satisfactorily for a single crystal. It amounts to 5.10^{14} ohm.cm. for ammonium alum with water of crystallization $(AlNH_4(SO_4)_2 + 12H_2O)$.

Substances which are dissociated into ions only slightly or practically not at all, even when fused, conduct all the worse when in the solid form. These are the substances whose lattices are composed of molecules, not of ions. As examples of these highly-insulating molecular lattices we may mention ice and sulphur.

The investigation of conduction currents in solid bodies is frequently hindered by an experimental difficulty. The atoms of electricity moving in the field reach the electrodes with difficulty or not at all. They often remain sticking to cleavage planes, etc., within the crystals. The accumulation of these charges produces an electric field whose direction is opposed to that of the original field between the electrodes and weakens the latter. That word of numerous meanings, polarization, is used to denote this disturbing effect.

24. Dependent Conduction of Electricity in Insulating Crystals.

In solid bodies which are good insulators at room temperature we can easily show the existence of dependent conduction of electricity also. To form the electricity carriers we use X-rays or the rays from radioactive substances. Sulphur, ice, calcite and many other substances lend themselves to experiment. Saturation currents are easily obtained. It is unknown whether the moving bodies are ions or electrons.

Other substances, e. g. diamond and zinc sulphide (ZnS), become dependent conductors even as a result of visible light. Fig. 66 shows a suitable form of apparatus for experiment. Diamond reacts even to the light of a match. Quantitative investigations have shown that the moving bodies are electrons.

The same is true for all crystals with a high optical refractive index (say $n > 2$). If the crystals conduct noticeably by themselves, the dependent current of electrons often causes a secondary phenomenon, namely a change in the specific resistance. A well-known example of this is furnished by the commercial selenium cell. These cells usually have electrodes in the form of long wire coils with fused selenium in its gray modification between them. The apparatus of fig. 67 may be used to show this alteration of the resistance of the selenium when exposed to light. A selenium cell is inserted between a neon lamp and the town mains. The lamp only lights up while the selenium is illuminated.

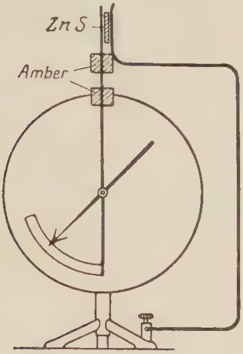

Fig. 66. Photo-electric conduction in highly-insulating crystals.

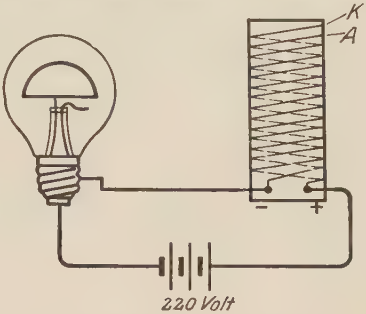

Fig. 67. Alteration of the specific resistance of commercial selenium preparations as a result of illumination. The neon lamp serves as a current indicator.

25. Concluding Remarks on the Mechanism of Conduction.

At the beginning of our discussion we drew a rough picture of the electric current in conductors. On p. 41 we said: "Atoms of electricity of opposite sign attract each other. Owing to their being free to move in the conductor they can follow this pull. They move through the conductor up to each other. They lay themselves so closely together that their lines of force do not appear externally at all. The charges neutralize each other."

The material nature of the atoms of electricity might be raised as an objection to this point of view. The conductor should increase in size at the place where the positive and negative atoms of electricity are united. With our present knowledge we can answer this objection as follows.

The increase in size can actually be observed. Imagine, for example, that solid silver chloride and solid lead chloride are pressed against each other and held together in a conducting circuit. In silver chloride it is only the positive silver ions that move, in lead chloride only the negative chlorine ions (p. 257). They meet at the boundary. Their charges neutralize each other, fresh silver chloride is formed, and the conductor increases in size.

In the conductors which are ordinarily used (metals), on the contrary, the state of affairs from the purely quantitative point of view is quite different. It is only the electrons that move; the positive metallic ions remain fixed. The mass of the electrons is very minute (p. 211). Their motion to and fro causes no appreciable change of volume in the conductors.

Thus closer investigation of the phenomenon of conduction has to a great extent justified our original primitive conception of the conduction current and has supplemented it merely as regards quantitative details.

26. The Connection between Induction Phenomena and the Mechanism of Conduction.

We found by experiment that relative motions only were involved in the phenomena of electromagnetic induction. Hence most induction phenomena can be considered from the point of view of a rigid secondary coil *at rest*. This leads us to Maxwell's interpretation of induction phenomena: a changing magnetic field is surrounded by closed ring-shaped lines of electric force (pp. 113—115).

Later, on p. 137, we came across an induction phenomenon which could not be reduced to the above scheme, namely, induction in conductors, some parts of which are in motion relative to other parts while the induction phenomenon is going on. We purposely left the mechanism of this phenomenon an open question, for it assumes a deeper knowledge of the mechanism of conduction. We now possess this deeper knowledge. We are now acquainted with two facts: (a) the phenomenon of the *convection current*; atoms of electricity moved by mechanical means represent an electric current, and, like cathode rays, they are deflected by a magnetic field, in a direction at right angles to that of their motion and to that of the magnetic field: and (b) the existence of a kind of "friction" between electricity carriers and molecules, e. g. between electrons and atoms of metal.

If now, possessing this information, we refer back to fig. 12b, p. 137, we have the following state of affairs: as a result of friction, the electrons situated in the sheet of metal between K and A take part in the motion of the metal strip. They form a convection current surrounded by a magnetic field. Hence they are deflected upwards by the external magnetic field $\mathbf{H}$. Electrons accumulate at K, so that K acts like the negative pole of a source of current.

Strictly speaking, the understanding even of eddy currents assumes a knowledge of the mechanism of conduction. This is particularly true if there are no external geometrical limits to the eddy currents, as e. g. in the case of an extensive sheet of metal between the poles of a magnet or electromagnet. If there were no friction between the electrons of the eddy current and the matter surrounding them we could not see how the mechanical reactions in the direction required by Lenz's law are transferred to the sheet of metal.

CHAPTER IX

Electric Fields in the Boundary Layer between Two Substances

Preliminary Remarks.

We first came across the formation of an electric field in the boundary layer between two substances on p. 61, in connection with the phenomenon which (with greater brevity than appropriateness) we call "frictional electricity". Our knowledge of the mechanism of conduction now enables us to deal with the matter in its wider aspects, and thereby we are finally led to discuss the third group of sources of current in general use, namely chemical sources of current or cells. All the phenomena concerned have been known experimentally for a long time, although they can only be interpreted in broadest outline. We cannot hope for a deeper understanding of these phenomena until the electric fields of molecules and atoms have been investigated to a greater extent than at present. There will yet be many changes in the explanation of the phenomena in the course of future years.

1. Cataphoresis and Convection Currents.

When any two bodies are in a close contact an electric field appears in the boundary layer (pp. 61—63). Fig. 1 is a crude diagrammatic picture of what happens in the case of a sphere immersed in a liquid.

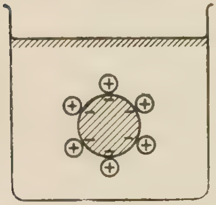

Fig. 1. Electric field in the boundary layer between a liquid and a sphere immersed in it: a rough schematic diagram.

The existence of this fundamental phenomenon was proved in a simple way on pp. 62—63, using a sphere which is not wetted by the liquid. We can subsequently separate the sphere and liquid completely and show that there are charges on both.

The phenomena known as cataphoresis and convection (or streaming) currents are essentially nothing but experimental variants of this experiment, which is shown in fig. 61, p. 62. They enable us to prove the existence of the electric field even in cases where the wetting of the boundary surfaces prevents them being separated subsequently.

(1) *Cataphoresis*. Imagine that the fluid in fig. 1 is penetrated by an electric field, and let this be indicated in fig. 2 by the two condenser plates or electrodes *K A*.

The electric field attracts the negatively charged sphere to the anode. The positive molecules of liquid slip over the sphere*. They leave the rear pole of the sphere at *b* and move to the cathode. The molecules of fluid which the sphere meets at *c* in front supply their places. The whole phenomenon is known as cataphoresis.

In spite of the continual advance of the liquid molecules the charge of the sphere does not rise beyond a stationary limiting value, on the average at least. The sphere behaves exactly like a piece of metal which is drawn over a supporting base of unlimited extent by means of an insulating handle.

In order to demonstrate cataphoresis we take, instead of the single sphere in the diagram, a great number of small bodies which, being particles in suspension, do not sink to the bottom of the fluid, or only do so very slowly. A typical form of apparatus is shown in fig. 3.

Two metal electrodes *K* and *A* dip into a glass U-tube filled with distilled water containing lycopodium powder (club-moss spores). If an electric field is produced by means of the induction machine, the grains move to the anode. Their motion can easily be followed with a projection microscope. *It follows that cataphoresis is merely a case of dependent conduction of electricity through a liquid, in which the electricity carriers of one sign are large enough to be visible.* In this respect it is exactly analogous to the dependent conduction of electricity through air by visible dust-like electricity carriers.

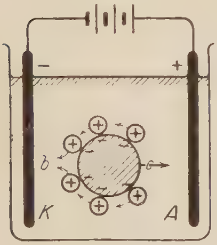

Fig. 2. Diagram illustrating cataphoresis.

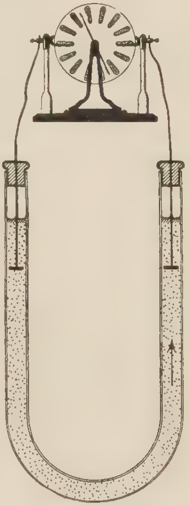

Fig. 3. Cataphoresis.

Cataphoresis is a widely distributed phenomenon, and is particularly often exhibited by suspensions of colloid particles. It enables us to detect the existence of electric fields between the boundary surfaces of very varied substances. All that is necessary is that one of the two substances should be liquid and the other obtainable in the form of a suspension or emulsion.

The interpretation of cataphoresis, although inherently so simple, often causes the beginner difficulties, as every teacher knows. The beginner does not think sufficiently in terms of atoms. He imagines the picture shown in fig. 4, where the sphere together with its coating of liquid appears to be an electrically neutral configuration, which is not capable of being affected by an electric field. The sphere and liquid are so to speak considered as being a spherical condenser, the outer boundary of whose field consists of a hollow sphere closed everywhere. The coating of liquid, however, is not a hollow conducting body. It consists of separate ions, as is shown in a very rough diagrammatic way in figs. 1 and 2. If these spheres, together with their liquid coatings, are brought into an electric field, the lines of force may very well reach the sphere, naïvely speaking, and attract it to the anode.

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* Cf. the third division of this section.

In the form of cataphoresis which we have hitherto described (fig. 3) we only see the suspended particles moving in the electric field. The movement of the molecules of liquid which have been charged so as to form ions is invisible. By suitable choice of the conditions of experiment, the phenomenon may be exactly reversed, so that only a motion of the fluid in the electric field is visible*. An example will immediately make this clear:

Imagine that solid suspended particles have by contact with their neighbours become united into a mesh or framework, forming a porous body like pumice stone, a heap of sand, a piece of fine felt, or the like. In fig. 5 we again have a glass U-tube filled with distilled water. It is divided in the middle by a porous plug of cotton-wool. An electric field is produced between the electrodes *K* and *A* by means of the town mains. After some time a rise of the liquid in the left-hand limb is noticeable. The water moves towards the cathode as a result of cataphoresis, but by no means all the water is charged: the greater part is carried along by the friction of the ions formed.

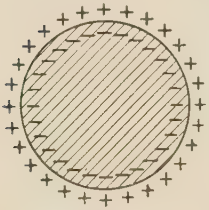

Fig. 4.
To illustrate cataphoresis.

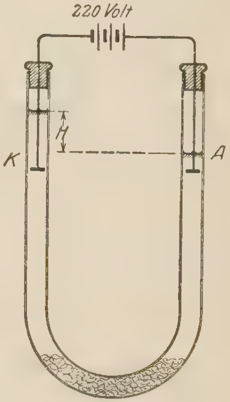

Fig. 5. Cataphoresis of water through a porous plug.

Other porous bodies may be used instead of cotton-wool. Even a single very narrow channel will do; this is the simplest case of all. The middle of the glass tube is made in the form of a fine capillary. Cataphoresis not infrequently occurs conspicuously and unexpectedly as a phenomenon accompanying electrolytic conduction. This point is mentioned in order to make our account of electrolysis complete.

Besides this, cataphoresis has practical applications. It may be used in practice for drying porous substances (the so-called electric endosmose method). It is essential that the liquid to be expelled should have a high specific resistance; otherwise a strong heating effect due to conduction currents, which is not wanted, is produced as well as the electric field intended. This makes the method wasteful, as is unfortunately the case in the example sketched in fig. 6. A piece of wet peat is placed between two electrodes of brass netting. The water begins to drop out in large quantities whenever the lower piece of netting is made the cathode.

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* [Strictly speaking, this is known as *electric osmosis or electric endosmose*, the term *cataphoresis* being then restricted to the case where the solid moves.]

Cataphoresis is also used in medicine. When it is desired to administer drugs whose molecules are not electrolytically dissociated into ions, attempts are made to pass them through the skin of the patient by means of an electric field.

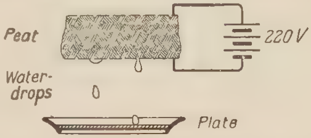


Fig. 6. The electric endosmose method.

(2) *Convection or streaming currents.* The essential features of the convection current are again illustrated in fig. 7. Two condenser plates are connected to a galvanometer but *not* to any source of current. The galvanometer shows a deflection whenever any electrically charged body or electricity carrier is moved in the direction shown by the arrow.

The origin of the charge on the carrier makes no difference whatever to the convection current. Hence the carrier may even receive its charge as a result of touching a liquid between the condenser plates. If a carrier charged in this way is moved, the resulting convection current is called a "streaming current". A convenient apparatus for carrying out this simple experiment is shown in fig. 8. Two platinum electrodes are arranged in a gas jar, one being vertically over the other. With this arrangement gravity is sufficient to give the electricity carriers the required motion in the direction of the arrow. Distilled water serves as the liquid and e. g. glass beads for the carriers. The beads are let fall from above, singly or several at a time, into the column of water. The convection current may be observed without any trouble.

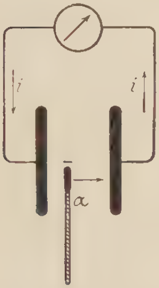


Fig. 7. Diagram of a convection current. Galvanometer of the type shown in fig. 16, p. 30, and fig. 27, p. 36.

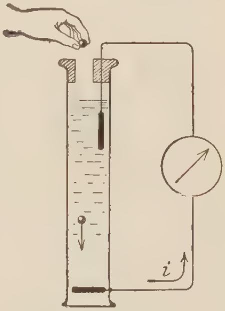


Fig. 8. Convection or streaming current in a liquid. Galvanometer as in fig. 16, p. 30, and fig. 27, p. 36.

In this way of carrying out the experiment we see the motion of the large visible carriers, not that of the water molecules passing over them (fig. 2). The reverse can be obtained by a simple variant of the experiment.

In fig. 9 the distilled water in the U-tube can be forced through the porous plug by means of a rubber bulb. Owing to internal friction the water carries along with it the molecular ions which are formed when it touches the walls of the pores. As soon as the column of water begins to move (i. e. even before the first electricity carriers reach the electrode; cf. p. 197), the galvanometer records the convection current as a "streaming current". Reversing the direction of the motion of the water reverses the streaming current.

(3) *The situation of the electric fields produced when solid and liquid substances touch.* To conclude these details about cataphoresis and convection currents, there is still a misunderstanding to be guarded against.

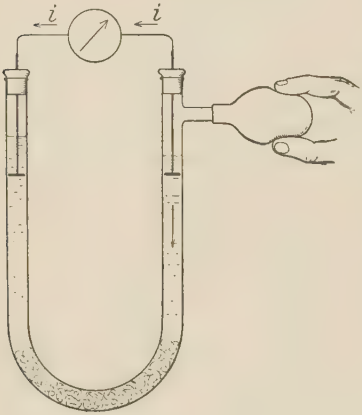


Fig. 9. Convection current caused by forcing a liquid through a porous plug.

We have always spoken of an electric field "in the boundary layer" between the liquid and the solid body. This expression, however, must not be taken too literally.

Of course the roughly diagrammatic figures 1 and 2 merely show the electric field between the solid body and the outermost layer of liquid, i. e. that immediately adjacent to the solid body. This simplification of the drawing, however, goes much too far, for there is no "external" friction between liquid and solid body, i. e. the molecules of the fluid cannot *slide* along the surface of the body. In this case there is only "internal" friction. The molecules of liquid touching the solid body *stick* to it. There are molecules situated deeper in the fluid which slip over the outer layer of liquid which sticks to the solid (fig. 53, p. 239). These deeper inner layers, however, can only be moved by the electric field if some of the ions produced by the contact of the liquid and solid are to be found in it. Thus the electric field must extend into the liquid for a depth of several molecules at least, possibly as a result of thermal diffusion of the ions. We shall return to this point later in connection with the electric fields between liquids and gases.

2. Electric Fields in the Boundary Layer between Two Metals.

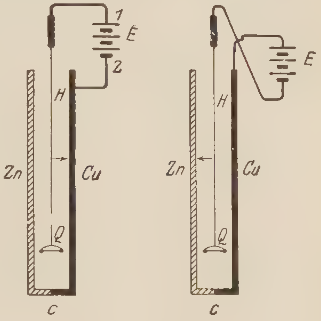
Electric fields in the boundary layer between two metals cannot be shown to exist by either of the two methods which we know so far.

Subsequent extension of the lines of electric force will not do, because the two metals cannot be completely separated at all points of their surfaces at the same moment. Hence the electrons rush across the bridge formed at the places which are last in contact, and the pressure remains as low as it was for molecular contacts.

Cataphoresis would require one of the metals to be in the liquid state. Apart from this, fields of only very small intensity (about 10^{-3} volt/cm.) can be produced in metals, so that the velocity of the metal particles in suspension would be too small.

A third method, however, which is illustrated in fig. 10, leads us to the desired goal. Once again we have a practical application of the fundamental experiment dealt with in fig. 73, p. 72. A sheet of zinc and a sheet of copper form a condenser with plates about 1 cm. apart. The metals touch at *c*, i. e. some of their superficial projections approach to within molecular distance, i. e. within about 10^{-8} cm. This causes the electric field whose existence we wish to prove to appear between the two metals. Its pressure will turn out to be about 1 volt; the zinc is positively charged, the copper negatively charged.

Thus **R**, the intensity of the electric field, is very high at the place where the metals touch, of the order of 10^8 volts/cm.; but higher up in the condenser, which is 1 cm. thick, it is very small, only about 1 volt/cm. A very fine hair-like platinum wire *H* in the middle of the condenser serves to show the existence of this feeble electric field. The wire is kept stretched by a thin semicircular piece of quartz at the bottom, just as in the unifilar voltmeter (fig. 81, p. 81).



a b

Fig. 10a, b. The occurrence of an electric field at the contact of two metals.

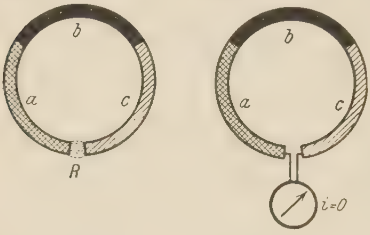
We connect a battery (giving about 50 volts) with the wire and the copper plate by means of two copper wires. This causes the wire to be deflected, the deflection being read by means of a projection microscope and scale. This deflection by itself proves nothing. The field between the metal plates Zn and Cu is not uniform; hence a deflection would necessarily appear with two condenser plates of the *same* metal if the wire were not situated exactly in the middle of the condenser. But the deflection observed changes in magnitude and direction when the charge on the wire is altered by interchanging the wires *1*, *2* which connect it with the battery (fig. 10b). This change of the deflection with the sign of the charge on the wire proves conclusively that there is an electric field of pressure ε between the two plates of the condenser. Or, in other words: the point of contact of the two metals acts like a cell, the pressure of which (ε) is added to the battery pressure E in fig. 10a and is subtracted from it in fig. 10b.

The magnitude of ε can be ascertained in a way which in theory is very simple. For example, the sheet of copper may be interrupted at c and cells of known pressure inserted between the ends. The deflections of the wire on the scale can be calibrated in this way.

In spite of many investigations the values of the contact pressure ε for pairs of different metals are much disputed. This is due to the difficulty of bringing two metallic surfaces satisfactorily into contact free from damp and films of gas; but there is no disputing the fact that contact pressures of the order of magnitude of a few hundredths of a volt up to about one volt do exist.

3. Metals as Conductors of the First Class. Thermo-electric Pressures.

At the junction of two different metals there exists a contact pressure ε (see last section). It follows that if several different metals a, b, c (fig. 11 a) are arranged in a chain or series, there will be a pressure between the free ends unless the arrangement of the single metals is completely symmetrical. If, however, we complete the chain and make it a "circuit", as e. g. in fig. 11 b, no permanent current flows in the circuit. No deflection is shown even by a sensitive galvanometer. Thus in a closed metallic circuit the sum of the contact pressures is equal to zero. This experimental result is considered so important that the metals have been distinguished from other conductors under the name of *conductors of the first class.*



a b

Fig. 11. Metals as conductors of the first class.

The phenomenon is less surprising now than it once was. We know that in metals it is only the electrons that move, so that it is only electrons that can pass through the boundary surfaces to the points of contact (pp. 246—250). During the passage of the electrons currents flow in the separate pieces of metal, since electrons are moving in the various metallic parts of the circuit. These electrons are very rapidly retarded by friction, however. Their original kinetic energy appears as the heat produced by the currents. A permanent current in the circuit of fig. 11 b would require that the electrons should be provided with a continual supply of energy, and this does not exist. After the initial passage of the electrons there are no chemical reactions from which energy might be supplied going on in the surfaces in contact.

The case is altered whenever we heat one of the junctions in the metallic conductor, i. e. supply it with energy. Then a current immediately flows in the conducting circuit. That is, when the junction of two metals is warmed an additional pressure ε_t is produced; this is called the *thermo-electric pressure.* In order to measure it the temperature of one junction is kept constant (by surrounding it with ice or water; cf. fig. 12) and values of the thermo-electric pressure are found for various temperature differences between the hot and cold junctions. Such values are collected in Table VIII.

Table VIII.

	Temperature of the cold junction: 0° C.			
	Temperature of the hot junction			
	100°	500°	1000°	1500°
	Thermo-electric pressure in millivolts			
Copper-Constantan	4·1	26·3	—	—
Silver-Platinum	0·72	6·26	—	—
Nickel-Platinum	1·5	5·55	9·59	—
Platinum-Platinum-rhodium(5%)	0·55	3·22	6·79	10·56

Thermo-electric pressures are small; temperature differences of as much as 500° give only 26.10^{-3} volts even in the favourable case of copper-constantan. Yet conduction currents i of say 100 amperes can be produced by thermo-electric pressures. We have only to take account of Ohm's law $i = E/r$ and make r, the resistance of the circuit, very small; for the quotient of two small quantities may have a very large value.

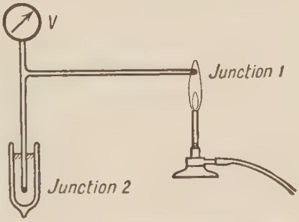
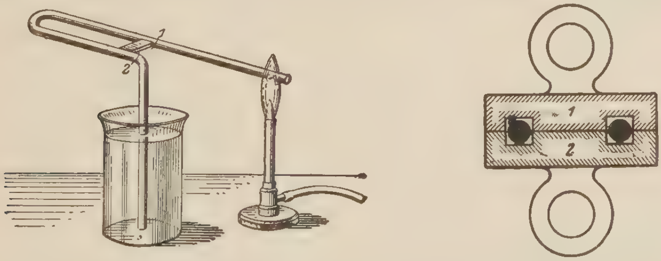


Fig. 12. Thermo-electric pressure. The junction 2 is placed in melting ice.

A suitable arrangement is shown in a perspective drawing in fig. 13a. A thick copper rod is bent into a U-shape and the two arms are joined by a short thick block of constantan soldered to the copper. The junction *1* is heated from the projecting end of the copper, which is placed in a bunsen flame. The other end is bent downwards so as to dip into cold water. This is to prevent junction *2* from becoming heated. The magnetic field



a

b

Fig. 13a, b. To illustrate the large currents produced by thermo-electric pressures.

of the current serves to show qualitatively how great the current is. Fig. 13b shows a section in profile of a piece of iron *1* which forms an iron core for the single half-turn of wire. A similar piece of iron *2* serves as armature. Core and armature stick so firmly together when the current passes that the armature will easily support ten pounds weight, a very remarkable feat for an electromagnet with only *one* turn.

Thermo-electric pressures play an important part in scientific measurement, in which junctions of suitable metals are much used under the name of "thermo-elements" or "thermo-couples". A thermo-couple is shown diagrammatically in fig. 12. As compared with ordinary mercury-in-glass thermometers, thermo-couples have the advantage of having a small mass and hence a small heat capacity, so that they rapidly come to equilibrium.

For example, changes in the temperature of minute insects can be accurately and conveniently followed by the use of a thermo-couple, whereas the application of the bulb of a mercury thermometer would disturb the temperature of the little creature's body.

Thermo-couples of very delicate construction are of special importance for measuring the energy of radiations of all kinds — light, X-rays, cathode rays, etc.

Finally, thermo-couples are used in the construction of a highly sensitive instrument for measuring alternating current, the essential part of which is a "crossed-wire thermo-junction". Only too frequently do investigations involving alternating currents come to nothing on account of the lack of sensitivity of the hot-wire galvanometer, whose lower limit in practice is a few hundredths of an ampere. In the hot-wire galvanometer the current heats a fine wire, which expands and thereby causes a pointer to turn by mechanical means (fig. 19, p. 12). In the crossed-wire instrument one junction of a thermo-couple is attached to a fine wire and the thermo-electric pressure observed by means of a voltmeter. In the practical form of the instrument (fig. 14) two fine wires of different metals, *1* and *2*, are looped together and soldered where they join. The halves of the wires *1* and *2* on the left together form the "hot wire"; those on the right the thermo-couple with the junction *a*.

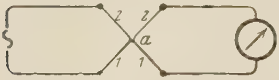


Fig. 14. The crossed-wire junction.

The thermo-electric pressures may be interpreted as the temperature coefficients of the contact pressures.

The so-called Peltier effect is a reversal of the thermo-electric phenomenon which is theoretically important. The junction of two metals is cooled by the passage of an electrical conduction current, provided the latter is in the same direction as the thermo-electric current which would be produced if the junction were heated.

4. Electric Fields in the Boundary Layer between Metals and Electrolytes. Conductors of the Second Class, Cells.

Electric fields are always formed in the places where a metal and an electrolyte are in contact. They can be shown to exist by means of the apparatus used in fig. 10 for two metals in contact. In fig. 15 a zinc plate *Zn* dips into an aqueous solution of a zinc salt, say of zinc sulphate. By means of an artifice part of the surface of the liquid is placed in a vertical position, being held up by the porous framework of a strip of filter-paper *Fl.*

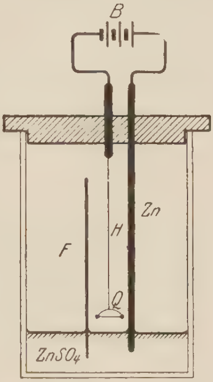


Fig. 15. Electric field in the boundary layer between metal and electrolyte.

Fl and *Zn* form the condenser. *H* is again a hair-like wire, *B* the battery for charging the wire. Just as in fig. 10, the deflection changes with the sign of the charge on the wire. The pressure ε between the metal and the electrolyte is of the order of one volt.

Metals become negatively charged, and the electrolyte accordingly becomes positively charged.

This experimental fact leads to a simple result: if two different metals are in contact with the same electrolyte, the measured pressure between the metals is the *difference* of their pressures with respect to the electrolyte; what we observe is $(\varepsilon_1 - \varepsilon_2)$.

Corresponding relations also hold when several electrolytes are connected in series. Thus for example in fig. 16 we have copper and zinc in aqueous solutions of their respective sulphates. A porous septum of unglazed earthenware or the like forms the boundary between the two electrolytes; this is to prevent the two liquids from mixing. Thus here the series consists of two metallic and two electrolytic conductors. A pressure of about 1·09 volts is observed between the two metals.

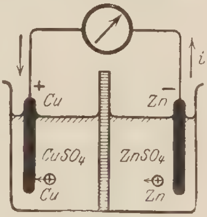


Fig. 16.
The Daniell cell.

When all the conductors in the series are metals, no current flows in the circuit when the latter is closed (fig. 11 b). On account of this fact, we call metals conductors of the first class. On the other hand, when electrolytes are included in series, their effect is such that a conduction current can flow even after the circuit has been closed. Thus electrolytes forming part of a closed circuit behave quite differently from metals; they are therefore called conductors of the second class.

The production of permanent electric currents by the insertion of suitable electrolytes in the circuit is not difficult to explain. Permanent currents require that energy should be permanently supplied to the atoms of electricity. When metals and liquids which conduct electrolytically are in contact, diffusion phenomena and chemical reactions may take place, and these serve as sources of energy.

The arrangement sketched in fig. 17 shows an example where diffusion acts as a source of energy. Here a loop of wire and two solutions of the same electrolyte differing in concentration form a circuit of three conductors. (In this case the pressure between the two copper plates *Cu* is of the order of a tenth of a volt.)

As regards chemical reactions acting as a source of energy we shall confine ourselves to one example which is particularly easy to understand, namely the Daniell cell shown in fig. 16.

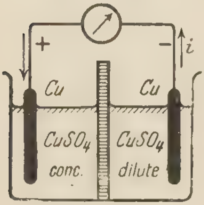


Fig. 17. A concentration cell.

Suppose the Daniell cell is giving current; e. g. let its zinc and copper electrodes be connected by a metallic wire. We then observe a permanent current flowing from the zinc to the copper; that is, electrons flow continually along the wire from the zinc to the copper. At the same time zinc goes into solution, actually in the form of positive ions, but of course all that we see is the zinc being dissolved. This dissolved zinc cannot be in the form of neutral zinc atoms, however, but only in the form of positive ions. Otherwise the zinc electrode could not contain the excess of electrons which enables it to keep on sending electrons through the metal wire to the copper electrode.

The positive zinc ions draw negative SO_4 ions out of the left-hand compartment into the right-hand one (cf. the lines of force diagram, fig. 7 a, p. 24). The positive copper ions in the left-hand compartment are deprived of their negative partners. They move to the copper plate and are deposited on it, so that they may unite themselves with the electrons which have moved up to the copper plate through the metal wire. During the passage of the current the zinc plate is dissolved and the copper plate becomes thicker. In the right-hand compartment the concentration of zinc ions and SO_4 ions rises. The solution soon becomes saturated and crystals of pure zinc sulphate separate out. In the left-hand compartment, on the contrary, the concentration of the copper ions and SO_4 ions falls. Hence in order that the source of current may continue in action a vessel with a stock of copper sulphate must be arranged so as to replenish the cell. For example, fig. 18 shows a conical supply vessel with crystals of copper sulphate. The so-called Meidinger cell shown here is a practical variant of the Daniell cell. The two electrodes are arranged not side by side but one above the other; this makes it possible to dispense with the porous septum, as mixing of the

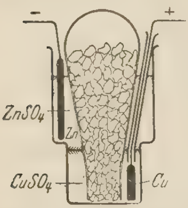


Fig. 18. A practical variant of the Daniell cell.

two sulphate solutions is sufficiently prevented by the difference in their densities.

Thus the final result in the Daniell cell is that zinc is transformed into zinc sulphate and copper deposited from copper sulphate. The formation of zinc sulphate in acid solution takes place with liberation of energy. When the reaction is made to take place in a calorimeter it gives a quantity of heat of $4{\cdot}40.10^5$ watt-seconds per mol of zinc. The deposition of copper, on the contrary, absorbs energy; it requires a supply of heat of $2{\cdot}34.10^5$ watt-seconds per mol of copper. The difference, $(4{\cdot}40-2{\cdot}34)10^5 = 2{\cdot}06.10^5$ watt-seconds, is therefore the energy which becomes available when one mol of metal is transformed. It is this energy that maintains the current.

Starting from these considerations the pressure of the Daniell cell may be calculated from chemical data. In the Daniell cell the copper and zinc ions are divalent, i. e. every ion carries two elementary quantities of electricity (pp. 233 to 235). Thus a mol of zinc going into solution or a mol of copper deposited from solution carries along with it 2.96500 coulombs in all. During this process the electric field of pressure ε does work amounting to $2.96500.\varepsilon = 2{\cdot}06.10^5$ watt-seconds, which gives the value 1·07 volts for ε instead of 1·09 volts as observed. This simple method of calculation, however, is admissible only for the few cells in which the pressure is practically constant with temperature.

In all "chemical sources of current" or cells the phenomenon which supplies the energy may be explained in a similar (though often less simple) way. The discussion is frequently made more difficult by the occurrence of products of secondary reactions at the electrodes. The liberation of the metallic ions often causes hydrogen to be evolved at the anode of the cell, and so on. The details belong to the realm of physical chemistry.

5. Polarization in Electrolytic Conductors.

According to what we have said in the last section, two conductors of the same metal in an electrolyte do not form a cell. There is no pressure between the two metals or electrodes. For apart from the electrolyte the arrangement is completely symmetrical, and hence diffusion phenomena or chemical reactions which might set the atoms of electricity moving in *one* direction cannot take place. As an example, we take as electrodes two platinum wires dipping into dilute sulphuric acid. The voltmeter gives no deflection.

The symmetry is at once disturbed, however, when we send a conduction current through the electrolyte (fig. 19b). Hydrogen is set free at the platinum wire on the left, the cathode, and oxygen at the wire on the right, the anode. As is well known, the greater part of the gases escapes in the form of bubbles. The gases liberated at the very beginning, on the other hand, form an invisible layer over the surfaces of the electrodes, which in certain circumstances is only one atom thick.

The symmetry is largely destroyed by these invisible layers of gas. The electrodes together with the electrolyte now form a cell, the pressure of which is called the polarization; it is of the order of a few volts and is therefore about as large as the pressure between two different metals in the same electrolyte.

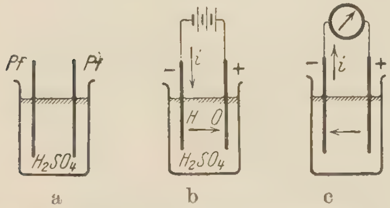


Fig. 19a—c. To show the existence of polarization.

The existence of polarization can easily be shown by a voltmeter (cf. fig. 19c). The arrows shown in figs. 19b and c indicate the direction of the current, i. e. the direction of the electrons or negative ions. The direction of the current due to polarization is opposite to the direction of the original current. Thus the electric field produced by polarization is opposed to the field originally produced by the source of current.

This can readily be understood as a result of the principle of energy. If the polarization and the original pressure were in the same direction, the original current would necessarily go on increasing indefinitely.

When we made the measurements for the current-pressure graph for electrolytes on p. 236 we found that it was not the quotient E/i that was constant, but $(E - \varepsilon)/i$. The current i, to be sure, did increase linearly with the pressure E in fig. 49, p. 236, but the straight line did not point in the direction of the origin; when prolonged, it cut the horizontal axis at a point corresponding to the small pressure ε. At that time we concluded that the voltmeter indicated a pressure which was ε volts too high (fig. 48, p. 236). The real pressure between the ends of the conducting column of liquid was taken as being only $(E - \varepsilon)$ volts. We now see the eventual justification of the interpretation which we gave on that occasion. In ε we recognize the polarization necessarily produced during the passage of the current.

Hitherto we have only considered polarization due to the electrodes being covered with gas. This, however, is merely a special case. The passage of a current through an electrolyte may produce asymmetry in many different ways, and each lack of symmetry gives rise to polarization. Several examples will be brought forward in the following sections.

6. Accumulators.

Polarization due to a layer of gas on the electrodes does not as a rule last long. This can easily be shown with the apparatus of fig. 19; we have merely to vary the lengths of the pauses between the experiments *b* and *c*.

There are cases, however, where the polarization can be maintained for a long time. In these cases it is a question of radical alterations in the electrodes as a result of the passage of the current. Accumulators are typical examples of this.

We shall explain the action of the well-known *lead accumulator* by a demonstration experiment. Two lead wires dip into dilute sulphuric acid, and thereby become covered superficially with a layer of lead sulphate ($PbSO_4$).

A battery B (giving about 6 volts) enables us to send a current through the "accumulator cell". The negative SO_4 ions then move to the anode and transform the latter into lead dioxide in accordance with the equation

$$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4 .$$

The positive hydrogen ions move to the cathode and reduce the latter to metallic lead, in accordance with the equation

$$PbSO_4 + H_2 = Pb + H_2SO_4 .$$

This process therefore produces marked asymmetry. A cell is formed with one electrode of lead and the other of lead dioxide; "the accumulator is being charged". After some minutes this "charging" process is interrupted and the electrodes are connected to a small glow-lamp which acts as a current indicator. The lamp lights up and the accumulator "is discharging"; the reverse reactions are now taking place in its interior, both electrodes being re-converted into lead sulphate. We have only to read the above equations from right to left. *Hence the accumulator may be called a reversible cell.*

Many reversible cells can be imagined. The Daniell cell may be mentioned as being particularly well known (fig. 16, p. 271). If the Daniell cell is giving a current, zinc is dissolved from the cathode and copper is deposited at the anode. If a current is sent through the cell from outside (fig. 20), the reverse process takes place: the zinc cathode grows and the copper anode is dissolved.

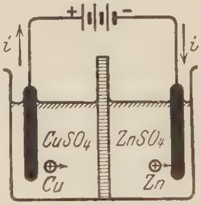


Fig. 20. The Daniell cell acting as an accumulator.

Hence a Daniell cell may in theory be used as an accumulator. The zinc lost while current is being supplied by the cell or the cell is discharging is made good by subsequent charging. This form of accumulator is of no practical importance because its life is too short. Its two electrolytes, the solutions of copper sulphate and zinc sulphate, get mixed by diffusion in spite of the earthenware partition. Copper ions finally reach the zinc, spoiling the cell.

The lead accumulator is still the most important type from the practical point of view. Various devices are used to give the lead plates extremely large surfaces. The anode or positive electrode is usually provided with a coating of lead oxide beforehand. When in working order an accumulator has a very constant pressure of 2·2 volts. A slightly higher pressure, namely 2·6 volts, is required to charge it. 2·02/2·6 gives the efficiency of the accumulator as about 78%.

7. Unpolarizable Electrodes and Cells. Standard Cells.

The occurence of polarization is very disturbing in many measurements on electrolytes. Often the currents used for the measurements can only have small pressures E, in which case ε may no longer be neglected as being small compared with E. We met with a case of this kind when verifying Ohm's law for electrolytes (cf. pp. 236, 274). Similar cases are particularly common in physiological experiments, such as those on nerve stimulation by electricity, etc.

We have found out how to do away with this difficulty to a great extent, however, and "unpolarizable electrodes" are now available. Ordinary polarizable electrodes consist of metallic conductors, i. e. actual metals, carbon, etc. *Unpolarizable electrodes consist of metals with suitable coatings. For example, an aqueous solution of a salt containing the metal of the electrode in the form of ions is a very convenient coating.* An unpolarizable zinc electrode is drawn as an example in fig. 21. A zinc rod dips into a glass tube containing an aqueous solution of zinc sulphate and closed at the bottom by a porous plug P saturated with a dilute solution of NaCl. This damp plug forms the connection with the electrolyte, say the end of a nerve indicated by N. It is advisable that the surface of the zinc rod should be amalgamated in order to preserve it from local action. When used as the anode, the zinc rod of this unpolarizable electrode is dissolved; when used as the cathode, it becomes thicker, owing to deposition of zinc. There remain only the changes in the concentration of the solutions. This lack of symmetry merely causes a vanishingly small polarization. This single example of an unpolarizable electrode will suffice.

Furthermore, electrolytic polarization plays a great, nay a decisive, part in the construction of chemical sources of current, or cells. Any number of cells may be devised, for any two metallic conductors unsymmetrically connected with electrolytes form a cell; but the vast majority of such cells are useless as practical sources of current. The occurrence of polarization when current is taken from them makes the pressure sink very rapidly. We connect e. g. two cells formed of zinc, arc-lamp carbon, and ammonium chloride solution with a small glow-lamp. The lamp gleams brightly at first, but goes out in a short time.

Covering the carbon anode with manganese dioxide (MnO_2) diminishes the polarization considerably. The manganese dioxide oxidizes a large proportion of the hydrogen liberated at the anode. If in addition the electrolyte is replaced by starch paste, sawdust, or the like, we have the dry cells (which are by no means absolutely dry!) used in pocket flash-lamps.

Cells of constant pressure must be constructed with "unpolarizable" electrodes. This condition is satisfied by "reversible" cells, as can easily be seen in the case of the Daniell cell (fig. 16, p. 271). In fact both its electrodes are unpolarizable in the sense indicated by fig. 21.

The most stringent requirements as to unpolarizability and constancy of pressure as time goes on are naturally made in the case of "standard (or normal) cells" (p. 15). In fact the international unit of electric pressure, the volt, is defined by means of a standard cell constructed in a definite way.

The Daniell cell does not satisfy these requirements at all. It will not do, for the reason which makes it practically useless as an accumulator; it does not last long enough. In spite of the partition, the copper ions of the concentrated solution of copper sulphate diffuse to the zinc cathode and cover it with copper "slime".

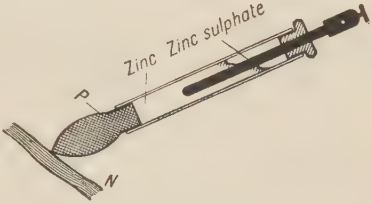


Fig. 21. An unpolarizable electrode.

This difficulty is avoided in the standard cadmium cells now in use. Instead of copper, a metal with an extremely insoluble sulphate (mercury) is used as anode (fig. 22) and the adjacent electrolyte consists of a thick paste of mercurous sulphate (Hg_2SO_4), mercury, and crystals of $CdSO_4$. The cathode consists of cadmium amalgam in a solution of cadmium sulphate, which is usually saturated and provided with an excess of crystals. Owing to the minute concentration of the mercury ions in the paste there is no danger of their diffusing to the cathode. The defect of the Daniell cell is thereby avoided; but of course another disadvantage has to be put up with in exchange in the standard cadmium cell, namely that only currents of a few hundred-thousandths of an ampere may be taken from the cell. Otherwise the concentration of mercury ions is used up, because its salt dissolves too slowly; this gives rise to polarization, owing to the change in the concentration of the electrolyte next to the anode. In carrying out modern scientific measurements the small load which the standard cell can bear does not involve any difficulty worth mentioning. So-called compensation methods are used, which prevent inadmissible amounts of current from being taken from the standard cell. These matters are discussed in elementary textbooks on physical measurements.

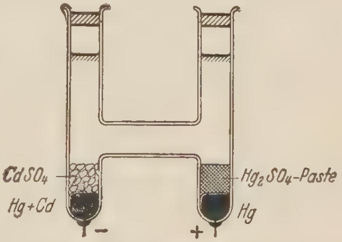


Fig. 22. The standard cadmium cell. Pressure between the electrodes at 18° C. = 1·0187 volts.

8. The Demonstration of Polarization by the Change in the Surface Tension of Liquids.

As is well known, cohesion between the molecules of a liquid is revealed by the phenomena of surface tension. The surface of a liquid behaves like a stretched indiarubber membrane. Surface tension e. g. causes a liquid to fall in single drops. The smaller the surface tension, the longer is the length of the undisturbed jet, and the later does it begin to break up. This is shown in detail in mechanics.

Surface tension is affected by electric fields. We can verify this assumption by a particularly simple experiment. Fig. 23 shows a glass tube with a narrow nozzle. When water is poured into the tube it at first runs out in a smooth jet. When H, the depth of the liquid, is decreased, i. e. when the pressure of the liquid is diminished, the formation of a jet is prevented by surface tension, and the water merely trickles out slowly in drops. We now connect the water by means of a wire with an induction machine at work. The water immediately begins to run out in a smooth jet again. The lines of force which run from the water in the nozzle to the other pole of the machine diminish the surface tension and thereby prevent the drops from becoming separate spheres.

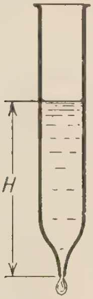


Fig. 23. Effect of an electric field on the surface tension of water.

This influence of an electric field on surface tension now enables the electric field due to polarization to be easily detected in liquid metals. This is done in the experiment sketched in fig. 24. A drop of mercury Hg and an iron nail are lying in dilute sulphuric acid with some potassium bichromate added to it. Along with this electrolyte the mercury and iron form a polarizable cell. The drop is much flatter than it would be in air, that is, the surface tension of the mercury is lowered by contact with the electrolyte. The cause of this diminution is the electric field which is formed in the boundary layer.

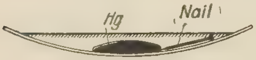


Fig. 24. Change of the surface tension of mercury owing to polarization.

We now connect the two electrodes of the cell, i. e. we let the point of the nail touch the drop of mercury. A current immediately flows, giving rise to polarization. The electric field in the boundary layer is diminished. The surface tension of the mercury rises. The drop slowly contracts and interrupts the circuit. The polarization does not last long (especially owing to the admixture of chromic acid, which oxidizes the hydrogen). After a short time the drop spreads again and the process begins once more. The drop pulsates like a heart.

The phenomenon whose principle has been illustrated here is commonly used in the construction of a simple voltmeter, the *capillary* *electrometer*. It is easy to construct a makeshift form, but the instrument can only be used for pressures under one volt.

The capillary electrometer consists essentially of two mercury electrodes of very different sizes, in dilute sulphuric acid (fig. 25). The small electrode is formed by M, the top of a thread of mercury in the slightly inclined glass capillary G. The meniscus comes to rest in a position such that the effects of gravity and surface tension balance each other. The latter is already diminished by the electric field in the layer where the mercury and the sulphuric acid are in contact. The equilibrium position of the meniscus is determined with the electrodes short-circuited. If an electric field is produced between the mercury electrodes, a current flows. At the top of the mercury column, which is small, the current density is several thousand times greater than at the large mercury surface, and a strong polarization effect is very rapidly developed at the former. If this electric field is opposed to the original field produced by contact in the boundary layer between the mercury and the sulphuric acid, the value of the surface tension is raised again. Hence the thread of mercury is pulled up the capillary again against gravity. The extent of the movement is not actually proportional to the pressure, but it is easy to calibrate the apparatus. The applicability of this simple instrument is unfortunately limited by the smallness of its range and the not inconsiderable amount of current which it consumes.

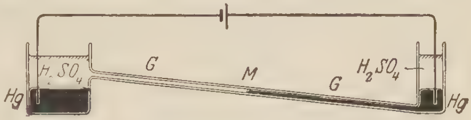


Fig. 25. The capillary electrometer.

9. Electric Fields in the Boundary Layer between Gases and Liquids.

Electric fields are likewise found in the boundary layer when gases and liquids are in contact. They can be shown to exist in exactly the same way as for the boundary layer between solids and liquids, by means of cataphoresis (pp. 263—265). For example, we may observe the motion of a small bubble of air B in the electric field of a condenser AK which is filled with water (fig. 26). In pure liquids the gases always turn out to be negatively charged.

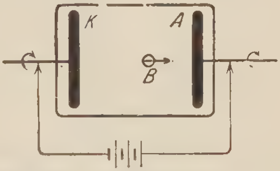


Fig. 26. Electric field at the boundary of a bubble of gas and a liquid.

In carrying out this experiment the main source of disturbance is the marked buoyancy of gases in liquids; this objectionable effect, however, can be got rid of by an artifice, namely by making the condenser, together with the fluid, rotate uniformly about its horizontal long axis. The bubble then remains in the longitudinal axis of the condenser and moves in the electric field up to the anode.

Bubbles moving to the anode must contain atoms of negative electricity. We might at first imagine that some of the gas molecules have been ionized by contact. This interpretation, however, is out of the question. In spite of many investigations, gaseous ions have never been proved to exist in gases after the latter have been in contact with a liquid. Hence in the case of bubbles the carriers of negative electricity must also be situated in the *liquid*, i. e. the whole electric field of the boundary layer must be contained in the outer layers of molecules of the *liquid*. Thus what is visibly seen moving to the anode in cataphoresis is a thin-walled *bubble in the liquid*, negatively charged and filled with gas. This corresponds to the sphere in fig. 2, p. 263. Positive ions of liquid glide invisibly over it to the cathode. There is no new principle involved in this interpretation. Even when liquids and solid bodies are in contact the electric field of the boundary layer extends into the liquid to a depth equal to the diameter of several molecules.

The following important experimental result agrees very well with this distribution of the electric field in the surface of the liquid.

By suddenly tearing the surface of the liquid apart, some of the charges which form the boundaries of the superficial field may be separated from the rest. We may let gases bubble vigorously through liquids, so that the bubbles fizz and burst on the surface of the liquid, or we may blow a liquid to pieces with some familiar form of atomizer, or we may let rapidly falling drops splash on to an obstacle (as e. g. in a waterfall). In all cases the same result is found: if the finely divided liquid is subsequently collected, it turns out to be positively charged, whereas countless ultra-microscopic carriers of negative electricity are found in the air. The "mobility" of these carriers is extremely small. There is no doubt that they consist of very fine particles of liquid dust from the outer, negatively-charged layer of molecules of the liquid surface which is blown to pieces.

In the free atmosphere strong vertical currents of air may tear large raindrops in pieces. The larger fragments soon reunite to form large drops. In this way there arise electric fields between positively charged drops and very fine particles of negatively charged water. The drops fall, while the particles remain floating high up in the air. The lines of force between the large drops and the particles are stretched and the pressure may rise to very high values (pp. 25—26). This process probably plays an essential part in the production of lightning.

We have confined our discussion to the case of pure, i. e. chemically homogeneous, liquids. In their case the surface is always the site of the negative charge. Complications arise when other substances are added, say salts or acids to water or non-noble metals to mercury. The details would lead us too far out of the way here.

10. Emission of Electrons from a Surface. The Microphone.

Throughout the whole of this chapter we have described the formation of electric fields in the boundary layer of various substances purely as an experimental fact. The details of the mechanism have not yet been cleared up. For example, we do not yet know why metals go into solution only in the form of positive ions and not as neutral atoms. The fact that "metals possess a solution pressure" (Nernst) is granted, but the connection between this fact and the electrical structure of the atoms of metals is wanting.

The electric fields in the boundary layer are ultimately due to those electric forces which unite atoms to form molecules and molecules to form liquid or solid bodies. To put it briefly, these are "molecular forces" i. e. essentially forces of electrical origin.

Another important problem arises in the same connection, namely *the emission of electrons and ions from the surface of a body.*

The question is similar to that of the escape of neutral molecules from a surface or their "evaporation": the escaping electron or ion must do *work*. The forces which hold it back are greatest in the immediate neighbourhood of the surface. Hence a large amount of work W_1 is done in the first piece of the path, s_1 (fig. 27). As the distance from the surface increases, the forces pulling the electron back decrease rapidly; for in general the effect of the molecular forces does not reach beyond a distance of the order of that between the molecules, i. e. of some 10^{-8} cm. Hence smaller amounts of work W_2, W_3, etc. are done in the subsequent elements of the path s_2, s_3, etc. The total work done in the whole path from the surface rapidly approaches the *limiting value* W_0, *the work required to extract the electron or ion from the surface.* If work equal to the work of extraction is done, the electron passes beyond the range of the forces pulling it back and is free.

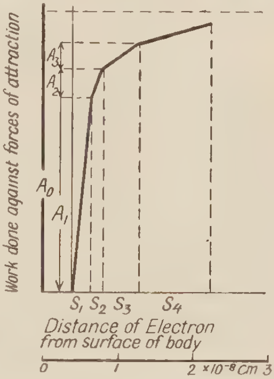


Fig. 27. To illustrate the work of extraction of an electron.

The work of extraction W_0 can be found from experimental data, at least as regards its order of magnitude.

For this purpose, two metal electrodes are placed opposite to each other in a high vacuum. **R**, the intensity of the electric field between them, is then raised until electrons are torn out of the cathode and fly over to the anode, forming a current. This happens when the field intensity is of the order of 10^7 to 10^8 volts/cm. In our calculations we take it as being 10^8 volts/cm.

The electric intensity **R** exerts the force $F = e\mathbf{R}$ on the electron of charge e. With this force acting along a path of length 10^{-8} cm. the field has to do work equal to W_0, the work of extraction of the electron. Work is the product of the force and the length of path, so that

$$W_0 = e.10^8 \text{ volts/cm.} \times 10^{-8} \text{ cm.},$$

$$\text{or } W_0 = e.1 \text{ volt}.$$

Inserting the value of the elementary quantity of electricity, $e = 1{\cdot}6.10^{-19}$ coulombs, we obtain $W_0 = 1{\cdot}6.10^{-19}$ watt-seconds. But this way of stating the result is not usual; in such a case we rather say shortly: "The work required to extract the electron (or, the energy of the electron) is one volt." This is obviously a loose mode of expression, for work (or energy) and volt are of different "dimensions"; but it has the advantage of clearness. "An electron of energy n volts" means an electron which has been accelerated by an electric field with a pressure of n volts.

In the experiment which forms the basis for this approximate calculation the electrons receive their energy from the external electric field attracting them. Other sources of energy may, however, take the place of this external field. As we already know, the thermal motion of bodies is one of these. At room temperature it is insufficient to enable electrons to escape in measurable quantities.

Thermal motion at room temperature only supplies the electrons with an energy of about $^1/_{30}$ of a volt.

At high temperatures, however, a vigorous emission of electrons takes place. *The thermal motion provides the electrons with kinetic energy by means of some mechanism whose nature has not yet been cleared up. At high temperature the kinetic energy exceeds the amount of work required to extract the electrons from the surface.* Quantitative investigations of this "thermal emission of electrons" has confirmed the fact that the work required to extract the electrons from metals is of the order of several volts.

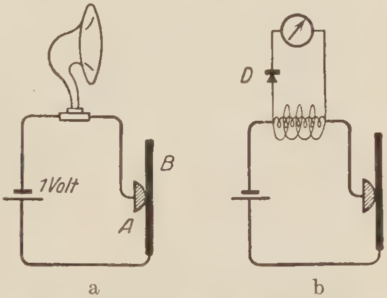
In many substances, and above all in incandescent salts, the work required to extract the positive ions is less than for the electrons. At lower temperatures such substances at first emit positive ions. The emission of electrons subsequently predominates at higher temperatures.

The emission of electrons from surfaces is important in connection with many physical problems. As a single example we shall name the *microphone*, which is of such great practical importance.

In essentials the microphone consists of two pieces of carbon which touch each other, say a sphere A and a diaphragm B (fig. 28a). Together with a cell and a telephone they form a circuit. Someone speaks opposite the diaphragm. The sound waves set the diaphragm in vibration. The resistance of the carbon contact is thereby altered, and the strength of the current varies, following the rhythm of the sound waves. The microphone is extraordinarily sensitive. A fly crawling over the diaphragm makes a loud noise in the telephone. In order to show the sensitivity to a large audience, the telephone is replaced by the primary coil of a small transformer (fig. 28b). The variations in the current induce an alternating current in the secondary of the transformer, following the rhythm of speech. This alternating current is observed by means of a moving-coil galvanometer with a small crystal detector D interposed to act as a rectifier. Ordinary speech even many yards away gives rise to large galvanometer deflections.

The effect of the microphone is to be interpreted as follows:

Every surface produced by mechanical means is very imperfect when regarded from the point of view of molecular magnitudes; it possesses numerous unevennesses and projections. Hence two carbon surfaces A and B loosely placed together approach each other to within 10^{-8} *cm., i. e. within distances comparable with the size of a molecule, at a few isolated places only.* It is only at these isolated places that the pieces of carbon are in contact in the electrical sense, and only there does **R**, the intensity of the field, reach the order of 10^8 volts/cm. which is required to tear out the electrons. Thus the stream of electrons can only pass from one piece of carbon to the other across a few "bridges".



a b

Fig. 28a, b. To show the microphone effect.

Carbon has a very high specific resistance. Hence in spite of their shortness the total cross-section of these few bridges offers a high resistance to the current, e. g. several hundred ohms. *That is, the greater part of the resistance of the whole circuit lies in the "bridges" of the microphone.* Squeezing the two pieces of carbon together to a slight extent brings other projections to within 10^{-8} cm. of each other. The strength of the field now reaches the order of 10^8 volts/cm. at these places also, and electrons are dragged from A to B there also. The number of "bridges" is increased. Doubling the number of bridges means halving the resistance of the microphone and approximately doubling the current, and so on.

So much for the carbon contacts of the microphone. Quite analogous considerations hold in the case of the contact of any two pieces of metal A and B, e. g. of copper. If one piece is loosely laid on the other, they also touch only at a few "bridges". In the case of carbon these bridges have a resistance of e. g. several hundred ohms; but in the case of copper, the geometrical relationships being the same, the resistance only amounts to a few tenths of an ohm, for copper has a specific resistance about 2000 times as small as that of carbon. When the pieces of copper are squeezed more closely together the number of bridges is likewise increased and the resistance reduced by several powers of ten.

Thus theoretically any metallic contact has the same effect as a microphone; but as a rule the alteration in the resistance is of no practical importance. If, for example, the total resistance of a circuit amounts to 10 ohms, it means practically no difference to the strength of the current in the circuit if the resistance of the contact varies between several tenths and several hundredths of an ohm. Hence the resistance of metallic contacts and its dependence on the pressure need only be taken into account in exceptional cases, namely in the case of circuits of extremely small resistance.

When pieces of carbon or metal are in contact the direction of the electric field is of no importance. The same electrical field strength is required to attract an electron from A to B or vice versa. When metals are in contact with sulphides or oxides of metals, etc. the strength of field required depends on the direction in which the current is passing. *These are the contacts which have the effect of a detector or rectifier.* We have made use of them several times in measuring feeble alternating currents. The details of their working would lead us too far out of our way here.

One word more in conclusion. Hitherto we have spoken of the work required to extract an electron from the surface of a body. In the limiting case the body may consist of a single molecule or even a single atom. In this limiting case the work of extraction becomes the work required to ionize the molecule or atom. It is also stated in "volts" for short*. Nowadays it is no use going into details unless in conjunction with the optical properties of atoms and molecules; for modern optics is really nothing but a special part of the theory of electricity.

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* [Strictly speaking, the number of volts gives the pressure of the electric field required to do this work on the electron; this pressure is usually called the *ionization potential* of the substance in question.]

CHAPTER X

Radioactivity

1. The Rays from Radioactive Substances.

Throughout this book our treatment has been based on atomic conceptions; these were particularly prominent when we were explaining conduction phenomena. We have taken the divisibility of material bodies and electrical substances into atoms as an undoubted experimental fact. An epoch-making advance in the development of modern atomic theory is marked by the discovery of radioactive phenomena. Doubts about the conceptions on which the atomic theory is based can no longer be entertained, as will be shown by means of convincing experiments in the present chapter. We have no intention, however, of giving an exhaustive account of radioactivity.

In 1895 W. C. Röntgen made his great discovery* known. Some months later, H. Becquerel, continuing Röntgen's work, came across the first radioactive phenomena†. He found that mysterious rays of a very penetrating nature are spontaneously emitted by pitchblende. These rays can be detected by the established methods for detecting X-rays, namely, by their effect on a photographic plate, by their causing fluorescence, and by their ionizing the air.

The well-known method depending on the decay of the field may be used to demonstrate the ionization effect. A plate condenser connected with a bifilar voltmeter is charged to 220 volts. Some pieces of pitchblende are placed on the lower plate (fig. 1). The deflection of the voltmeter falls with a velocity which is easily measurable (by means of a stop-watch). A blank experiment shows that the observed conducting power of the air is really caused by the pitchblende.

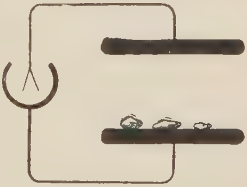

Fig. 1. To detect the radioactive radiation from pieces of pitchblende.

Using the ionization effect as an indicator, several dozen radioactive elements which are chemically well-defined have been discovered in the course of time. Further, we have learned to distinguish between three kinds of rays which are physically very different. These are denoted by the Greek letters α, β, γ.

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* [I. e. that of X-rays.]

† [Subsequent developments were due to Crookes, the Curies, Rutherford, Soddy, and others.]

From a purely external point of view these three kinds of ray are distinguished from each other by their different powers of penetration. We shall illustrate this briefly. A commercial preparation of radium is shown in fig. 2. Several milligrams of $RaBr_2$ in the form of a horny powder are contained in a vulcanite capsule with a thin mica cover.

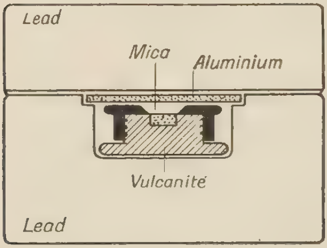

Fig. 2. A preparation of radium bromide in a vulcanite capsule and a thickwalled lead box with a brass lid.

For experimental purposes the under surface and sides of the capsule are surrounded with a layer of lead 2 cm. thick. On the top, an aluminium cover 2 mm. thick is first laid over the mica window and then another lead cover 2 cm. thick is put on.

We shall first proceed to show the γ-rays, which are the most penetrating of the three. The ionization of the air is again observed by means of the decay of the field. This is done by means of a large ionization chamber (fig. 3) containing about 30 litres. It is a cylindrical condenser with a large window F about 0·1 mm thick. In small ionization chambers too little of the penetrating radiation is absorbed. The ionizing effect of the γ-rays can be shown even at a distance of several metres. It begins as soon as the thick lead cover is lifted off the radium preparation. That is, the rays can penetrate through several metres of air in addition to 2·1 mm. of aluminium. 13 mm. of lead or about 50 mm. of aluminium merely weaken them by half. They exceed X-rays in penetrating power. So much for the γ-rays.

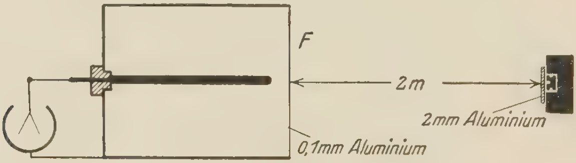

Fig. 3. To show γ-rays.

In order to show the β-rays, the aluminium cover is also removed and the preparation placed about 20 cm. from an ionization chamber. We obtain an ionization which is cut down to half when about 0·5 mm. aluminium is interposed. That is, the penetrating power of the β-rays is considerably smaller than that of the γ-rays. The stronger absorption causes a rise in the number of ions formed per unit length of path. Hence the amount of ions can easily be measured by the galvanometer and the lengthy method of observation with the stop-watch avoided; see fig. 4.

Quantitative determinations, e. g. of the coefficient of absorption, require more elaborate apparatus. Among other things it is important that "secondary radiation", which forms a troublesome source of error, should be prevented. All bodies on which γ- or β-rays impinge themselves become the starting-point for fresh β-rays, as can be shown with the apparatus sketched in fig. 5. Direct rays (continuous arrows) cannot reach the condenser (ionization chamber). They are cut off laterally by the lead cover. In spite of this the galvanometer still shows a measurable current, which is due to the "secondary radiation" formed in the air. The secondary radiation is considerably increased, when a metal plate (e. g. a sheet of lead) is brought into the neighbourhood of the rays. A few secondary rays are indicated by dotted arrows.

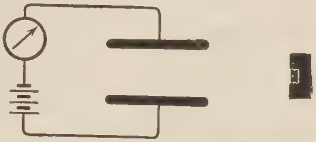

Fig. 4. To show β-rays. Galvanometer of the type shown in fig. 16, p. 30[*].

Finally we come to the α-rays. To show them we require to remove the mica window. They become indetectable after traversing even less than 10 cm. of air. As will be shown later, they have a perfectly characteristic range for every radioactive element. α-rays ionize the air very strongly, but the insertion of only 41μ of aluminium[†] stops them completely.

The tremendous differences in absorbability of the three kinds of rays are due to their different physical natures.

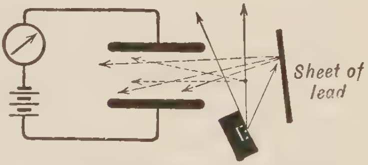

Fig. 5. Secondary radiation.

It has been found possible to identify γ-rays with X-rays of very short wave-length. By the methods which have been developed in connection with X-rays, the wave-length has been ascertained to be of the order of 10^{-10} cm. (or less). The details belong to the theories of optics and of atomic structure.

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[*] In order to obtain saturation currents (p. 206) it is advisable to use a source of current with a pressure of as much as 3000 volts. The most convenient one is a good practical condenser with glass plates and a capacity of about 10^{-8} farad, connected with a small induction machine. Once the condenser has been charged to the required pressure, it acts in almost the same way as a high pressure accumulator, i. e. its pressure remains practically constant, for the small quantities of electricity which pass through the galvanometer are negligible in comparison with the charge of the condenser. Short-circuiting is guarded against by including a carbon resistance of about 10^7 ohms in series. This source of current, which is as cheap as it is convenient, is used e. g. in the experiments shown in figs. 4, 5, 12, 13, 15, and 17 of this chapter. The use of high pressure batteries, which, though commonly employed, are a continual source of trouble, is thereby avoided.

[†] [See footnote, p. 228.]

β-rays are electrons of very high velocity, or very fast cathode rays. Their velocity is measured by their deviations in magnetic and electric fields. The method is similar to that described on p. 223. Velocities up to 99·6% of the velocity of light have been observed.

There is no difficulty in showing the magnetic deviation qualitatively. A suitable arrangement is sketched in fig. 6. For a change, the indicator for the β-rays takes a somewhat different form in this case. The β-rays are made to affect a spark gap. The secondary of a small induction coil is connected with a small condenser C (a Leyden jar) and two curved metal electrodes K. The pressure is adjusted to be so low that sparks just fail to pass. β-rays are then allowed to fall on the spark gap. The ionization of the air calls forth a play of sparks which can be seen at a distance. The β-rays can only reach the spark gap by passing through between the poles of the electromagnet NS. If the circuit of the electromagnet is closed the β-rays are deviated to one side, namely upwards out of the plane of the paper. The sparks cease. The process may be repeated as many times as we like.

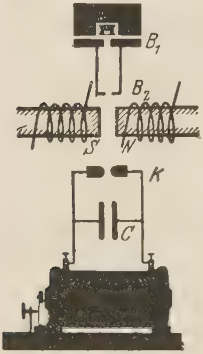

Fig. 6. Magnetic deviation of β-rays.

α-rays are likewise deviated by magnetic and electric fields, but in the same direction as positive rays are. Measurements using the method explained for positive rays (p. 223) have shown that α-rays have an atomic weight of 4, i. e. identical with that of the inert gas helium. Their charge is two elementary quantities of electricity, i. e. $2.1 \cdot 6.10^{\ 19} = 3 \cdot 2.10^{-19}$ coulombs. *α-rays are therefore divalent positive ions of helium.* Their velocities go up to 2.10^9 cm./sec. The positive rays of the luminous discharge do not reach anything like this velocity. The measurements, like those on positive rays, are not well adapted for lecture experiments.

To summarize this brief survey, we may say that

$$\alpha\text{-rays} = \text{positive rays},$$

$$\beta\text{-rays} = \text{cathode rays},$$

$$\gamma\text{-rays} = \text{X-rays}.$$

Thus it has been possible to fit these rays, which were so mysterious when they were first discovered, into the framework of known facts. In spite of this, however, rays of radioactive origin have acquired special significance in connection with the electrical conception of the nature of the universe, as will be obvious from the sections which follow.

2. The Observation of Single Electrons and Ions.

At an earlier stage both cathode and positive rays were investigated as mass phenomena only. All the time we were observing dense swarms of the quickly flying electrons. The investigation of α-rays prepared the way for a great advance in this respect. Single cathode and positive rays can now be detected, i. e. single electrons and ions are accessible to direct observation. Three different methods must be mentioned.

(1) *The scintillation method.* Numerous crystals exhibit a peculiar optical phenomenon, known as triboluminescence, when they are mechanically injured. Ordinary lump sugar is a well-known example of this: if we crush it in the dark, we see feeble flashes of bluish light. Crystals of zinc sulphide containing copper and manganese exhibit the phenomenon in a greatly enhanced degree. When they are scratched with a needle or ground to powder bright points of light are seen at the fractures.

The impact of a single α-ray has the same effect as mechanical injury. The point of impact of each single projectile is marked by a bright flash of light.

It is convenient to make the observations through a pocket lens magnifying six times. Fig. 7 shows a small apparatus* for showing the effect. The screen S, consisting of crystal powder stuck on to cardboard, is placed at the bottom, the magnifying-glass at the top, and a trace of radioactive substance R (about 10^{-4} mgm. of radium bromide) at the side. The phenomenon is remarkably fascinating even to an unlearned observer, but unfortunately it can only be seen by one at a time. The resemblance between this phenomenon and the appearance of twinkling or scintillating stars has given the method its name.

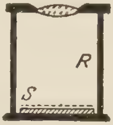

Fig. 7. Screen S with magnifying glass for observing scintillations (spinthariscope).

β-rays, i. e. very fast electrons of radioactive origin, can be observed in the same way; but the phenomenon then produces a very much feebler light and can be recognized with certainty only by practised observers.

(2) *Condensation tracks.* The scintillation or triboluminescence phenomenon only enables the points of impact of the single corpuscles or rays to be recognized. The condensation track method (due to C. T. R. Wilson) gives a picture of their whole path. The principle of this method is a simple one. If damp air is cooled, water vapour condenses, and a mist of water-droplets (not bubbles!) is formed. The formation of these droplets is favoured by the presence of so-called "condensation nuclei". All kinds of foreign bodies mixed with the air have this effect, such as dust, products of combustion (as in a London fog), and above all, ions. Ions give rise to fog even when the amount of cooling is trifling. Fig. 8 shows a glass chamber K in front of the condensing lens of a lantern. The front glass wall can be moved in the direction

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* [Invented by Crookes and usually known as a *spinthariscope.*]

of the arrow, thus acting like a piston and enabling the air to be suddenly rarefied and thereby cooled. The practical details of the apparatus for moving the piston are of no consequence here. A small sliding window F is mechanically connected with the piston. This enables a few α-rays to be shot into the chamber immediately after the rarefaction has taken place. The picture shown in fig. 9 appears on the screen. The α-ray tracks, which are as straight as an arrow, are marked by a continuous chain of mist particles. The molecules of air along the track which have been struck and thereby ionized serve as condensation nuclei. The picture lasts for a few tenths of a second, and is then blown away by air currents. The charged drops of water are extracted by an electric field (not indicated in the figure) and the process can begin afresh. The practical details of the method are of no consequence. Their object is merely to avoid disturbing eddies of air during the rarefaction.

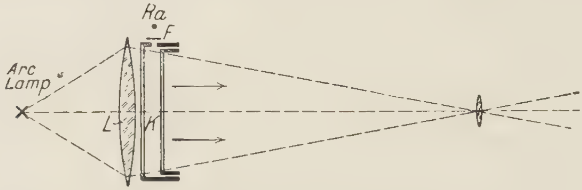

Fig. 8. Chamber for producing condensation tracks in front of a lantern, very diagrammatically.

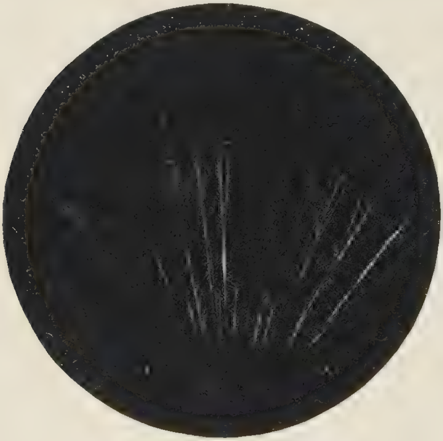

Fig. 9. α-rays from Thorium C and Thorium C′ in air.

The tracks of fast electrons, i. e. of β-rays, are obtained in exactly the same way. They give us a picture which is essentially different. Instead of the perfectly straight α-ray tracks we have paths which are irregularly bent and frequently tangled. The mass of a β-particle is $4 \cdot 1800 = 7200$ times smaller than that of an α-particle, so that we can easily understand why β-rays do not pass through the molecules of air in as straight a line as α-rays do.

The condensation track method is by no means confined to corpuscular rays of radioactive origin. Cathode rays of small velocity, e. g.

those emerging from the aluminium window of a discharge tube (fig. 34, p. 222), may likewise be investigated by this means.

Condensation tracks have also been used with very great success in the investigation of the formation of ions by X-rays. Imagine that the radium preparation in fig. 8 is replaced by an X-ray tube. A small bundle of rays must be selected by means of suitable screens with holes. The condensation tracks are as shown in fig. 10, which illustrates the typical crooked trajectories of the electrons. Along their path the X-rays split off electrons from single molecules at wide intervals. These electrons fly off to the side in the form of cathode rays, far beyond the boundaries of the bundle of X-rays. It is these cathode rays that produce the strong ionization of the gas. The electrons which are split off in this way by X-rays (γ-rays) are said to be split off "photo-electrically"*. The details belong to the realm of electro-optics.

Fig. 10. Paths of electrons split off by X-rays.

Another variant of this method remains to be mentioned in conclusion. A small grain of a preparation emitting α-rays is placed on a photographic plate. While passing through the layer of gelatine the α-rays liberate from the molecules of silver bromide which they strike some neutral silver atoms or "silver nuclei". In the subsequent photographic development these act as "condensation nuclei". During the process of reduction in the developer numerous silver atoms are deposited on them, until visible grains of silver are produced. The photograph finally obtained is reproduced in fig. 11. It enables us to follow out the straight tracks of the α-rays very clearly.

(3) *The electrical counting method*. We shall describe Geiger's counting apparatus, which is the most convenient of a whole series of practical

---

* [The term *photo-electric effect* is applied not only to this phenomenon but also to the emission of electrons caused by visible or ultra-violet light.]

variants. The principle underlying it is a simple one. A single corpuscular ray produces enough ions along its track to set up a brief luminous discharge. The details are shown in fig. 12a. *A* is a metal point, e. g. a gramophone needle, inserted through an insulator into the metal cylinder *K*. In the other end of the cylinder there is a hole of about 2 mm. diameter opposite the point to let the rays in. The point and the cylinder are connected with a source of current with a pressure of about 2000 volts, but this is too low for independent conduction to take place through the air. The circuit also includes a galvanometer of high internal resistance. It is very convenient to use a match with a bifilar voltmeter (fig. 24, p. 14) in parallel.

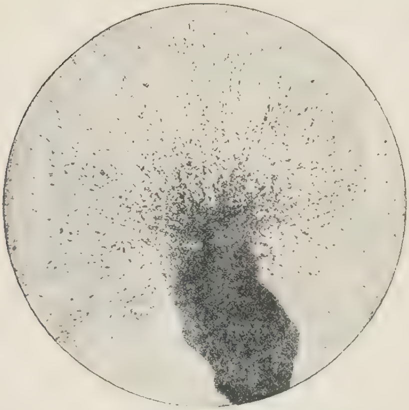

Fig. 11. α-ray tracks in a photographic plate, magnified about 500 times.

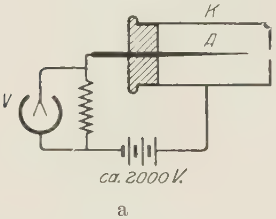

Fig. 12a. Geiger's counter for α- and β-rays.

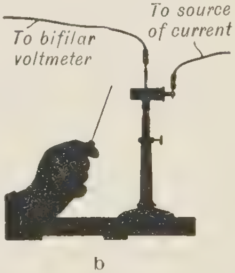

Fig. 12 b. A wire with a coating of Radium A is brought near an electrical counter.

If a ray of corpuscles travels through the hole into the cylinder, a small, barely visible spark is seen at the point, and the measuring instrument shows a deflection of short duration.

According to other experiments, α-rays produce about 20000 pairs of ions per centimetre of their path, β-rays about $^1/_{200}$ times as many. Hence α-rays give large deflections, β-rays only small ones. Thus the rays can easily be distinguished.

A telephone may also be used to detect the brief impulsive currents

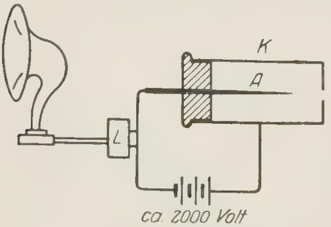

Fig. 13. Counting apparatus with amplifier *L* and loud-speaker (a relay and bell etc. may also be used).

due to the formation of the small sparks (see diagram of the connections in fig. 13). A loud-speaker and amplifier *L*, such as are familiar to "listeners-in", make the crackling noise easily perceptible in a large auditorium. The method is not confined to the rapid corpuscular rays of radioactive origin. The demonstration of single electrons and ions is an experimental problem which has been completely solved, and no observer will be able to ignore the impression made by these experiments.

3. The Direct Evaluation of the Universal Constant *N*, the Number of Molecules in a Gram-molecule (Avogadro's Number*).

An α-ray consists of an atom of helium with two positive charges (He^{++}). This assumption is based on the determination of the atomic weight by electrical methods, i. e. in the final analysis, on the deviation of the α-rays in electric and magnetic fields. The identification of rays with doubly-charged helium atoms is carried out by the method (due to Rutherford) illustrated in fig. 14; the latter is very diagrammatic from the quantitative point of view, but not from the qualitative.

A is a glass tube which is blown so as to have very thin walls and yet be completely gas-tight. The inner wall of the tube is covered with a thin layer of radium bromide, containing one gram of the metal radium. The glass tube is surrounded on the outside by a second tube *B*, which has thick walls, and the space between the tubes is exhausted till a high vacuum is produced. After the lapse of a year 156 cubic millimetres of helium (measured at 0° C. and atmospheric pressure) i. e. $2{\cdot}76.10^{-5}$ gm. of helium, are found in the space between the tubes. The success of this experiment depends on one condition only: the thickness of the wall of the inner glass tube must be less than the range of α-rays in glass, i. e. the α-rays from radium must be able to pass through it. This experiment disposes of any doubts about the chemical nature of the α-rays.

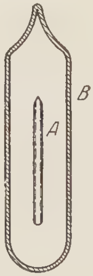

Fig. 14. Apparatus for identifying α-rays with helium.

Another experiment is required to ascertain n_α, the number of α-rays emitted by 1 gm. of radium in a year. This is done by means of one of the counting methods described in the last section. A very convenient way is to use the electrical counter with photographic registration.

In actual practice, of course, a minute but known fraction of a gram of radium is used instead of a gram. Also, the count is not kept up for a year on end but only for a few hours at most.

We find that $n_\alpha = 4{\cdot}29.10^{18}$ per gram of radium per year. Thus the $2{\cdot}76.10^{-5}$ gm. of helium are composed of $4{\cdot}29.10^{18}$ atoms. The number of molecules is also $4{\cdot}29.10^{18}$, for helium is a monatomic gas (from c_v/c_p, the ratio of the specific heats); a molecule of helium consists of only a single helium atom. The mass of the single helium molecule is $6{\cdot}44.10^{-24}$ gm. 4 gm. helium form the gram-molecule or mol of helium (cf. pp. 233—234). Thus a mol of helium contains

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* [Sometimes called *Loschmidt's number.*]

$$\frac{4}{6{\cdot}44 \cdot 10^{-24}} = 6{\cdot}22.10^{23} \text{ molecules.}$$

$\boldsymbol{N}$, the universal number of molecules in a gram-molecule, which is identical for all substances, is here found by direct evaluation to be $6{\cdot}22.10^{23}$ for a well-known gas. How does this value agree with other observations?

By equation (8), p. 235, $\boldsymbol{N}$ is connected with e, the number of coulombs in the elementary quantity of electricity, by the equation

$$\boldsymbol{N}e = 96500 \text{ coulombs}.$$

If we substitute the value $6{\cdot}22.10^{23}$ for $\boldsymbol{N}$, it follows that $e = 1{\cdot}55.10^{-19}$ coulombs. The elementary quantity of electricity e has been determined by direct measurement on charged droplets floating in an electric field as $1{\cdot}59.10^{-19}$ (pp. 83—85). The agreement exceeds all expectations. $\boldsymbol{N}$ and e, two fundamental constants of atomic theory, are nowadays known accurate to one or two per cent.

4. The Decay of Radioactive Atoms. Atoms of Electricity as Essential Constituents of the Elements.

In all *chemical* experiments the atoms of an element behave as the ultimate possible divisions of matter, as unchangeable and indivisible entities.

The facts relating to *electricity* carry us further. We find it necessary to assume the existence of atoms of electricity within the chemical atoms. In support of this assumption we need only refer to the shortening of lines of electric force by the atoms of a dielectric, the many and varied cases of the formation of ions, and the molecular currents in paramagnetic and diamagnetic substances.

The phenomena of *radioactivity* now show us that the atoms of electricity are absolutely essential constituents of the atom. *When they emit radiation, the atoms of a radioactive element lose electrons in the form of β-rays or atoms of positive electricity with the α-rays during the process of radiation. Thereby they cease to exist any longer as atoms of the element in question. They become atoms of another element with a lower atomic weight.* We shall illustrate this fundamental fact of "radioactive disintegration" or "radioactive transmutation of the elements"* by two examples.

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* [This now well-established theory was put forward by Rutherford and Soddy in 1903.]

(1) *The decay of thorium emanation.* Thorium emanation is an inert gas of atomic weight 220, which belongs to compartment 86 in the periodic table of the elements. It invariably accompanies preparations containing thorium.

Thorium itself is a metal of atomic weight 232, occupying compartment 90 of the periodic table.

Thorium emanation is obtained by letting a current of air pass over a thorium preparation spread over a considerable area, as in the piece of apparatus marked *a* in fig. 15. About 100 gm. of thorium oxide (a white powder) are spread over the bottom of a glass tube. Air is slowly introduced from the left. The air which streams off to the right contains small quantities of thorium emanation. Of course these quantities are minute, if we use the word quantity in the usual sense. Their presence could not be detected by any process of micro-analysis, even if we were dealing with a very reactive element instead of a practically reactionless inert gas. Even the method of spectrum analysis, which is so highly sensitive, fails here. The only means of detecting these minute quantities of gas is their radioactivity. Thorium emanation emits α-rays. The number of α-rays emitted per second is a relative measure of the stock of atoms of thorium emanation existing at that instant, just as the number of deaths per day is a relative measure of the population of a city. A relative measure for the α-rays is very easily obtained by using their ionizing effect. The right-hand side of fig. 15 shows a suitable form of apparatus. The air containing thorium emanation is led into a cylindrical condenser and the saturation current (fig. 14, p. 206) read off on a moving-coil galvanometer *A*. The intensity of current observed follows a very characteristic course; it diminishes according to the exponential curve represented in fig. 16, and falls by half its value in

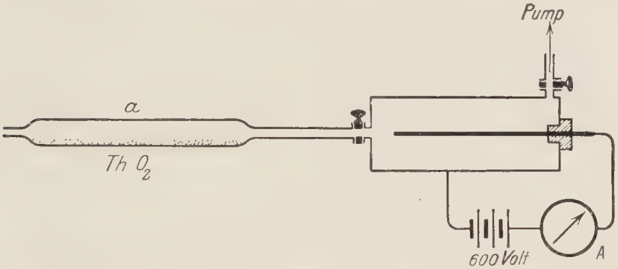

Fig. 15. Measurement of the half-period of thorium emanation. Galvanometer as in fig. 16, p. 30, fig. 27, p. 36, and fig. 68, p. 66.

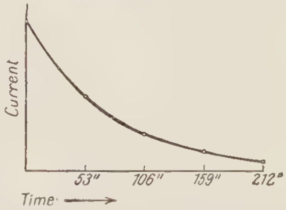

Fig. 16. Decay of thorium emanation as time goes on.

every 53 seconds. That is, the stock of atoms emitting α-rays is diminished by half every 53 seconds. This period of 53 seconds is called the *half-period* of the thorium emanation; it is a constant which is characteristic of this element and of this element only, a fact which enables us to identify atoms of this element at any time.

The half-period tells us absolutely nothing about the length of the life of a single individual atom of thorium emanation. For an individual atom it may have any greater or smaller value whatever. All that is meant is that among a sufficiently large number of atoms n, $n/2$ individuals on the average "disintegrate" every 53 seconds and are lost to further observation by electrical methods. The disintegration of the single atom takes place with the hurling forth of *one* α-ray, i. e. a single positively charged helium atom (a He^{++} ion) of atomic weight 4. The remainder of the atom has only the atomic weight $220 - 4 = 216$. It is a new chemical individual, an atom of the element thorium A, which cannot be observed under the conditions of our experiment.

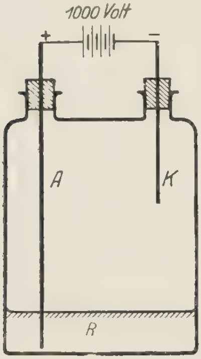

Fig. 17. Method of obtaining a deposit of radium A on the wire K. R is an aqueous solution of a radium salt.

Strictly speaking, the phenomenon takes place in a somewhat more complicated way. Thorium A is a radioactive element with the very small half-period of 0·14 seconds. It disintegrates into thorium B, emitting another α-ray in the process. That is, unless refined methods of observation are employed, the effect is as if *one* disintegrating atom of thorium emanation were to give off *two* α-rays. Thorium B, the element which is left after the thorium A has disintegrated, emits β- and γ-rays only; it decays slowly, having the half-period 10·6 hours. Our measuring arrangements are not sensitive enough to detect its radiation, so that it can no longer be detected electrically.

(2) *Some facts about the radium disintegration series.* Radium emanation (or radon) is an inert gas of atomic weight 222 (compartment 86 in the periodic table). It invariably accompanies preparations containing radium and is very con-

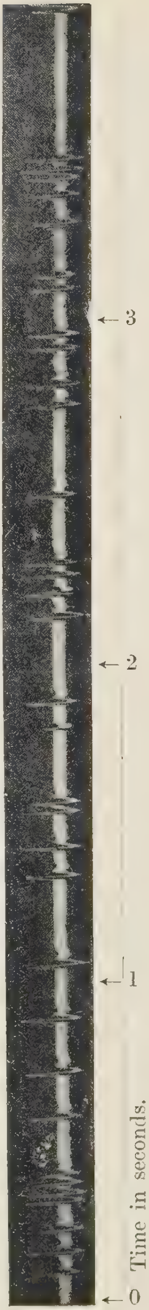

Fig. 18. Variation of the emission of α-rays with time. Photographic record of the deflections of the unifilar voltmeter V in fig. 12, p. 292.

veniently obtained by collecting it from above aqueous solutions of radium salts.

Radium emanation disintegrates with emission of α-rays with a half-period of 3·85 days, giving rise to a new element of atomic weight 218, known as radium A. Radium A is a solid at room temperatures. It is obtained as an extremely thin deposit on an electrically charged wire K in air containing radium emanation (fig. 17).

It is surprising to find that the wire must be negatively charged; that is, radium A is positively charged although it is produced by the emission of a positively charged ion (He^{++}). As a matter of fact, however, a whole set of very slow β-rays, which are often called δ-rays, escape at the same time as the α-rays; hence the positive charge on the atom of radium A.

Let the formation of the deposit of radium A on a wire be interrupted after $^1/_4$ minute. We carefully bring the wire near an electrical counter (figs. 12, 13, p. 292) and find that the α-rays have a range of 4·8 cm.

After a few minutes β-rays also appear to a noticeable extent, for radium A has a half-period of 3 minutes, and even within these few minutes a new element, radium B, has been formed. It is a metal of atomic weight $218 - 4 = 214$, which is chemically indistinguishable from lead; radium B, however, is radioactive and disintegrates, emitting β-rays, with a half-period of 27 minutes.

We wait still longer. After about three quarters of an hour α-rays of range 6·9 cm. appear. These belong to the element radium C. This is a metal which chemically is practically identical with bismuth (compartment 83) and which disintegrates with a half-period of 19·6 minutes.

We cannot follow out the series any further in a lecture experiment. The series ends with lead of atomic weight 206, such as is found even in commercial lead.

Ordinary lead is a mixture of elements, with an atomic weight of 207·2 (cf. p. 224).

In one respect observation of radioactive decay by means of the electrical counter is even more instructive than the method shown in fig. 15, p. 295, for measuring the total ionization current. The measurement of the current gives the half-period in the form of a statistical average which is true for large numbers of individual atoms. The electrical counter, on the other hand, enables the disintegration of a single atom to be observed. We see the single elementary processes following each other in a way which is often surprisingly irregular. Fig. 18 is an example chosen at random. Here rapid successions alternate with long pauses; the idea of the half-period has no meaning unless very large numbers of individual atoms are involved. The half-period tells us absolutely nothing about the length of the life of a single individual; in this it resembles statistical tables for the length of human lives.

So much for our two examples of radioactive disintegration of atoms. The radioactive elements at present known have been discovered by using the half-period and the nature of the emitted radiation as distinctive properties. It has been found possible to arrange them in three large genealogical trees and to assign to the various members their places in the periodic table of the elements.

Atomic disintegration involves the fact that a radioactive element can really never be observed by itself. Of course its products of disintegration can be removed, but this is only a help for the time being. Disintegration goes on irresistibly, and the products of disintegration accumulate again in varying amounts according to their half-periods.

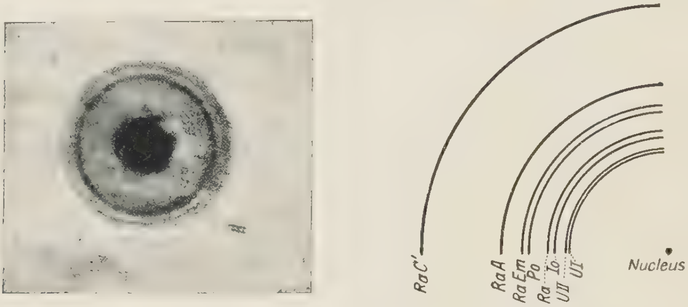

Fig. 19. Pleochroic haloes in fluor spar, due to α-rays from members of the radium family.

What we have just said is well illustrated by inclusions of small grains of uranium minerals in natural crystals, e. g. in fluor spar. Uranium is a radioactive element of atomic weight 238·2 (compartment 92 of the periodic table), which forms the starting-point of the radium series. The α-rays cause fluor spar to become coloured at the place where they stop, i. e. at the end of their range. Each of the products of disintegration which emits α-rays has its own characteristic range. Hence in the course of millions of years the nucleus containing uranium is surrounded by a system of concentric hollow spheres, which are coloured violet. In a thin section they appear as rings. Fig. 19 shows a micro-photograph magnified 500 times, together with an explanatory drawing. Each separate ring has the name of the corresponding element beside it. All the members of the radium family or radium series which emit α-rays (except Radium C) are represented.

5. The Artificial Disintegration of Atoms. The Question of the Ultimate Constituents of Matter. The Electrical Nature of Mass.

The radioactive disintegration of atoms has done away with the chemical dogma that chemical elements cannot be transmuted. The loss of essential constituents transmutes an atom of one element into an atom of another element. These essential constituents are the atoms of electricity, namely, atoms of negative electricity, or electrons (atomic weight $^1/_{1800}$), and atoms of positive electricity, with which, on the other hand, the mass of a helium atom (atomic weight 4) is associated. That this is the actual state of affairs in the case of the radioactive elements has been proved beyond all doubt. Is the same true of the other elements, which do not exhibit spontaneous or "radioactive" disintegration? Today the question can be answered with certainty in the affirmative. Atoms of various light elements can be disintegrated by means of powerful disturbances from outside, namely, by bombarding them with α-rays from radioactive substances. Occasionally, but rarely, it happens that the atom struck fails to resist the impact of the atom striking it. Besides electrons and the well-known helium α-rays, a new kind of atomic constituent occurs among the fragments of the disintegrated atom, namely monovalent positive atoms of hydrogen, or protons. These appear as α-rays of abnormally long range. The atomic weight of the protons has been determined in the usual way (cf. p. 223).

The details of these important investigations would lead us too far out of our way here. Their results have led to the revival of the old question about the ultimate constituents of matter. There can scarcely be any doubts about the answer any longer. *The ultimate constituents of matter are the atoms of negative and positive electricity, the electron and the proton.* The atomic weight of the electron is $^1/_{1800}$, that of the proton is 1. The proton is a monovalent positive hydrogen ion. An electron and a proton together form a neutral hydrogen atom. Four protons are required to build up an atom of helium, and so on. *In modern atomic theory only two real substances are recognized, namely, the atoms of positive and negative electricity. In the ultimate analysis all matter is composed of these two constituents.*

This point of view demands that we should abandon our traditional ways of thinking. Simple-minded people are most intimately acquainted with the mechanical phenomena of life, which are easily grasped and fit in with preconceived ideas. In their attempts to interpret physical phenomena, these people strive after an understanding of them in terms of mechanical conceptions and mechanical similes. The overwhelming pressure of facts has forced the new physics to inaugurate the opposite method, in which mechanical phenomena are regarded as something secondary which must ultimately be interpreted electrically, i. e. in terms of the electric field and atoms of electricity.

In conclusion we shall briefly illustrate what is meant by these tendencies of thought in connection with the mechanical conception of mass.

An electron flying along as a cathode ray contains a definite amount of energy. When it impinges on an obstacle, this energy is transformed into heat and may be measured in calories (fig. 16, p. 208). It is found to be proportional to the square of the electronic velocity u. Hence this energy is called the *kinetic* energy $\frac{1}{2} m u^2$, and m is called the mass of the electron. This train of thought arises from purely mechanical considerations throughout. We observe an energy of an electron which is proportional to u^2, and regard it as being carried by a mass in the sense of classical mechanics.

The experimental facts may however be interpreted *electrically*. A moving electron may be regarded as a *current*. In the plate condenser of fig. 20 an electron is moving in the direction of the arrow with the velocity u. Then the ammeter records a convection current of eu/d amperes; the strength of the current is proportional to u, the velocity of the electron.

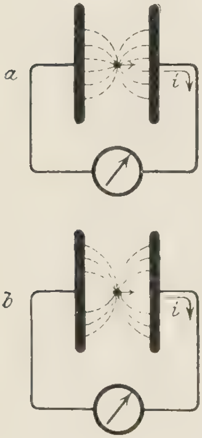

Fig. 20a, b. The change in the electric field of an electron with its velocity.

Now the experimental result above gave the energy of the flying electron as being proportional to u^2. Hence the energy of a flying electron is also proportional to i^2. An energy of a current proportional to i^2 has long been known to us. The magnetic energy of a current is

$$W_m = \frac{1}{2} L i^2, \qquad \text{(equation 5, p. 140)}$$

where L, the factor of proportionality, is not a mechanical quantity, but a purely electrical one. L, the coefficient of self-induction, is of the dimensions volt-seconds/ampere. L is a measure of the inertia with which the magnetic field of the current opposes every change (pp. 168—175). L always depends on the geometrical configuration of the magnetic field. Previously we managed to calculate L for the simple case of the uniform magnetic field of a long coil, from equation 4, p. 136. The magnetic field of a moving electron is very complicated. It is caused by the displacement current, by the alteration in space and time of the electric field as the electron moves. A snap-shot of the electric field is diagrammatically shown in fig. 20a.

In order to calculate L, we have to make assumptions about the shape and size of the electron. The actual calculation is easily carried out. We may accordingly interpret the mass of an electron in a purely electrical way as the inertia of the energy of a magnetic field.

The following very important experimental result favours the electrical interpretation of the mass of the electron. *The mass of an electron is by no means constant. The numerical value* 9.10^{-28} *is merely a lower limit, which is true only for electrons whose velocity u is small compared with c, the velocity of light. When higher velocities are reached, increased values of the mass are obtained* (Kaufmann 1901). This was indicated by the measurements in Table III, p. 212, but we ignored the matter at the time.

The high velocities of the β-rays from radioactive substances have enabled us to supplement the earlier measurements. We thus obtain the numbers reproduced in Table IX.

Table IX.

Velocity of the electron in fractions of the velocity of light, $c = 3 \cdot 10^{10}$ cm./sec.	Mass of the electron in multiples of $9 \cdot 10^{-28}$ gm.
0·1	1·001
0·2	1·02
0·3	1·05
0·4	1·09
0·5	1·16
0·6	1·25
0·7	1·40
0·8	1·67
0·9	2·29
0·95	3·20
0·990	7·09
0·998	15·82

The mass m increases rapidly as the velocity of light is approached. The relationship experimentally observed may be represented in the form of an equation as follows:

$$m = \frac{m_0}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}, \tag{1}$$

where m_0 is the limiting value of the mass when the electron is at rest.

The mechanical interpretation of mass is at a loss when faced with this experimental fact. A mass whose numerical value increases with the velocity is a downright paradox from the mechanical point of view.

If, on the other hand, we interpret mass electrically, the effect of velocity on mass is not surprising. For the coefficent of self-induction depends on the configuration of the magnetic field, and this again is conditioned by the displacement current of the electric field. For small electronic velocities the electric field at any instant is of the form sketched in fig. 20a. When the velocity of light is approached the configuration of the electric field is distorted (fig. 20b). For the changes in the electric field are themselves propagated only with the velocity of light (p. 320). To this distorted field of the flying electron there corresponds a different distribution of the displacement current and an increase in the energy. The above train of thought enables us to understand the increase of the mass m, qualitatively at least, from the electrical point of view.

On pp. 134—136 the importance of the coefficient of self-induction and of the inertia of the magnetic field were urgently emphasized. The full bearing of the experimental results obtained on that occasion is now revealed. Mechanical mass becomes an electrical problem; it no longer appears as an attribute of material bodies, but as an attribute of energy.

CHAPTER XI

Electric Waves

1. Preliminary Remarks.

Broadly speaking, we have discussed the electric field under the following heads:

(1) *The electric field at rest.* The ends of the lines of force are considered as atoms of electricity (see diagram, fig. 1a).

(2) *The slowly changing electric field.* The two plates of the condenser are joined by a conductor. In fig. 1b it is a long coiled-up wire. The electric field decays; but the self-induction of the conductor still makes the phenomenon take place "slowly", that is, we may still regard the time which the alteration of the field beginning at α takes to appear at β as very large. The strength of the electric field still has practically the same value at both α and β; this is shown in the figure by the equidistance of the lines of force between α und β.

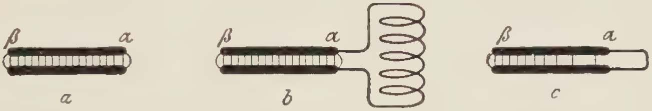

Fig. 1.a Electric field at rest, b and c decaying electric field in a condenser.

In this chapter we finally have as the last case:

(3) *The rapidly changing electric field.* In fig. 1c the conductor is short and its self-induction small. The field decays "rapidly", i.e. the smallness of the time taken by changes in the field to traverse $\alpha\beta$ must now be taken account of. The decay of the field caused by the conductor is much further advanced at α than at β; this is shown by the differing distances between the lines of force. It follows that the electric field has a finite (though of course very high) velocity of propagation.

This finite velocity of propagation makes the occurrence of electric waves possible.

The term "electric waves" is on everyone's lips in these days of broadcasting, but an understanding of the fundamental phenomena is not so common.

In order to simplify the discussion we begin by consider two phenomena which will be of subsequent use to us. Starting from previous developments we discuss (1) convenient methods for producing alternating currents of very high frequency, on pp. 304—308: (2) the production of forced electrical oscillations, on pp. 309—311. With these methods at our disposal, we go on to something new, namely the electric dipole, on p. 311. Finally, on p. 315 we come to our real subject, electric waves in wires and free radiation.

2. The Production of High Frequency Alternating Currents by Undamped Oscillations.

We have previously come across two ways of producing alternating currents: (1) by using a *generator*, i. e. a coil made to revolve in a magnetic field by mechanical means (fig. 1, p. 148): (2) by using the *oscillations in a circuit consisting of a condenser and a coil* (fig. 17b, p. 182).

As a rule, ordinary alternating current generators in practical use have a frequency of 50 cycles per second. A frequency of 500 cycles per second is much less common, while 10^4 or 10^5 cycles per second would require special constructional features which are difficult to carry out in practice. The alternating currents supplied by generators are constant as time goes on and quite undamped, all the amplitudes of successive currents being the same (fig. 2a, p. 149).

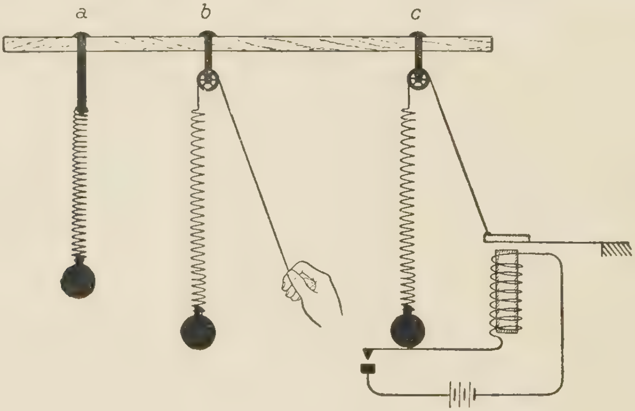

Fig. 2a—c. Automatic regulation of a mechanical pendulum.

The production of alternating currents by means of oscillations has the advantage of being very convenient. Only a modest outlay on apparatus is required. In addition, extremely high frequencies can be obtained by using condensers of small capacity and coils of small self-induction. There is no difficulty in attaining a frequency of 10^8 cycles per second. But a circuit like this, consisting of a condenser and a coil, has the great disadvantage that its oscillations are always damped, i. e. the amplitudes of the current decrease more or less rapidly (fig. 18, p. 183). This drawback, however, may be got rid of comparatively simply; several good practical methods of doing so have been devised. Just as in mechanics, these methods depend on the principle of *automatic regulation*.

We begin by considering a mechanical example. Fig. 2a shows a simple pendulum, consisting of a spherical bob suspended by a spring. Once an impulse is given to it, it executes vertical oscillations which slowly die away, being damped by the resistance of the air. The tension in the spring at the turning points becomes smaller at every oscillation.

If the pendulum is to execute undamped oscillations, the tension of the spring must be raised to its initial value at regular intervals of time. This can be done by hand, by pulling a string (fig. 2b). We stretch the spring every time the sphere reaches its lowest position.

The intervention of a human being may easily be dispensed with. The pendulum can itself be made to cause a force to stretch its spring every time the right position is reached. Fig. 2c shows a simple way in which this "automatic regulation" can be carried out. The hand is replaced by the armature of an electromagnet, the circuit of which is completed by the pendulum every time it reaches its lowest position.

Now *an oscillatory electric circuit* can be made to oscillate without damping in a corresponding way. We illustrate this in the first instance for quite slow oscillations, such as we can conveniently follow with a moving-coil ammeter. We take the circuit used in fig. 19, p. 183; its frequency is 0·5 cycles per second, i. e. the pointer of the ammeter takes 2 seconds to complete a vibration.

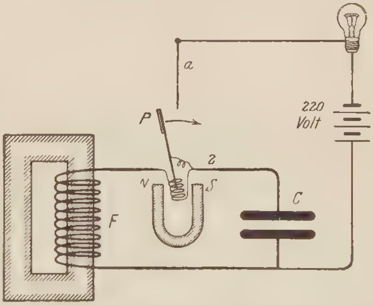

Fig. 3. Automatic regulation of an oscillatory electric circuit of very low frequency by means of a mechanical switch.

We make use of this slowly vibrating pointer in the construction of a simple form of self-regulating apparatus. The pointer actuates a switch which connects the condenser with the town mains at regular intervals at the proper instant. The pressure which has been diminished by damping is thereby raised again to its initial value of 220 volts.

Fig. 3 shows an arrangement for carrying out this simple idea. The pointer of the moving-coil ammeter itself forms one half of the switch. The other consists of a vibrating metal wire *a*. The pointer is insulated from the ammeter coil, but connected by a movable metal band with the lead 2. If the pointer approaches the wire *a* from the left and touches it, connection with the negative pole of the town mains is established and the pressure of the condenser is again raised to 220 volts. If the pointer approaches the wire from the right, this does not occur, for the left-hand side of the pointer has an insulating layer of paper *P* pasted on it.

This primitive arrangement works perfectly well; we find that the oscillations are undamped, that the ammeter pointer vibrates uniformly, and that the alternating current flows without alteration as time goes on. An incandescent lamp inserted in the lead to the mains lights up every time the switch *Pa* makes contact. We see that the charge of the condenser is restored to its full value every time. The period of oscillation of the circuit may be reduced in the usual way by diminishing the capacity of the condenser.

So much for the self-regulating principle as applied to an oscillatory electric circuit.

The use of a switch with mechanical inertia is of course restricted to low frequencies. With higher frequencies the lever of the switch is too inert to follow the rapid alternations of the current any longer. Even with mechanical switches which are well constructed in every way we cannot go beyond frequencies of more than about 1000 per second.

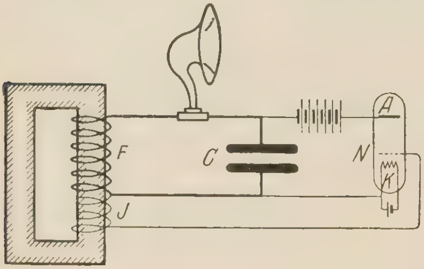

Fig. 4. Automatic regulation of an oscillatory electric circuit of audible frequency by means of a valve.

The perfecting of the switch, however, is merely a technical matter. Modern electrical engineering has put switches free from mechanical inertia at our disposal, for it has produced the well-known three-electrode valve (fig. 19, p. 213). One of these valves is diagrammatically shown on the right-hand side of fig. 4. The conduction current passes in the form of a swarm of electrons from the glowing cathode *K* to the anode *A*. On the way it passes through the third electrode *N*, which takes the form of a network or grid. The current is interrupted, or figuratively speaking, "the switch is open", whenever the grid has a negative charge; "the switch is closed", whenever the grid is discharged or has a positive charge.

We shall first illustrate the use of the valve in the case of an oscillatory electric circuit with a frequency of about 1000 cycles per second. *F* is a coil with an iron core (with a coefficient of self-induction *L* of about one henry). *C* is an adjustable condenser (with a maximum capacity *C* of 2.10^{-9} farad). A secondary coil *J* is wound on the iron core of the primary *F*. Its ends are joined by conductors to the cathode and grid of the valve. This coil is the actual regulator; it corresponds to the ammeter coil in fig. 3. It does not, however, actuate the switching process *mechanically* by its motion, but opens and shuts the valve in a purely electrical way; it serves to charge the grid of the valve alternately positively and negatively, by means of the pressures induced in it by the alternating magnetic field of the coil F.

A loud-speaker inserted in the circuit FC is used to detect the oscillations. To begin with, suppose that the self-regulating apparatus is not brought into use. The condenser is charged from the town mains and then allowed to discharge itself through the primary coil F. A clicking sound which dies away quite rapidly is heard. On the other hand, with automatic regulation the sound lasts indefinitely. The vibrations are undamped and a constant alternating current flows in the circuit FC. Adjustment of the variable condenser causes the frequency to be altered. By turning round the knob of the condenser a large range of audible sounds may be traversed. These electrical circuits of audible frequency form a useful and convenient adjunct to many acoustical investigations.

As a second example we take an oscillatory circuit with a frequency far above the range of human audibility. This circuit is shown half-diagrammatically in fig. 5. The primary coil F has a much smaller coefficient of self-induction than before, say 3.10^{-3} henry. This coil, like the secondary coil J, is wound on a cardboard cylinder about 10 cm. in diameter. The primary has several hundred turns but no iron core. The adjustable condenser and connections are the same as in fig. 4. By equation 3, p. 184, the frequency amounts to about 10^5 cycles per second.

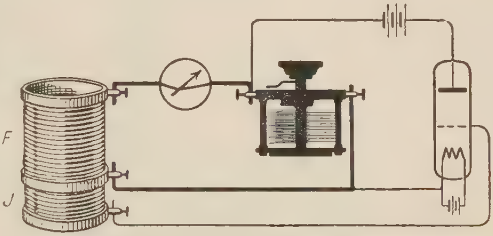

Fig. 5. Production of undamped electric oscillations of frequency 10^5 cycles per second.

A hot-wire ammeter is used to record the alternating current. We see that an alternating current of several amperes flows in the circuit formed by F and C. This current must therefore bridge the air-gap of a few millimetres between the plates of the adjustable condenser as a displacement current. Previously we could only obtain displacement currents of this order of magnitude at the frequency of practical alternating currents if the length of the lines of force did not exceed a few hundredths of a millimetre and condensers with a surface of about 100 square metres were available.

The apparatus is absolutely noiseless in its working, without either the purring of a mechanical generator or the disturbing noise of sparks in the switches of damped oscillatory circuits. This brilliant experimental arrangement is due to the German engineer Alexander Meissner.

By reducing the capacity and coefficient of self-induction still more, alternating current of frequencies up to 10^8 cycles per second can be obtained in the same way without any particular trouble. But as the frequency is raised a defect becomes more and more prominent; the essential parts of the oscillatory circuit, the condenser and the coil, become insignificant compared with the quite unimportant auxiliary apparatus for automatic regulation. It becomes quite impossible to trace out the circuits, and with high frequencies, moreover, the capacity of the condenser formed by the electrodes of the valve can no longer be neglected. In fig. 6 we show without comment an automatically-regulated oscillatory circuit for frequencies of about 10^8 cycles per second. Here, as in the case of the old induction machines described in textbooks, even the specialist can scarcely extract the simple essentials from purely incidental matters.

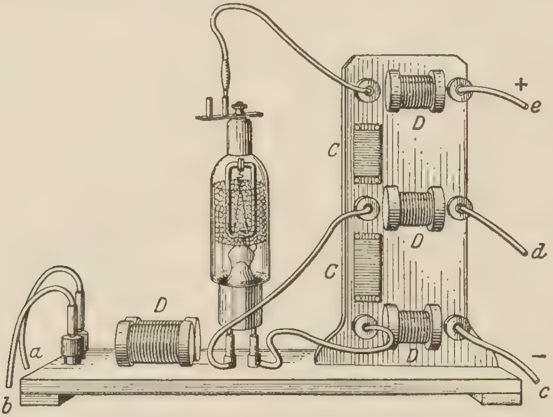

Fig. 6. Production of undamped electrical oscillations with a frequency of about 10^8 cycles per second. Between *c* and *e* there are about 800 volts, between *d* and *c* about 20 volts; *a* and *b* are leads for the heating current, *D* choking coils, *C* block condensers.

It is no good wasting time over picture-puzzles of this kind. It is much better to leave all subsidiary details entirely on one side and to make the apparatus so simple as to be immediately understood. This problem is solved in the following section.

A word in conclusion lest we should be misunderstood. Our treatment in no way pretends to do justice to the practical importance of the valve or to the multiplicity of ways in which it is used to produce oscillations. Our intention is merely to make the main features of a piece of apparatus, which we require in what follows, intelligible by discussing a single practical form of it.

3. Forced Electrical Oscillations.

As in the case of automatic regulation, we start with a mechanical example. On the right-hand side of fig. 7 we again have a simple pendulum *f*, consisting of a spherical mass suspended from a spiral spring. With the aid of self-regulating apparatus it is made to execute undamped oscillations. The automatic regulation is essentially the same as in fig. 2c above, but the practical details are somewhat simpler. The electromagnet and string are done away with: the spring compresses itself when the circuit is completed, as the turns mutually attract each other owing to their magnetic field. We merely have the vibrating spiral which we met with in fig. 13, p. 139.

At the top the spiral is suspended from a horizontal bar. At the other end of the bar there hangs a row of similar pendulums from rod supports of differing lengths. Owing to the varying lengths of their springs some of the pendulums have greater frequencies of their own, some smaller, than the self-regulating pendulum at the right-hand end. We shall henceforth call the latter the "exciter" for short.

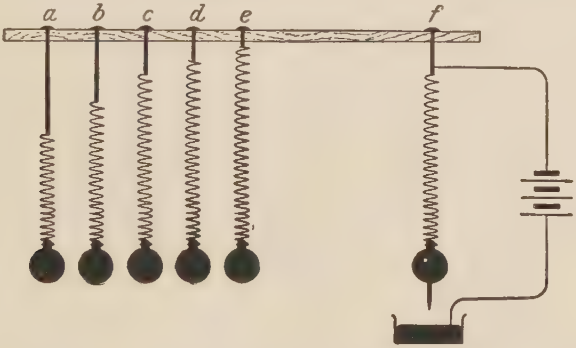

Fig. 7a—f. Forced mechanical vibrations; f exciter, a to e resonators.

If we make the exciter oscillate, the horizontal bar carrying it executes small motions in the vertical plane (here that of the paper) owing to mechanical reaction. The springs of the pendulums on the left are periodically stretched by these motions. Their bobs begin to vibrate up and down, not with their own proper frequencies but with the frequency of the exciter. They execute "forced" vibrations.

The amplitude of the forced vibrations is in general small. It only reaches a large value in the case of those pendulums whose own frequency approaches the frequency of the exciter. The pendulum *c*, whose frequency coincides with that of the exciter *f*, vibrates most strongly. The diagram of fig. 8 shows this graphically. The pendulums which are stimulated to make forced vibrations are usually referred to as "resonators". The distribution of the amplitudes of the forced vibrations recorded in fig. 8 is called the resonance curve and is fully dealt with in mechanics. In particular, the way in which it depends on the damping of the resonators is shown. The more strongly damped the resonators are, i. e. the more rapidly their oscillations die away, when an impulse is given to them ("excitation by an impulse"), the flatter does the resonance curve become. It is only when the damping is small that those resonators whose frequency coincides with the frequency of the exciter are sharply marked out. So much for the mechanical example.

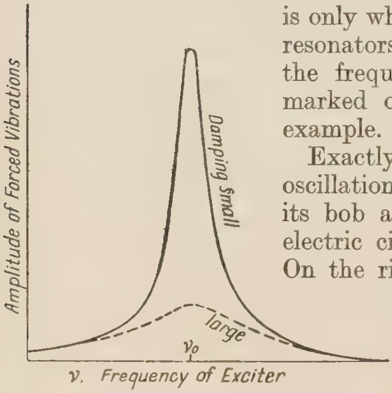

Fig. 8. Resonance curve for small and large damping.

Exactly the same is true of forced electrical oscillations. Instead of the pendulum with its bob and spring we have the oscillatory electric circuit with its coil and condenser. On the right-hand side of fig. 9 we see an undamped oscillatory circuit acting as exciter, with its self-regulating apparatus. To the left is the resonator circuit.

In the case of the mechanical pendulums of fig. 7 it is the horizontal bar which, by its small motions in the plane of the paper, is responsible for transferring energy from the exciter to the resonator. Here it is the lines of force of the magnetic field of the exciter circuit that induce pressures and currents in the coil of the resonator circuit.

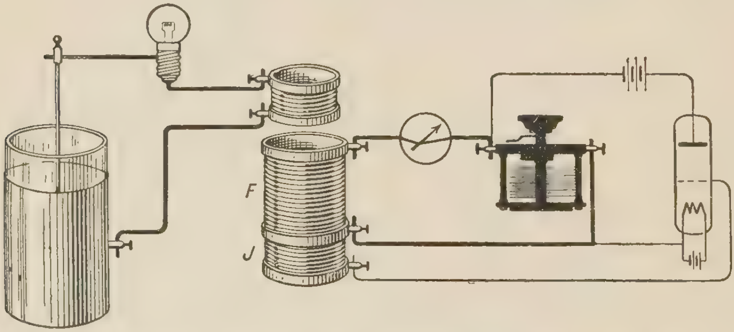

Fig. 9. Forced electrical oscillations, the resonator being on the left, the exciter on the right.

For our resonator we take the apparatus shown on the left-hand side of fig. 9, consisting of a coil wound on a cardboard tube, joined to a small Leyden jar as condenser. As an indicator of the forced oscillations, i. e. as a cheap form of alternating current ammeter, we make use of a small incandescent lamp, which we have previously graduated roughly: white heat means about 0·5 ampere.

We bring the two circuits so close together that some of the lines of magnetic force of the exciter penetrate the coil of the resonator. We then make both circuits have approximately the same frequency, by slowly varying the capacity of the exciter circuit by means of the adjustable condenser. We soon obtain "resonance", shown by the lamp lighting up brightly.

Now the coil and Leyden jar form a delightfully simple example of an undamped oscillatory circuit. But we will push the tendency towards purely diagrammatic apparatus still further. We proceed to reduce the coil and the condenser to their simplest forms. This is done in the apparatus shown in fig. 10. Now we have only a single copper wire bent into a circle about 30 cm. in diameter. At its middle point in front of the wooden handle there is a small incandescent lamp to indicate the current. At each end of the wire there is a condenser plate of the size of a visiting-card. The two plates can move without touching each other, being placed so as to face each other with a distance of about 5 cm. between them. We bring this resonator circuit near the exciter circuit shown in fig. 6. By bending the copper wire we can make the frequency of the resonator sufficiently close to that of the exciter. The lamp lights up brightly. An alternating current of about half an ampere with a frequency of about 100 million cycles per second flows in our circuit.

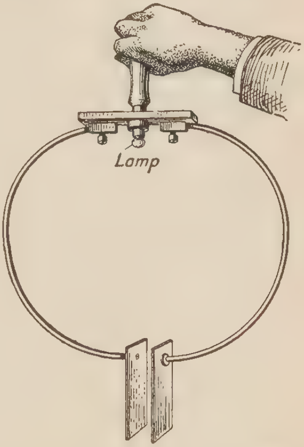

Fig. 10. A very simple closed oscillatory circuit for detecting forced electrical oscillations. The incandescent lamp serves as an indicator of alternating current in the bent wire.

Cf. the experiments shown in fig. 16, p. 30, and fig. 10 above. In fig. 16 the destruction of the field happens *once* and the quantity of electricity involved is of the order of 10^{-8} coulomb. In fig. 10 above the destruction of the field takes place about 10^{8} times per second and we accordingly observe currents of the order of one ampere.

Thus we have solved the problem which was raised at the end of last section. We have an undamped oscillatory circuit of absolutely ideal simplicity. All disturbing accessories have been got rid of.

4. The Electric Dipole.

The forms of practical apparatus which are now at our disposal lead us to something new and important, namely the electric dipole.

In mechanics the simple pendulum consists of an inert mass and a spiral spring. The oscillatory circuit consisting of coil and condenser corresponds to it in electricity. We worked out the analogy in detail on p. 181 and now merely refer the reader to fig. 11.

In the simple mechanical pendulum there is a clear-cut distinction between the mass and the force exerted by the spring. The mass of the bob can be chosen to be so large that the small mass of the spring can be neglected compared to it.

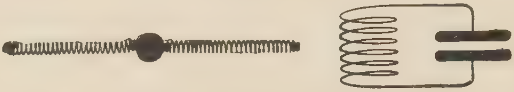

Fig. 11. Mechanical pendulum and oscillatory electric circuit.

But in mechanics we know of numerous bodies capable of vibration in which there is no question of a separate localization of the mass and the force of the spring. The typical example is the stretched string. Every element of length of the string represents a piece of inert mass as well as a piece of stretched spring.

Exactly the same is true for electrical oscillations. In the ordinary oscillatory circuit, say that in fig. 11, we can clearly distinguish the coil as being the seat of the inert magnetic field and the condenser as being the seat of the pressure. Thus we may e. g. neglect the fact that strictly speaking every two adjacent turns of the coil actually form a condenser of finite capacity. But there are other electrical bodies capable of vibration in which this separate localization is just as impossible as in the mechanical case of the vibrating string. The electric dipole is an extreme case of this kind. We now proceed to consider it.

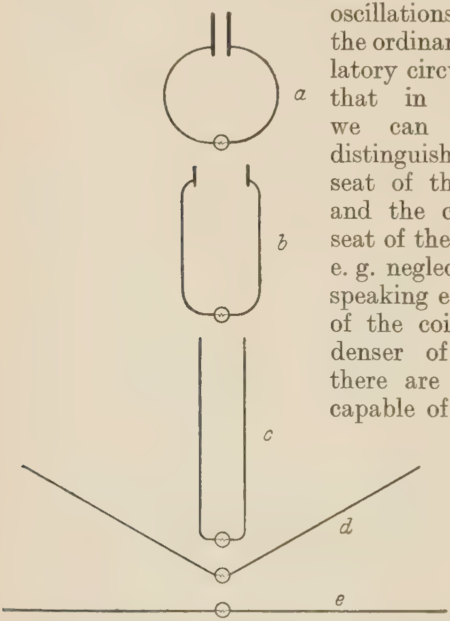

Fig. 12. Transition from the closed oscillatory circuit to the electric dipole.

We refer back to the

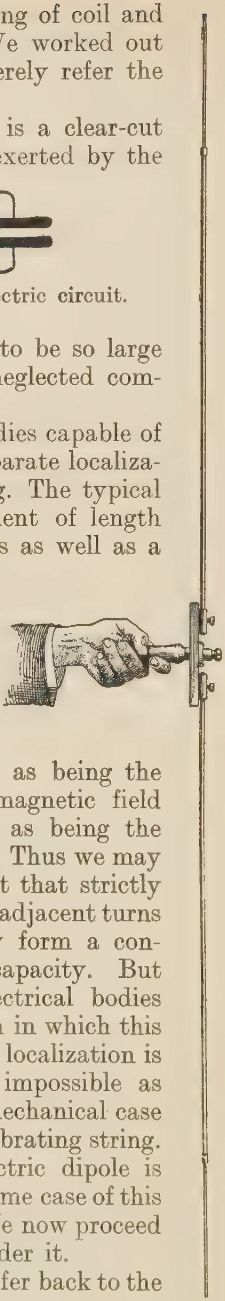

Fig. 13. An electric dipole about 2 m. long.

simplest of our oscillating circuits, that shown in fig. 10. The current which flows through the bent copper wire and the lamp as a *conduction current* must bridge the gap of 5 cm. between the condenser plates as a *displacement current*. We proceed to increase the range of this displacement current systematically, and at the same time we continually diminish the size of the condenser plates. We make the transition sketched in fig. 12. By trial we find that it is possible to compensate for the gradual shrinking of the condenser plates by lengthening the two halves of the bent wire. The lamp lights up again, showing that an alternating current flows just as before.

In the limiting case we obtain fig. 12e, a straight wire, with the lamp shining brightly in the middle. This is an *electric dipole*. Fig. 13 shows a dipole as it is actually made; the hand gives an idea of its size.

The lamp may without any trouble be replaced by a practical hot-wire ammeter, which records a current of about 0·5 ampere. The exact length of the dipole does not matter; 10 cm. more or less at either end make no difference, for in spite of this the dipole responds to the frequency of the exciter (about 10^8 cycles per second), which we must imagine as being about half a metre away in fig. 13. Thus the dipole is a strongly-damped resonator (cf. last section and fig. 8).

This electric dipole is analogous to the string in mechanics. The vibrating string appears to the eye as in fig. 14, which shows the two extreme positions. At each point the ordinate of the figure directly indicates the amplitude of the particle of the string vibrating up and down at that point. The amplitudes of the mechanical vibrations are measured in centimetres, positive upwards, negative downwards. They are greatest in the middle of the string and vanish in either direction as we approach the ends of the string.

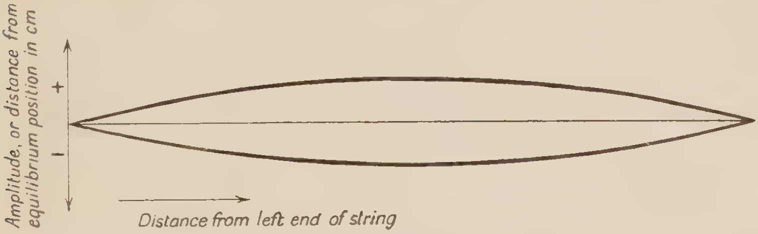
Amplitude, or distance from equilibrium position in cm
+
−
Distance from left end of string

Fig. 14. Diagram of a vibrating string.

The electric dipole is an exact analogue of this. Now, however, the ordinate in fig. 15 means the strength of a conduction current, measured in amperes. An ordinate measured upwards means a motion of the electrons in the wire towards the right, an ordinate measured downwards the corresponding motion towards the left. Here, as in all conduction currents, the displacements involved are quite trifling in amount. An approximate calculation shows that the electrons in the middle of the dipole only oscillate backwards and forwards through about $^{1}/_{10}$ of the diameter of an atom on either side (cf. p. 249).

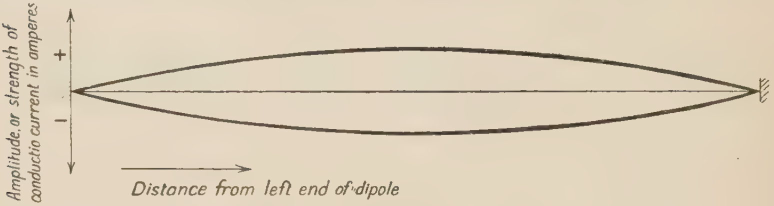

Fig. 15. Distribution of conduction current in a dipole.

The current is strongest in the middle and decreases towards both ends. The lamps to the left and right of the centre of fig. 16 only glow with a yellowish-red light. The meaning of the term "vibrations of a dipole" is illustrated very clearly by figs. 15 and 16: the conduction current in a dipole is distributed along the length of the dipole in exactly the same way as the amplitudes of the stationary waves of a transversely vibrating string.

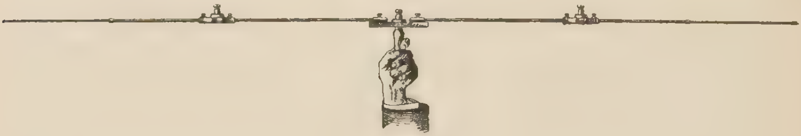

Fig. 16. Dipole with three lamps for showing the distribution of conduction current in it.

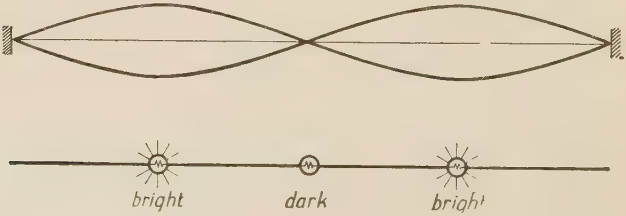

Fig. 17. A dipole oscillating in its first harmonic.

This agreement goes still further. The string in fig. 14 is vibrating in its *fundamental tone*. In fig. 17 we see a string excited so as to vibrate in its first *harmonic*. Its stationary vibration now has a "node" in the middle as well as at the two ends.

Below it a dipole about 6 metres long is drawn diagrammatically. It is made up of two of the dipoles previously used. The distribution of the current is recorded by the incandescent lamps included in it. The lamp at the node in the centre remains dark. This dipole is oscillating in its first harmonic. We can go on in this way fixing more dipoles together until we obtain a dipole 9, 12 or more metres in length.

Just as in the case of a string in mechanics, a dipole may of course be excited by a self-regulating arrangement so as to execute undamped oscillations. This is done e. g. by the apparatus shown in fig. 18. In this way undamped alternating currents of frequency 10^8 cycles per second can be produced just as well as with the apparatus sketched in fig. 6. As compared with the latter, the self-regulating dipole has the advantage that the diagram of the connections is clearer but the disadvantage that previous knowledge of the vibrations of a dipole must be assumed.

So much for the dipole. It has yielded us an important result, namely the fact that the distribution of a conduction current in a wire represents a stationary wave, both in its fundamental and its first harmonic.

To this distribution of the conduction current there corresponds a definite distribution of the electric field. *The path of the conduction current must be made into a closed circuit by a displacement current* constituted by the rapid alternations of the electric field with time. Our next problem is the investigation of this electric field and its alterations in time. This leads us to consider progressive electric waves *in wires* as well as *free* electric waves or radiation proper.

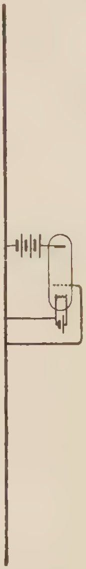

Fig. 18. Dipole with automatic regulation.

5. Stationary Electric Waves between Two Parallel Wires.

The electric lines of force of a straight opened-out dipole must somehow pass in wide curves between different points of the dipole. On the way they impinge on the wall of the room, the observer, and so on. We shall not venture to begin with these relationships for a straight open dipole, for which they are by no means simple. We shall first simplify the conditions in such a way that there can be no doubt about the paths of the lines of force. For this purpose we return a little along the way which led us to consider the dipole.

In passing from the closed oscillatory circuit to the open dipole we had the intermediate form shown in fig. 19a. We may briefly describe it as a dipole which has not been opened out. If we bring this body near the exciter familiar to us from fig. 6, the lamps immediately show how the *conduction current is distributed.* The lamp in the middle shines most brightly; the loop in the conduction current is situated in the middle.

In the case of this form there can be no doubt about the paths of the lines of electric force between the two limbs. The distribution of **R**, the electric intensity, is shown graphically in fig. 19b. There must again be maximum values attained twice during every vibration. The upper ordinates refer to the instants in which the upper limb is positive, the lower to those in which it is negative. The ordinates can be interpreted either as electric intensities in volts/cm. or as displacement currents in amperes; for the regions of high intensity are at the same time regions where the displacement currents are large.

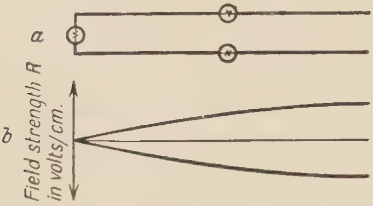

Fig. 19a, b. A dipole which has not been "opened-out", and the distribution of the electric intensity between its limbs.

One or more dipoles of equal lengths may be attached to the ends of a dipole (fig. 17). This is done in fig. 20a, in which the ends of the separate dipoles are marked by transverse strokes. The distribution of the field is shown underneath: the loops, in which the intensity of the field and the displacement current attain their greatest values, and the nodes, in which they are zero (fig. 20b).

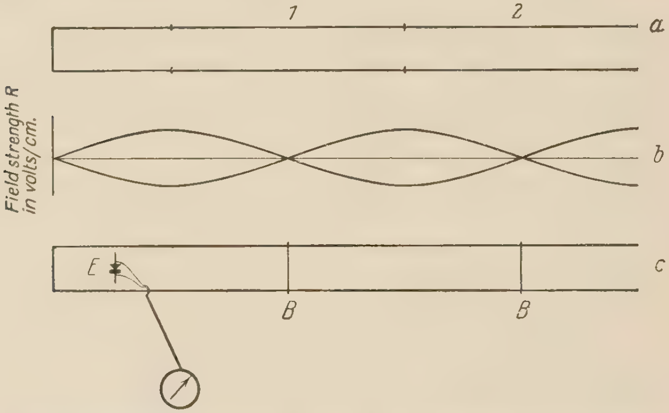

Fig. 20. Stationary electric waves in wires. An incandescent lamp inserted at the left-hand end of the dipole is not shown in the figure.

Now we can measure the distribution of this field in an extremely simple and accurate way. We shall describe two methods:

(1) We observe the magnitude of the displacement currents by means of a short piece of wire or "receiver" placed between the limbs

(E in fig. 20c). We are already familiar with this apparatus from fig. 27, p. 189. The displacement current produces a conduction current in the piece of wire or receiver. It is an alternating current. In order to record it we transform it into direct current, for alternating current ammeters are not as a rule sufficiently sensitive. A crossed-wire thermo-junction or crystal detector, such as is shown in the middle of E, serves to rectify the current; it is joined by leads to a direct current galvanometer. If we make the receiver pass along the limb, the nodes, i. e. the places where the displacement current is zero, can be traced out very accurately. This method can always be used. At the nodes thus found the parallel wires may subsequently be connected by a wire B or by the fingers (cf. fig. 20c). As an incandescent lamp inserted at the left-hand end of the dipole shows, this does not disturb the stationary waves in the least.

The separate wire rectangles divided off in this way may be regarded as closed resonators whose own frequency has been tuned to that of the exciter. We generally make use of this point of view when we start experimenting on a double conductor spread out in any way without previously knowing the frequency of the exciter or the length of the corresponding dipole. It is easy to show that the single rectangles are independent of each other: the double conductor can be cut behind each bridge and removed without disturbing the oscillations in the remaining rectangles.

(2) We stretch out the two limbs in a long glass tube, in which the air (or better, neon) can be highly rarefied (fig. 21). An independent discharge through the gas then takes place in the regions of high intensity (the loops). What we see is the light from the positive column of the luminous discharge. The successive stretches of alternately bright and dark gas give us a remarkably clear picture of the whole distribution of the field between the wires.

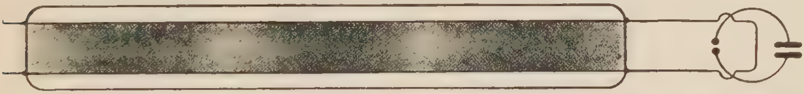

Fig. 21. Apparatus for making the distribution of the field due to stationary electric waves in wires visible.

This method requires that the electric intensity should have fairly high values. This is most simply obtained by using a damped exciter, say the circuit with spark-gap sketched in fig. 21. The time-averages of current and pressure are smaller than those which can conveniently be produced by an undamped oscillatory circuit. But the instantaneous values of current and pressure are very high, because each single oscillation is actually completed during the brief time taken by a spark to pass.

These experiments prove conclusively that the *electric field* situated between the two wires and excited by high-frequency alternating currents is in the form of a *stationary wave.*

6. The Importance of Stationary Electric Waves in Wires. The Propagation of Electric Fields with the Speed of Light.

The discovery of electric fields in the form of stationary waves is of quite outstanding significance in connection with our knowledge of the electric field. It shows without any doubt that the electric field is propagated with a *finite velocity* and that progressive electric waves must exist.

In order to understand this we again begin with a mechanical case, say with the production of electric waves in strings or rods.

We begin by recalling how elastic waves arise. The essential point is that the elastic disturbances are propagated with a finite velocity.

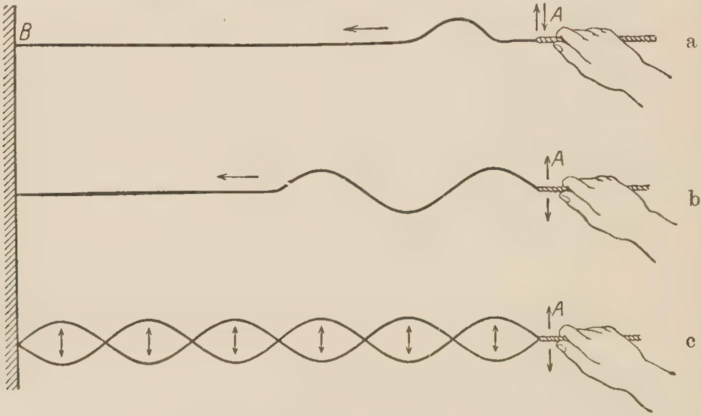

Fig. 22. Progressive and stationary mechanical waves in a rubber tube.

We fasten a rubber tube (to act as a vibrating string) to the wall at *B* (fig. 22). We jerk the other end *A* with our hand. We see the elastic disturbance passing along the tube as an "impulsive wave" (like that of an explosion) (fig. 22a).

We then move the end *A* in our hand up and down in a periodic vibration.

At first we see the snaky form of a progressive wave passing along from *A* to *B* (fig. 22b). Reflection takes place at the fixed end *B*. The original and reflected wave-trains now move in opposite directions. Their superposition generally gives a form which changes in a complicated way.

It is otherwise if the length of the "string" amounts to an exact number of quarter-wavelengths of the progressive wave. We then get the definite form of a stationary wave (fig. 22c). Nothing more is to be seen of the initial progressive wave which was the cause of the whole phenomenon.

The total number of quarter-wavelengths must be *even*, if the end *B* is fixed as in fig. 22. It must be *odd* if the end *B* is free to vibrate as in e. g. a rod held at *A*.

The stationary wave and the initial progressive wave have the same wave-length λ. Hence we can find $\lambda/2$ very accurately from the distance between the nodes of the stationary waves.

When n, the frequency of the exciter (i. e. of the hand in this case) is given, the wave-length depends only on u, the velocity with which the elastic disturbances are propagated along the string. We have

$$n\lambda = u; \tag{1}$$

hence we can calculate u, provided both λ and n are known.

The same is true for any stationary wave, no matter in what branch of physics it occurs. The formal relationship involved is perfectly general. Hence it also holds for the stationary electric waves between the parallel wire conductors of figs. 20 and 21. *The existence of stationary electric waves proves the existence of progressive electric waves between two wires.* These "waves in wires" move out from the exciter along the two wires. At the ends of the wires they are reflected. The superposition of the two opposing waves gives rise to progressive waves in the wires, provided the length of the wire is an exact number of quarter-wavelengths.

For conductors open on one side as in fig. 20 the total number of quarter-wavelengths must be odd; in the contrary, for conductors closed on both sides it must be even.

These stationary waves occupy the whole length of the wire if the exciter supplies an *undamped* alternating current. When the alternating current is damped and dies away the formation of stationary waves is confined to the neighbourhood of the ends. At a greater distance from the ends the combination of direct and reflected portions of waves no longer gives rise to wave-trains of limited extent.

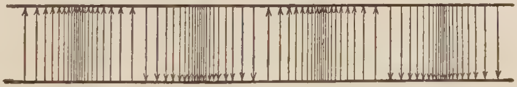

Fig. 23. Snapshot of a progressive electric wave. The arrows give the direction of the electric intensity, their closeness the magnitude of the intensity in volts/cm.

Fig. 23 serves to illustrate the term "progressive electric waves in wires". Figuratively speaking, it represents a snapshot. All the lines of force have ends; the lines of force pass in straight lines between points of the two wires which are immediately opposite each other. The arrowheads indicate the direction of the field, i. e. the direction in which an electron moves in the field. The intensities of the field in volts/cm. are indicated by the varying density of the lines of force. This whole picture is to be imagined as moving horizontally with a large velocity. To an observer *at rest* the progressive wave appears as a periodically alternating displacement current. An observer moving with exactly the same velocity as the wave always sees an electric field at rest distributed as shown in fig. 23.

Another representation which in itself is equally valid is shown in fig. 24 below. Crests mean electric fields directed upwards, troughs those directed downwards. The amplitude denotes the intensity of the field at the point in question in volts/cm. This method of representation, however, does not let us see the course and longitudinal extension of the lines of force.

In our mechanical example (fig. 22) a knowledge of λ, the wave-length of the stationary waves, enables us to calculate the velocity of the progressive waves, provided we know n, the frequency of the exciter.

We proceed in exactly the same way with electric waves in wires. Once we have ascertained n, the frequency of the alternating current in the exciter, we calculate the velocity u from equation (1),

$$u = n\lambda\,.$$

The former is easy to find if the exciter is a closed oscillatory circuit. We can then use equation 3, p. 184:

$$1/n = \tau = 2\pi\sqrt{LC}\,.$$

Measurements of this kind have been carried out by various experimenters with great care. The results agree with one another. The velocity of electric waves in wires is 3.10^{10} cm./sec. It is equal to the velocity of light, that fundamental constant which is generally denoted by the letter c. For electric waves in wires we have $c = n\lambda$, or $\lambda = c\tau$, where τ denotes the period of the alternating current.

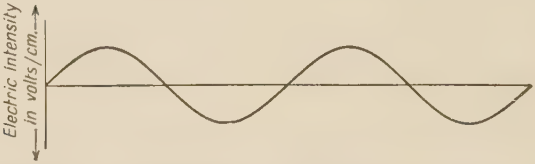

Fig. 24. Another way of representing the instantaneous state of a progressive electric wave.

Thus the discovery of stationary electric waves in wires has brought with it two fundamental results:

(1) *The condition of space which we call an electric field is propagated with a very high but finite velocity.*

(2) *The numerical value of this velocity is that of the velocity of light,* 3.10^{10} cm./sec. Although light and electricity affect our senses in such different ways, the two phenomena are closely related to each other.

The significance of the velocity of light in connection with *electric* phenomena, however, was not revealed by the measurements we have referred to. Many years previously it had been noticed that (in the units and notation of the present book)

$$\frac{1}{\sqrt{\mu K}} = \frac{1}{\sqrt{1{\cdot}256.10^{-8}\,\frac{\text{volt-sec.}}{\text{amp.-cm.}}\,.\,0{\cdot}884.10^{-13}\,\frac{\text{amp.-sec.}}{\text{volt-cm.}}}} = 3.10^{10}\ \text{cm./sec.}$$

The existence of a close relationship between electrical phenomena and optical phenomena was immediately inferred from this fact.

7. Direct Measurement of the Velocity of Progressive Waves in Wires.

The account given in last section leaves us in an unsatisfactory frame of mind. It is true that the detection of electric fields in the form of stationary waves between parallel wires is an important experimental fact. But we were not really investigating these stationary waves for their own sake. Their real purpose was to help us to prove that electric waves in wires are propagated with the velocity of light.

Why then, the reader will ask, do we not measure the velocity of electric waves in wires directly? Why do we go out of our way to consider stationary waves? Why do we not simply measure the time taken for the waves to traverse a known distance, as we do with light?

The answer is easily given. Direct measurements of this kind have been repeatedly carried out by electrical engineers, but they require apparatus which is not usually available in physical laboratories. It is necessary to have a double telegraph wire about 1000 km. long, which forms a ring and returns to the starting-point. The rest is easy. The times of the departure and arrival of single waves are photographically recorded by means of measuring instruments of sufficiently low inertia and then the records are measured. Oscillographs of all kinds are used as measuring instruments (see p. 9).

Measurements of this kind have confirmed the previous result in giving the velocity of light; but with an important limitation. *The velocity of light merely turns out to be an upper limit for high frequency alternating currents and double leads of small resistance.* This brings us to the second point in respect of which the account given in the previous section was unsatisfactory.

Let us consider the method for measuring the velocity directly, as described above. At bottom what we do is just this: we connect a source of current, in the simplest case a battery of pressure E, with a long double conductor (fig. 25). Previously we would have regarded this in the following way. The double conductor forms a condenser of capacity C. The source of current has to charge this condenser until there is a pressure E between any two points of the double conductor. Thus a quantity of electricity CE is required. This quantity of electricity flows in the form of a conduction current; electrons move into the lower conductor in the figure and out of the upper conductor. To charge a condenser by means of such conduction currents, however, may take a considerable time in certain circumstances, namely when the conductors through which the conduction currents flow have a high resistance.

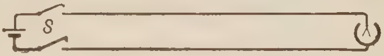

Fig. 25. Measurement of the velocity of propagation of an electric field between two badly-conducting wires.

For example, we replace the two wires in fig. 25 by strings 3 m. long. The electric field certainly does *not* appear at the electrometer at the other end after a lapse of $3.10^2/3.10^{10} = 10^{-8}$ second. It takes several seconds for the electrometer threads to be set in motion and several seconds more for the full pressure E to be recorded.

Now metal wires of course have ever so much smaller an ohmic resistance than the strings. On the other hand, however, the self-induction of long double metallic conductors is very considerable (cf. p. 190). Self-induction allows the conduction current to start only in a "sluggish" way. Hence at bottom the same must be true in this case as for the double conductor of string. The direct measurements of velocity in double telegraph wires are in complete agreement with this. With the alternating currents used in telephony, i. e. with frequencies of several hundred cycles per second, we often get velocities which do not greatly exceed 2.10^{10} cm./sec.

In laboratory experiments on short stationary waves in wires, on the other hand, the material of the conductor no longer has any noticeable effect. From the experimental point of view there is no doubt that for waves in wires the velocity of light is the limiting value obtained when we pass to high frequencies.

Now why does the effect of the nature of the conductor become of no importance when the frequency is high? The answer is immediate. In the case of low frequencies it is the *conduction currents* that make the field advance. The electric field existing at the beginning of the double conductor has to move the electrons onwards against the frictional resistance in the wires. In the case of high frequencies it is the *displacement currents* that play the chief part in driving the field along. In this case there is a strong *displacement current* at the beginning of the double conductor *as well as* an electric field. The magnetic field of this displacement current induces lines of electric force *between* the immediately succeeding pieces of wire, and so on.

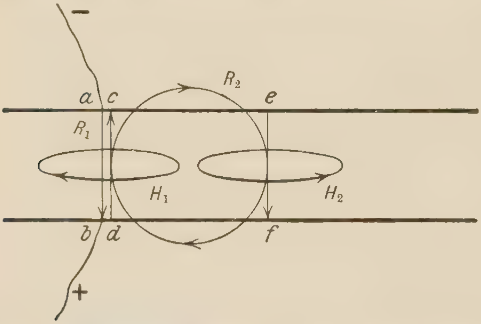

Fig. 26. Inductive effect of the displacement current in the propagation of electric waves in wires.

In order to get a clear idea of what takes place, let us break up the phenomenon into separate stages, referring to the rough diagram of fig. 26.

The figure shows the two metal wires of a double horizontal conductor. An electric field is produced between the points a and b by a source of alternating current not shown in the figure. At the instant represented the electric intensity $\mathbf{R}_1$ is in the downward direction and is decreasing, thereby forming a displacement current. The latter gives rise to a magnetic field indicated by the circular line of force $\mathbf{H}_1$. Since this magnetic field changes with the time, it is surrounded by closed lines of electric force. In the *absence* of the wire the circle $\mathbf{R}_2$ would indicate one of its lines of electric force, but in the *presence* of the wire there remain only the two rectilinear pieces of lines of force, cd and ef. The electric field breaks down in the conductors between ce and df. Now the field cd weakens the original electric field ab further, and the field ef appears in its place. *The original electric field ab has been moved along through the interval of path ae by the inductive effect of its displacement current.* The field ef in its turn, regarded as a displacement current, is surrounded by another magnetic field $\mathbf{H}_2$, and so on.

Thus the phenomenon does not really take place *in* or *between* the wires at all, but in the air, or if we pump away the air, in a vacuum. Hence in the case of high frequencies what goes on in the wires is of quite subordinate importance. This is why the nature of the wire conductors makes no difference when the frequencies are high.

8. The Displacement Current of the Dipole. The Radiation of Free Electric Waves.

By what we have said in last section the double wire conductor plays a perfectly subsidiary part for high frequencies. It holds the electric field together and makes the induction effect of the displacement currents act in one direction only. Thus it hinders the propagation of progressive waves in all directions. A pipe holding a sound-wave together is a close analogy in acoustics.

Seeing that the wire conductor plays such a minor part, it may be left out altogether. This in no way disturbs the *essential* phenomenon, i. e. the inductive effect of the displacement current. We thus obtain electric waves advancing freely in space, and we come to our last question, which is one of particular interest: the emission of free electric waves.

The dipole again forms our experimental starting-point. We briefly remind the reader how the conduction current is distributed in the dipole. The "loop" of the current is in the middle (figs. 15 and 16, p. 314).

To this distribution of the conduction current there corresponds a definite distribution of the displacement current. Corresponding points of the two halves of the dipole must somehow be connected by lines of electric force sweeping round in wide curves. A rough sketch is given in fig. 27: this is for the case where the two halves of the dipole have their maximum charge.

We shall now proceed to investigate the distribution in *space* of this displacement current of the dipole S. This is done by the method with which we are already familiar. We bring a short piece of wire E, which we shall again call the "receiver", to the point under observation. It transforms the displacement current at that point into a conduction current, which is an alternating current with the same frequency as the dipole. By means of a small rectifier (crystal detector or crossed-wire thermo-junction inserted in the wire E) the alternating current is transformed into a direct current, which may conveniently be measured by a moving-coil galvanometer G.

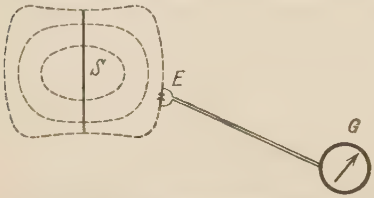

Fig. 27. The displacement current of a dipole; a snapshot of the distribution of the electric field.

Let the dipole S in fig. 27 again be about 1·5 metres in length and let it be excited so as to execute forced undamped oscillations. The receiver need then be only as long as one's fingers. It enables the displacement current to be easily detected even at a distance of several metres from the dipole. The lines of electric force spread out from the dipole far into external space.

An extensive investigation of the space distribution is meaningless unless in the open or in a large hall. Walls, floor, observer, auxiliary apparatus, etc. must be removed by a multiple of its length from the dipole, otherwise they distort the far-reaching lines of electric force. A lecture room is too small for a dipole 1·5 metres long.

In order to avoid this difficulty we use a much smaller dipole, e. g. one 10 cm. long. With the practical apparatus at present available these small dipoles cannot be made to oscillate without damping, so we must content ourselves with damped oscillations. We go back to the spark gap with which we previously became acquainted in connection with closed oscillatory circuits (p. 185). A convenient practical form is shown in fig. 28a.

The dipole consists of two similar thick brass rods. Their plane end faces are covered with sheet magnesium. When brought to a distance of about 0·1 mm. from each other they form the spark gap. A long thin flexible double conductor (electric bell cord) connects the spark gap with a source of alternating current (say at 500 volts and about 50 cycles per second, supplied by a small transformer). Two small choking coils are included at a and b. They prevent the high frequency alternating current of the dipole from entering the double conductor. The spark gap scarcely makes any noise; we merely hear a gentle humming. The dipole is supported by a wooden pillar half a metre long. For short let us henceforth call it the "transmitter". We can turn the transmitter round, tilt it, or move it about in any way we like while it is working.

The apparatus for detecting the displacement current remains the same as in fig. 27. This time, therefore, the receiver E has about the same length as the transmitter. Thus this receiver is a little too large for investigating the immediate neighbourhood of the transmitter, as the finer details of the distribution of the field are obliterated. This disadvantage of the relatively long receiver is however outweighed by its great sensitivity.

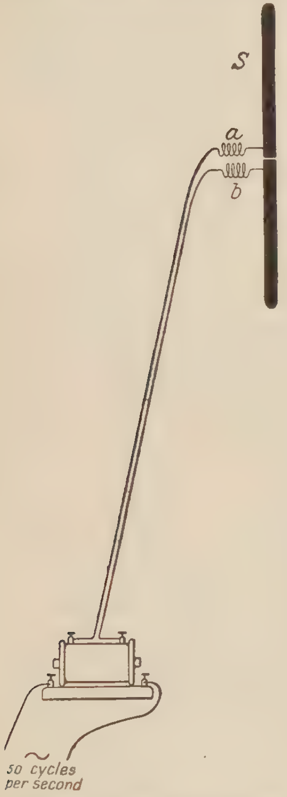

Fig. 28a. A small dipole acting as a transmitter: a and b are choking coils.

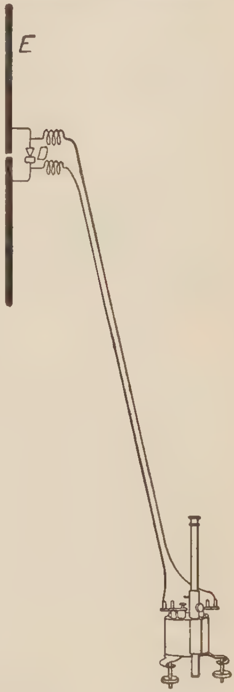

Fig. 28b. A small dipole acting as a receiver; D is a crystal detector.

The receiver also forms a dipole. It reacts to the alternating field of the transmitter by executing forced vibrations. Approximate equality of the lengths of the two dipoles means that they are in tune or resonance with each other.

The receiver (fig. 28b) on its thin leads of fine wire is just as easy to move as the transmitter. Hence the whole region of the displacement current of the transmitter can be thoroughly investigated in the simplest way imaginable.

We begin by looking for *radial* components of the electric field or displacement current *in the neighbourhood of the transmitter*. That is, we arrange the transmitter and receiver in the relative positions shown in fig. 29. We carry out these observations for different azimuths φ. We find radially directed displacement currents in all azimuths in the neighbourhood of the transmitter. Their current strengths, however, rapidly diminish as the distance r increases. Even when it is only twice or three times the length of the dipole they become imperceptible.

Further, we look for *transverse* components of the displacement current *in the neighbourhood of the transmitter*. We use the orientation shown in fig. 30. These transverse components increase rapidly with the azimuth φ: but even for $\varphi = 0$, i. e. in the direction transverse to the axis of the dipole, they have quite perceptible values.,

Then follows the investigation of the transverse components of the displacement current at *a greater distance from the transmitter*, say at a distance equal to six times the length of the dipole. Now it is no longer possible to detect any component of the field transverse to the direction of the axis of the dipole, i. e. for $\varphi = 0$. A displacement current appears only as the angle φ increases. When $\varphi = 90°$ the displacement current reaches its maximum value. At any point it is in a direction transverse to the line (r) joining that point to the dipole.

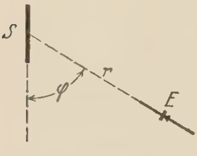

Fig. 29. Mapping out the field of the dipole: radial components of the electric field in the neighbourhood of the transmitting dipole S.

Hitherto the transmitter and the receiver have always been in the *same* plane, namely in the plane of the paper in figs. 28—30. Now we slowly rotate either the transmitter or the receiver out of the plane of the paper: the displacement current decreases. It vanishes whenever the long axes of the transmitter and of the receiver are at right angles to each other. **R**, the intensity of the electric field, is a vector. Hence by the experiments which we have just performed it lies in the planes which pass through the longitudinal axis of the transmitter.

Thus *at greater distances* our observations lead us to a very simple picture of the electric field, which may be represented graphically as in fig. 31. The direction of the arrows gives the direction of the electric intensity **R** for various points of observation at equal distances r from the dipole. The number of parallel arrows denotes the magnitude of the intensity in volts/cm. Figuratively speaking, the whole forms a small *section of a snapshot* of the field of the sender.

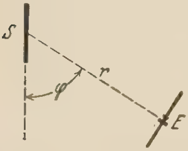

Fig. 30. Transverse components of the field of the dipole.

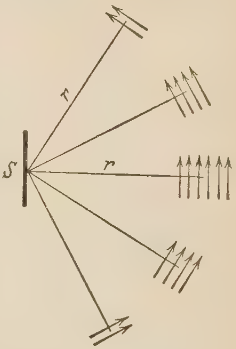

Fig. 31. Distribution of the transverse components of the field of the dipole in various directions.

But what does the complete "snapshot" look like? The necessary extensions are easily carried out. To begin with, two facts are certain:

(1) The field drawn in fig. 31 originates from the transmitter. It has traversed the distance r in empty space*.

---

* To make sure, let us once more remind the reader that the absence of the molecules of the air makes no difference of any importance to electric phenomena in space (p. 26).

(2) The field alters periodically, with the frequency of the transmitter. What our receiver measures in the first place is of course not the electric intensity, but the displacement current. The snapshot of fig. 31 must shortly thereafter make way for a similar picture with the arrows reversed, i. e. with the direction of the field reversed, and so on in continual alternation.

These two facts enable us to extend the snapshot of fig. 31 as shown in fig. 32.

Now we have in addition a third fundamental fact: lines of electric force cannot begin and end anywhere in empty space. In empty space there can be no lines of electric force other than closed ones. We have to complete the lines of force by making them into closed curves. This is done in fig. 33. Thus we finally obtain the complete "snapshot" of fig. 34. It shows the electric field of the sending dipole, the immediate neighbourhood of the sender being excluded. *This is the radiation field of the dipole, which was discovered by Heinrich Hertz.* The snapshot shows the emission of an electric field in the form of a wave advancing freely in space. The amount of the electric intensity in volts/cm. at any point is indicated by the density of the lines of force at the point in question. For comparison the reader should refer back to the snapshot of a progressive electric wave in a wire given in fig. 23 (p. 319).

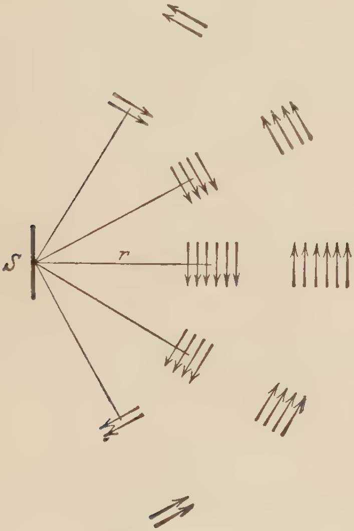

Fig. 32. Reversal of the field of the electric dipole at different times and places.

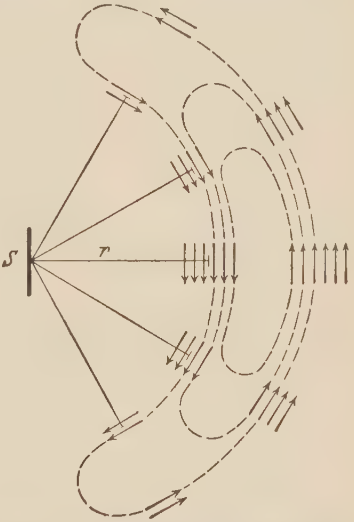

Fig. 33. The lines of electric force made into complete closed curves.

Any kind of progressive wave may always be detected by physical means by the observation of *stationary* waves. We explained this previously in detail for electric waves in wires, beginning with a mechanical example. We shall proceed in the same way here with the exprimental detection of electric waves progressing freely in space. We shall take our mechanical example from *acoustics*. It is represented in fig. 35, which shows a small whistle with a high note acting as a transmitter (S). A sensitive gas flame G serves as receiver. When undisturbed it burns with a long steady thread-like flame. When affected by sound waves it takes on a broad flickering form.

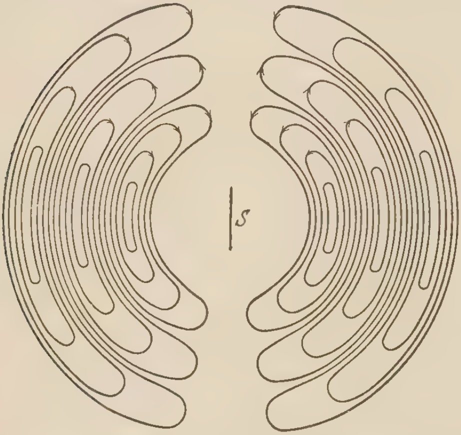

Fig. 34. Snapshot of the distribution of the electric field round a dipole: the Hertzian radiation field of a dipole.

The progressive sound waves pass beyond the flame to the "mirror" M, i. e. any plate of metal, by which they are reflected. The reflected wave advances to meet the original wave. Hence a system of stationary waves with nodes and loops is produced between the mirror and the transmitter. We move the receiver, i. e. the flame, through these stationary waves in the direction of the double arrow; this can be done steadily by sliding it along a rail. The nodes are indicated by the flame burning quietly, the loops by its flickering irregularly. The distance between every pair of neighbouring nodes or loops gives half the wave-length, a few

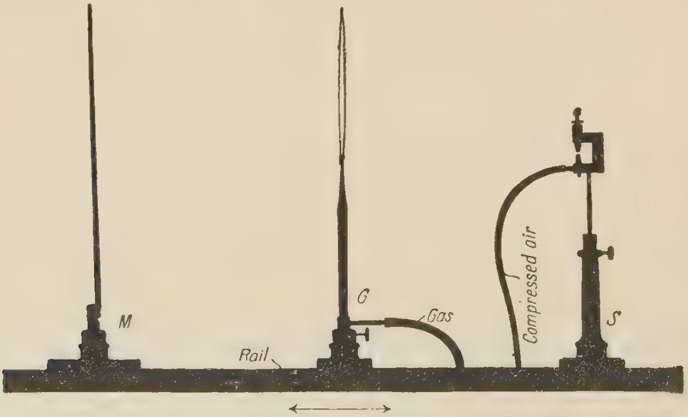

Fig. 35. Measurement of the wave-length of short sound waves.

centimetres in our case. The frequency of the note of the whistle can be calculated from the equation $n = u/\lambda$ (u, the velocity of sound, is 333 metres per second).

Now for the corresponding experiment with electric waves. The whistle is replaced by the small dipole acting as transmitter, the flame by the small receiver. The mirror and the sliding rail remain as before (fig. 35). The receiver is moved by stages from the mirror up to the transmitter. At the same time relative values of the displacement current are observed on the galvanometer. The results of a measurement of this kind are shown in fig. 36. The nodes of the stationary electric waves are clearly marked by the minima of the displacement current. The distance between the nodes turns out to be 18 cm. The wave-length of the stationary waves, and hence also of the initial progressive electric wave, comes to about 36 cm. in this example; n, the frequency of the dipole, is $\frac{3.10^{10}}{36}$ = about 8.10^8 cycles per second.

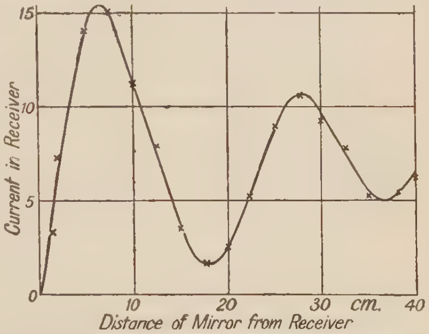

Fig. 36. Measurement of the wave-length of the dipole shown in fig. 28a.

The experiment has one defect of minor importance. The stationary waves are well formed only in the neighbourhood of the mirror. Further away the minima of the displacement current become flatter and flatter. The reason for this is the strong damping of the oscillations of the transmitter. The single train of waves emitted by a spark is but a short one, something like the curve shown in fig. 29, p. 220. At a greater distance from the mirror the high reflected amplitudes from the beginning of the single wave-train have superposed on them the small amplitudes at the end of the same wave-train still on their outward journey. Thus only badly-defined minima are produced.

Stationary waves from larger transmitting dipoles may be detected in a similar way. The undamped dipole about $1^1/_2$ metres in length which we used previously is very suitable. A few square metres of sheet metal on the wall of the room will suffice for a mirror. If the "mirror" is at a sufficient distance several distinct nodes can be found by means of the finger-length receiver of fig. 27, p. 324. Doubling the distance between the nodes, we find that the wave-length λ is about 3 metres.

The diagram of the waves radiated from a dipole, which we sketched in fig. 34, is therefore completely justified by experiment. An electric dipole sends out into space free electric waves which vibrate in directions transverse to the direction of propagation.

Nowadays it is essential to memorize the diagram of the field of a dipole (fig. 34). It is just as indispensable as the diagram of the lines of electric force of a plate condenser and that of the lines of magnetic force of a long coil.

The lines of force diagram of the dipole still requires to be supplemented in two respects:

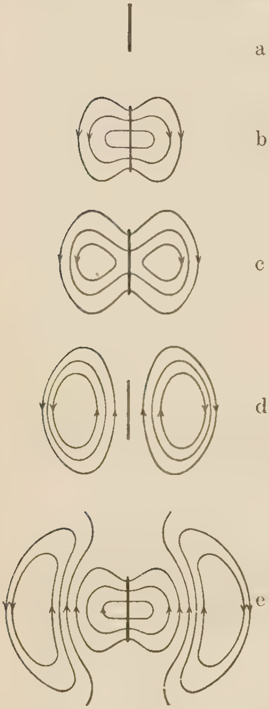

Fig. 37a—e. Five snapshots of the electric field in the neighbourhood of a dipole.

In fig. 34 the field in the immediate neighbourhood of the dipole is wanting. There the field varies according to the state of the charge on the dipole at the time in question. We shall confine ourselves to a brief description by means of figs. 37a—e.

a. Before the oscillation begins both halves of the dipole are uncharged; hence there are no lines of electric force passing between them.

b. The conduction current has begun to flow upwards. By the end of a quarter-vibration it has charged the upper half of the dipole negatively and the lower positively. Discharging lines of force now run far out between the halves of the dipole.

c. *During* the second quarter-vibration the charge of both halves of the dipole again decreases, and so on. It has already sunk to about half. The outer part of the field has advanced further. At the same time the lines of force begin to become constricted in a peculiar way.

d. Here at the *close* of the second quarter-vibration both halves of the dipole are again uncharged. The constriction of the lines of force is complete.

e. In the third quarter-vibration the conduction current flowing downwards has caused the upper half of the dipole to become positively charged and the lower half negatively charged. At the close of the third quarter-vibration the diagram is now the same as case b, except for the reversal of the directions of the arrows, i. e. of the field.

Further, we have still to mention the magnetic field of the dipole. In its case also we shall confine ourselves to a short description by means of a sketch, namely fig. 38.

The magnetic field of the dipole consists of concentric circles. The lines of magnetic force run in planes perpendicular to the long axis of the dipole. The density and direction of the lines of magnetic force alternate in the form of waves. The magnetic field advances along with the electric field.

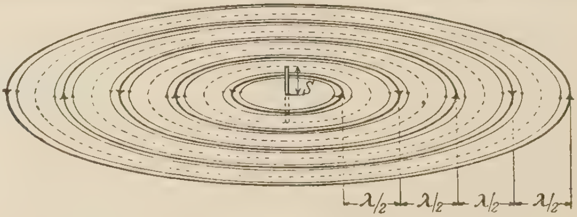

Fig. 38. Lines of magnetic force of a dipole.

Every change in the electric field, being a displacement current, produces lines of magnetic force. All the lines of magnetic force produced give rise to fresh closed lines of electric force by induction. The advance of the whole "electromagnetic" wave depends on this intimate linking together of the electric and magnetic fields, which we briefly illustrated in the case of waves in wires. There is nothing essential to add to the primitive description we gave on that occasion.

9. Half-free Electric Waves. Wireless Telegraphy.

We now know of two kinds of electric waves: waves in wires and free waves. In waves in wires the ends of the lines of electric force are connected to a conductor, namely the wires of the double conductor, at both sides (fig. 23, p. 319). In free electric waves the lines of electric force form closed kidney-shaped curves and are quite free, not being joined to any kind of conductor.

Half-free waves occupy an intermediate position. We shall give a brief description of their shape and of the way in which they are produced.

In the case of stationary waves in wires the double conductor can be bridged by conducting cross-wires *at the loops of the conduction current*, without interfering with the formation of stationary waves in any way. Similarly, no disturbance is caused by *cutting off* the double conductor immediately behind one of these conducting cross-bridges (p. 317).

In the case of the stationary waves of the dipole a *conducting plane* of arbitrary extent takes the place of the conducting bridges. If it is placed in the *loop* of the conduction current, i. e. in the middle of the dipole, it does not affect the stationary wave in the dipole in any way. Similarly, no disturbance is caused if the dipole is cut off behind the conducting plane, i. e. if one half of the dipole is left out. We thus obtain the arrangement of fig. 39, which shows a half dipole S on an extensive conducting plane B. The loop of the conduction current is situated at the foot of the dipole, where it is attached to the conducting plane.

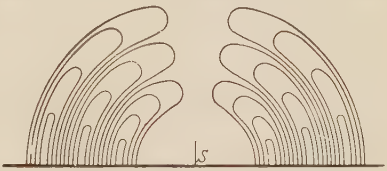

Fig. 39. Half-free electric waves on a conducting plane.

The radiation field of this half dipole is also included in the figure. The way in which it arises from the radiation field of the whole dipole (fig. 34, p. 328) requires no further explanation. We have the snapshot of the half-free wave before us. The lines of electric forces form closed kidney-shaped curves in their upper parts only; below they end on the conducting plane. The electric waves are guided on one side by the conducting surface.

Half-free electric waves play an extremely important part in wireless telegraphy practice.

The principle of wireless telegraphy is very simple. It is illustrated for an idealized limiting case in fig. 40. The shaded arc represents the curved surface of the earth. S is the so-called transmitting antenna, consisting of a vertical wire, the upper half of a dipole. Any practical source of alternating current (×) is made to cause a strong conduction current to flow in this wire. To the right of this we then have the emitted electric field of half-free waves. The waves are guided by the conducting surface of the earth and follow its curvature. E is the receiving antenna, again a vertical wire, the upper half of the "receiver" which we have often used (p. 190). The electric field of the half-free waves breaks down in the wire of the receiver. The displacement current of the advancing wave produces a conduction current in the receiver wire. In theory this conduction current deflects an ammeter or activates some other current indicator (×) of sufficient sensitivity. Thus signals of any kind can be transferred from the transmitting station to the receiving station.

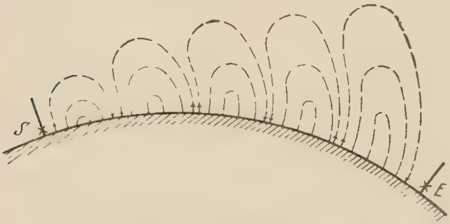

Fig. 40. To illustrate wireless telegraphy with long waves over good-conducting parts of the curved surface of the earth.

The electric waves which are used in practice in wireless telegraphy and telephony usually have wave-lengths between 0·3 and 15 km. The alternating current source of the transmitter must accordingly have a frequency between 10^6 and 2.10^4 cycles per second (equation 1, p. 319; $n = c/\lambda$).

In the antennae used in practice the simple form of the half dipole is often departed from. The upper parts of the dipole wires are replaced by transverse wires extending horizontally, or by umbrellas (fig. 9, p. 26). In this way antennae which do not extend so far in a vertical direction can be tuned to the frequency of the source of alternating current used. Very high current strengths in the antenna may thus be obtained.

At the receiving station resonance phenomena are made use of in many ways. Instruments for detecting the feeble conduction currents in the receiving antenna have been brought to a marvellous degree of perfection. This advance is due above all to the invention of the three-electrode valve (fig. 19, p. 213). Nowadays waves which reach the receiving station with a field intensity of only 3.10^{-7} volts/cm. can easily be detected, using ordinary receiving antennae.

To-day we can send electric waves right round the globe. The circumference of the earth, 40000 km. in length, is traversed in $\frac{40000 \text{ km.}}{300000 \text{ km./sec.}} = 0{\cdot}13$ seconds. The velocity of progressive electric waves can now be measured directly (by dividing the length of path by the time taken to traverse it).

The guiding of the half-free waves by the surface of the earth (fig. 40) is, as we mentioned, an idealized limiting case, in which we assume that the surface of the earth has a high electrical conductivity. This is approximately true for sea water, at least for long waves. When stretches of dry land are traversed, on the contrary, the propagation of the waves is considerably more complicated, especially for shorter waves. Along the path of the waves there arise all possible kinds of hybrids between half-free and free waves. The lines of electric force no longer reach the ground at right angles, but obliquely, and so on.

Perfectly free waves have recently been employed in so-called "short-wave" transmission, wave-lengths from about 10 to 50 metres being used. Dipoles excited so as to vibrate in harmonics (fig. 17, p. 314) serve as transmitting antennae. The waves are no longer guided by the earth's surface, but reach their destination at the antipodes by a roundabout path, being reflected once (or perhaps several times?) by the upper layers of the atmosphere. The upper regions of the atmosphere appear to be strongly ionized by some kind of radiation from external space which makes them conduct well. This enables them to act like a mirror, just as conducting metal plates do. But this matter, which is also important from the point of view of meteorology, still remains to be fully cleared up*.

Apart from the above we have altogether refrained from going into details, especially those of a technical nature. The achievements of electrical engineering in the realms of telegraphy and telephony, wireless or otherwise, are perfectly marvellous. We cannot do anything like justice to them within the scope of a physical textbook, but refer our readers to the extensive literature devoted to technical matters.

10. The Essential Identity of Electric Waves and Light Waves. The Whole Spectrum of Electric Waves.

We have already seen that in two respects electric waves behave in the same way as light waves. Electric waves are propagated in empty space with the velocity of light. They are also reflected by metallic mirrors just as light is. This reflection gives rise to free stationary electric waves.

But the similarity between electric waves and light waves extends still further. Light waves are refracted by transparent surfaces, the deviation at the boundary being given by the well-known law of refraction $\sin\alpha/\sin\beta = n$ (fig. 41). There likewise exist substances which are "transparent" to electric waves,

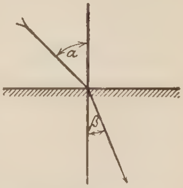

Fig. 41. The law of optical refraction.

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* [This conducting region in the upper atmosphere is often referred to as the *Heaviside layer*.]

namely all insulators. Qualitatively speaking, electric waves are refracted by the substances permeable to them, just as light waves are. Electric waves may be deviated by prisms and accordingly made to "converge" by lenses, and so on. The lens effect, for example, may be shown by means of quite makeshift apparatus.

In fig. 42 S is once more the small dipole transmitter which we have repeatedly used above, and E the corresponding receiver. Both are placed with their long axes vertical. A large glass flask Fl, filled with benzene or the like, acts as a cylindrical lens. When it is placed at a suitable spot in the path of the waves it considerably increases the effect of the waves on the receiver.

A "spectrometer" of sufficiently large dimensions may be used to determine the refractive index n quantitatively. It is diagrammatically sketched in fig. 43. HH indicate two concave parabolic mirrors. The transmitter and receiver are placed in their focal lines. The prism consists of paraffin, sulphur, or some similar substance. The value of n, the refractive index, is found from the deviation by means of well-known formulae in optics; it was determined by Hertz for several substances in his classical experiments. n turns out to be equal to the square root of K', the relative dielectric constant of the substance of the prism. This relationship, $n = \sqrt{K'}$, had already been predicted by Maxwell from his equations. It plays a large part in the general theory of dispersion.

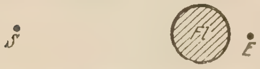

Fig. 42. Effect of a lens on electric waves.

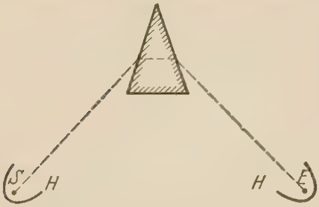

Fig. 43. Spectrometer for measuring the index of refraction of electric waves.

Finally, we must refer to the polarization of light. We can produce light waves whose vibrations are confined exclusively to one plane. The same is true for the electric waves emitted by the dipole. These also are "linearly polarized" to a great extent. Their electric intensity **R** always lies in a plane passing through the long axis of the dipole. We showed this previously by means of our movable receiver: the receiver failed to respond whenever its long axis was at right angles to that of the transmitter.

Hertz devised another very striking experiment on the polarization of the waves from a dipole, namely the so-called grid experiment. Transmitter and receiver are placed parallel to each other. A grid of metal wires about 1 cm. apart is then inserted between them. At first the wires are set at right angles to the axis of the dipole and to the direction of the field: this scarcely weakens the waves at all. The grid is then rotated through 90°; it now turns out to be quite opaque to the waves. The combined effect of the wires parallel to the direction of the field is like that of an impervious metal wall.

An exactly analogous experiment may be performed in optics. We must, however, use long invisible infra-red waves ($\lambda \sim 100\,\mu$). If we were to use the short waves of visible light ($\lambda \sim 0{\cdot}5\,\mu$) we should require wire grids of a fineness that cannot be produced in practice.

The experiments described here are merely an introduction to the subject. A tremendous development has followed. Electro-optics has taken its place on a level with classical optics. The identity of light with short electric waves is established beyond all doubt. Modern physical research is centred on the connection between the emission and absorption of light and the elementary electrical processes going on in the atom.

What is true of visible light is also true for the invisible kinds of light, e. g. X-rays, the ultra-violet, and the infra-red. All these consist of displacement currents, and they are distinguished from each other only by the numerical values of their wave-lengths, i. e. the distances between successive repetitions of the alternations in direction of the electric intensity **R**.

To the range of visible light belong the electric waves which have wave-lengths between $0{\cdot}8\,\mu$ and $0{\cdot}4\,\mu$. In bright sunshine their electric intensity amounts to several volts/cm. Our eyes still react to displacement currents in which the electric intensity only reaches about 3.10^{-6} volt/cm.

We previously mentioned 3.10^{-7} volt/cm. as the limit of sensitivity of a modern wireless receiver, i. e. an intensity about ten times as small as for the eye. At first sight the eye seems to come out of the comparison badly. This opinion, however, can no longer be maintained when we compare the dimensions of the eye with those of an antenna; for even the smallest antenna usual in broadcasting reception has linear dimensions about a hundred times as great as those of the pupil of the eye. Our eye is therefore a receiver for electric waves which can very well bear comparison with the most perfect products of modern electrical engineering.

This brief survey must suffice us. Once again it shows the overwhelming importance of the electric field in connection with our knowledge of nature.

MISCELLANEOUS EXAMPLES

The problems marked * require the use of the differential calculus.

1. A primary cell is connected in series with a 5-ohm resistance, the leads having a total resistance of 0·1 ohm. The cell has a pressure on open circuit of 1·3 volts and an internal resistance of 2 ohms. Find the current flowing in the circuit and the pressure which then exists between the terminals of the cell (p. 16).

2. Two resistances r_1 and r_2 are connected in parallel and a pressure E_a is applied to the combination. The resistances are then connected in series and a pressure E_b is applied. If the resistance r_1 is 2 ohms and the pressure E_b has to be four and a half times as great as E_a in order to make the total current the same in both cases, find the resistance r_2 (p. 16).

3. A certain ammeter has a resistance of 1 ohm, and the full scale deflection is obtained when a current of 0·05 ampere flows through the ammeter. Find what resistance must be connected in parallel with the ammeter (forming a "shunt") in order that the ammeter may read up to (*a*) 10 amperes (*b*) 100 amperes (p. 16).

4. A moving-coil voltmeter has a resistance of 1 ohm, and its maximum reading direct is 150 millivolts. How can this instrument be adapted to measure pressures up to (*a*) 15 volts, (*b*) 150 volts? (p. 16).

5. The conductor in fig. 58, p. 247 has a resistance r, and the voltmeter a resistance r_v, where r_v is small compared with r. How must the quotient E/i obtained by direct observation be corrected in order to give the resistance r accurately? (p. 16).

6. A battery has a terminal pressure of 2·0 volts per cell, which falls to 1·9 volts at the end of the discharge. The load consists of 500 incandescent lamps, each of 32 candle-power, taking 1·2 watts per candle-power. The pressure is kept constant at 250 volts. If the internal resistance of each cell is 5×10^{-3} ohms, find the number of cells in circuit (*a*) at the beginning of the discharge, (*b*) at the end of the discharge (p. 16).

7. Two condensers of capacity C_1 und C_2 are connected in series. What is their combined capacity C? If $C_1 = 10^{-3}$ microfarad (small Leyden jar), and $C_2 = 2$ microfarads (practical paper condenser), what is the value of C? (p.37).

8. A plate condenser of capacity 10^{-10} farad is connected in parallel with a charged electrostatic voltmeter. The pressure of the voltmeter is thereby caused to fall by 10 per cent. Find C_V, the capacity of the voltmeter (p. 37).

9. A variable condenser has semi-circular plates of radius 5 cm., 1·1 mm. apart. How many plates must it have in order that its capacity may reach 5×10^{-10} farad? (p. 38).

10. Suppose that the Leyden jar in fig. 9, p. 310 is replaced by a variable condenser and that in its position of maximum capacity (180° on its scale) it is in resonance with the exciter circuit on the right. If the condenser is then filled with a liquid of relative dielectric constant K', through what angle on the scale must the condenser be turned back in order to obtain resonance again? (p. 38).

11*. The plates of a condenser of capacity C are joined by a resistance of r ohms. How long does the pressure take to fall to half its original value? (p. 40).

12*. A neon lamp lights up at pressures varying between 160 and 220 volts. The current increases linearly with the pressure and the current-pressure graph if produced backwards cuts the x-axis at 157 volts. The increase of current is represented by the equation $i = (E - 157)/7{\cdot}63 \times 10^3$.
The condenser shown in fig. 18, p. 31 has a capacity of 50 micro-farads. How long would a neon lamp remain bright in the experiment illustrated? (p. 40).

13*. What must the capacity of a condenser be in order that the pressure may be reduced by 20 per cent in 10 seconds by a resistance of 10^{12} ohms? (p. 40).

14. When the model induction machine described on p. 51 makes 10 revolutions per second it gives a current of 10^{-8} ampere. There is a pressure of 220 volts between the condenser plates A and K, and the area of each plate is 10·5 sq.cm. What is the distance (d) between the plates A, K and the rotating plates α, β? (p. 67).

15*. A cylindrical condenser of length l consists of a hollow cylinder of internal radius R and a wire of radius r_0 placed along its axis, l being large compared with R. What is the capacity of the condenser? (p. 69).

16*. A cable contains a core of copper wire of cross-section 2 sq.mm. It is insulated by a layer of caoutchouc (relative dielectric constant 3) 5 mm. thick and is enclosed in an outer covering of lead. What is its capacity per kilometre? (p. 70: see solution of ex. 15).

17. A negatively-charged sphere of radius 1 cm. is situated well away from all the walls of the room. The pressure between it and the walls is 10^5 volts. How many electrons are there on one square centimetre of the surface of the sphere? (p. 70: see p. 85 for the charge on an electron).

18. On a certain day the electric intensity in the open was found to be 300 volts per metre. What was the quantity of electricity on one square metre of the earth's surface? (p. 70).

19. A direct current motor has an efficiency of 80 per cent. If it works at a pressure of 220 volts and is intended to lift a weight of 1·5 kg. through a height of 2 metres per second, what is the current required? (p. 74).

20. The anticathode of an X-ray tube (A in fig. 18, p. 213) consists of a hollow water-cooled cylinder. If 100 c.c. of water are evaporated per hour when the tube is working, and the current through the tube is 10 milliamperes, what is the working pressure of the tube? (p. 75: latent heat of evaporation of water = 539 calories per gram).

21. A plate condenser of capacity C is permanently connected with a source of current at pressure E. The distance between the plates is decreased to $1/n$ of its value. What is the effect on the energy of the condenser? (p. 79).

22. What must the radius of a water-drop be in order that it may float in the electric field of the earth (the intensity of which may be taken as 3 volts/cm.) with a charge equal to that of the electron? (p. 85).

23. A moving-coil galvanometer has a resistance of 100 ohms. When a condenser of capacity $^1/_{10}$ micro-farad charged to 10 volts is discharged through

it, it gives a ballistic deflection of 10 scale-divisions. How many volt-seconds correspond to a ballistic deflection of one scale-division? (p. 102).

24. The galvanometer in ex. 23 is to be used for measurements with a secondary coil *J* of resistance 100 ohms. How must the condenser be connected up? To what pressure must the condenser be charged to give a ballistic deflection of 10 scale-divisions? (p. 102).

25*. Suppose that the secondary coil *J* in fig. 4b, p. 102 is rectangular and 5 cm. broad, with 1000 turns, and that the primary coil used is as shown in fig. 5a, p. 104, with a current of 10 amperes flowing in it. With what velocity must the coil *J* be pulled out of the slit in the primary coil in order that a constant pressure of 0·01 volt may be produced during the motion? (p. 102).

26. A coil of cross-section 1000 sq.cm. is placed with its opening facing north and south (magnetic) and is then turned through an angle of 180° about a vertical axis. This causes the magnetic field of the earth (the horizontal component of which is 0·2 ampere-turns per cm.) to induce an integral pressure of 10^{-3} volt-second between the ends of the coil. How many turns must the coil have?
If the integral-pressure is 2·25 times as great when the same coil is placed horizontally and turned through 180° about a horizontal axis, what is the magnetic dip (the angle α between the earth's magnetic force and the horizontal)? (p. 104).

27*. Imagine the condensers in fig. 50b, p. 54 charged to 1000 volts. Let the plates of the left-hand condenser be kept at a constant distance apart. In the right-hand condenser, on the other hand, let the upper plate move backwards and forwards relative to the lower 100 times per second, so that the distance between the plates, which is 1 mm. in the state of rest, varies periodically by $\pm$ 0·5 mm. If the area of the plates is 100 sq.cm. and the pressure 500 volts, find the strength of the alternating current which flows in the condenser between the two condensers. (p. 117).

28. Let the switch in fig. 19, p. 118 be moved backwards and forwards 20 times per second. The total pressure available (E) is 200 volts. If the 5-watt incandescent lamp is to burn in the normal way, what must the capacity of the condenser be? (p. 118).

29. A conductor 1 ft. in length moves with a constant velocity of 30 ft. per second in a field of strength $\mathbf{H} = 7950$ ampere-turns per centimetre. The motion is at right angles both to the length of the conductor and to the field. What is the pressure in volts induced in the conductor? (p. 138).

30*. Suppose that the strip of metal in fig. 12, p. 137 is 10 cm. broad, and imagine it prolonged both ways and made into a flat circular cylinder of diameter 1 metre. The magnetic field **H** is to be supplied by a long coil 50 cm. long, with 1000 turns, carrying a current of 1·2 amperes. At what rate must the circular cylinder rotate about its axis in order to produce a pressure of 1 millivolt between *K* and *A*? (p. 138).

31. The inductance of a circuit is found to be 8 millihenries. Find the energy in foot-pounds stored up in the magnetic field when the current in the circuit is 5 amperes (p. 140).

32. The experiment in fig. 18, p. 31 is to be carried out with a condenser of 100 microfarads and a pressure of 220 volts. The same lamp is then to be used for the experiment of fig. 11, p. 135 and is to shine as brightly as during the discharge of the condenser in the former figure. The current interrupted in fig. 11, p. 135 is to be 10 amperes. What should the coefficient of self-induction of the coil be? (The resistance of the coil is assumed to be so small that the lamp remains quite dark when the current is flowing steadily) (p. 140).

33. A source of current is inserted in the circuit of fig. 14, p. 139. The turns of the coil have a diameter of 5 cm. and there are 20 turns on every 10 cm.

along the coil. What must the current in the coil be in order to lift a weight of 1 gm. previously at rest on a support? (p. 142).

34. A square wire frame with sides 10·2 cm. long is set parallel to the lines of force of a magnetic field in which the magnetic induction is 5×10^{-5} volt-sec./cm^2. A current of 10 amperes is flowing in the frame, and the latter is to be held in position by a lever 80 cm. long. What force must be applied at the other end of the lever? (p. 145).

35. If the magnetic field **H** in the interior of the liquid in fig. 52, p. 239 is regarded as uniform to a first approximation, and if the current is 8·2 amperes and the shortest distance between the electrodes 2 cm., what is the tangential pressure keeping the electrolyte in rotation? (p. 145).

36*. If the coil used in fig. 3a, p. 169 has an inductance of 50 henries, what must its resistance be if the current reaches half its final maximum value 10 seconds after it is switched on? (p. 169).

37. A pressure of effective value 150 volts is applied to an inductance of 0·3 henry. What will the effective current be if the frequency is 100 cycles per second? (p. 174).

38. For what frequency will a current of 12 amperes be taken from a 250-volt supply by a circuit having a resistance of 13·7 ohms and an inductance of 0·05 henry? (p. 174).

39. A non-inductive circuit has a resistance of 10 ohms. How much inductance must be added to the circuit in order that the effective value of the current may not exceed 2 amperes when a simple harmonic pressure of effective value 80 volts with a frequency of 50 cycles per second is applied to the circuit? (p.174).

40. If the self-induction of a circuit is 0·1 henry and its resistance 10 ohms, at what frequency will an increase of 1 per cent in the self-induction affect the impedance more than a decrease of 50 per cent in the resistance? (p. 174).

41. A direct current and an alternating current of 50 cycles per second flow simultaneously through a copper wire of resistance 5 ohms. i_0, the maximum value of the alternating current, is equal to i, the strength of the direct current, but is to be reduced to the value 0·1 i by coiling up the wire, say on an iron core. What must the inductance of the coil be? (p. 174).

42*. A wire wound in a coil has a higher resistance for alternating current than for direct current, owing to self-induction. The effect of self-induction is to be destroyed by inserting a capacity. If the frequency of the current is 1000 cycles per second, what must the capacity be? (p. 174).

43. The electrolytic iron whose hysteresis loop is drawn in fig. 9, p. 132 is to be used to construct a transformer like that in fig. 7a, p. 176. The current in the primary coil is to be 2 amperes. How high may the number of turns in the primary be without affecting the sine-form of the alternating pressure in the secondary circuit? (p. 176).

44*. An electricity carrier is at rest before being subjected to an electric field. According to what law does the velocity of the carrier rise from zero to its full value when the field is switched on? (p. 200).

45. A tungsten lamp requires about 0·8 watts per candle-power (Hefner). How many electrons must impinge on the anode a in fig. 16, p. 208 per second, the pressure being 1000 volts, in order that a may shine as brightly as a 10-candle-power tungsten lamp? (It is assumed that the radiation and conduction losses are the same for the anode and the lamp filament) (p. 208).

46. Electrons enter a uniform magnetic field at right angles to the lines of force and are deflected into a circular path. What must the value of the magnetic

intensity **H** be in order that they may traverse a complete circle in 10^{-8} seconds? (p. 211).

47. A beam of cathode rays is emitted by a Wehnelt cathode, the surface of which is vertical and faces east and west. The magnetic field of the earth deflects the rays into a circular path with a radius of curvature of 1 metre. At the place of observation the intensity of the earth's magnetic field is 0·2 ampere-turns per cm. What is the velocity of the cathode rays, and what cathode fall of potential is required to produce them? (p. 221).

48. If 96540 coulombs of electricity liberate 1 gm. of hydrogen, and if the chemical equivalent of copper (cupric) is 31·65, find the number of hours for which a current of 4000 amperes will have to flow through a solution of copper sulphate in order to deposit one ton of copper (p. 234).

49. In calibrating an ammeter by means of a copper voltameter it was found that the gain plate increased in weight by 5·9 gm. per hour. The average reading of the ammeter under test was 4·5 amperes. The electrochemical equivalent of copper (cupric) is 0·328 mgm. per coulomb. What is the percentage error of the ammeter? (p. 234).

50. In the experiment of fig. 19, p. 31 water is decomposed by the discharge of a condenser. If the condenser is charged to a pressure of 220 volts, what must its capacity be in order that 10 c.mm. of hydrogen may be produced? (p. 234: 1 c.mm. of hydrogen at N.T.P. weighs $9{\cdot}0 \times 10^{-8}$ gm.).

51. The zinc plate of the Daniell cell in fig. 18, p. 272 weighs 100 gm. What is the maximum quantity of electricity obtainable from this cell? (p. 234: the atomic weight of zinc is 65·4 and its valency 2).

52. Two platinum electrodes dip into a vessel of copper sulphate solution. Ordinary alternating current at 50 cycles per second and effective current 10 amperes is passed through the solution for 10 minutes. What is the maximum amount of copper which will be found on one of the electrodes at the close of the experiment? (p. 234).

53. If the strip of filter-paper in fig. 48, p. 236 is 50 cm. long and 5 cm. broad and if when uniformly wetted with a solution of common salt (5 per cent by weight) it becomes 0·1 gm. heavier, what is the resistance of the wet strip in ohms, assuming that the density of the salt solution is unity? (p. 236: for the specific resistance, see Table V).

54. In fig. 52, p. 239 the outer cylinder (A) has a diameter of 10 cm. and the inner (K) of 8 cm. The liquid is 1 cm. deep and contains 20 per cent of zinc sulphate by weight. Neglecting polarization at the electrodes, what is the approximate value of the current between K and A? (p. 239: for the specific resistance, see Table V).

55. What quantity of water will be brought to the boiling-point in 15 min. in an electric kettle taking 4 amperes on a 250-volt circuit, if the efficiency of the kettle is 85 per cent and the initial temperature of the water is 12° C? (p. 241: 1 calorie = 4·184 joules).

56. Two resistances, r_1 und r_2, are connected in series and a pressure of 10 volts is maintained continuously between the ends. It is found that the heating of the resistance r_2 is three times that of r_1. If the resistance of r_1 is 2 ohms, find the current flowing in the circuit.

The two resistances are now connected in parallel and the same pressure is applied across them. Find the current through each resistance. What is now the ratio of the heat developed in r_2 to that developed in r_1? (p. 241).

57. The field winding of a shunt motor has a resistance of 150 ohms at 15° C. The motor is started and after it has been running for some hours the field

current is found to be 1·4 amperes. The motor is connected to a 250-volt supply. What is the average temperature of the field coils if the temperature coefficient of the resistance is ·004 per degree C? (p. 247).

58. Constantan wire 0·01 mm. in diameter is to be wound so as to make a resistance of one megohm (10^6 ohms). How many metres of wire are required? (p. 248).

59. If the copper U in fig. 13a, p. 269 is 20 cm. long and has a cross-section of 1 sq.cm., and the constantan block is 3 cm. long and has a cross-section of 3 sq.cm., and if the difference of temperature between the points marked 1 and 2 in the figure is 500° C., what is the current in amperes? (p. 269).

60. In the experiment shown in fig. 15, p. 295 the initial value of the current is 5.10^{-9} ampere. How many α-rays must initially be emitted per second by the disintegrating emanation? (The range of the α-rays is about 5 cm.; cf. p. 282) (p. 285).

SOLUTIONS AND ANSWERS TO EXAMPLES

1. The current $= 1{\cdot}3/(2 + 5 + 0{\cdot}1) = {\cdot}183$ ampere. The pressure between the terminals of the cell = current × *external* resistance $= 0{\cdot}934$ volt.

2. The effect of connecting two resistances r_1, r_2 in parallel may be found as follows. Let I be the total current, i_1, i_2 the currents passing through the resistances r_1, r_2 respectively, and R the resistance of the combination. Then $I = i_1 + i_2$, and $E_a = RI = r_1 i_1 + r_2 i_2$. Hence

$$R = \frac{r_1 i_1}{i_1 + i_2} = \frac{r_1 i_1}{i_1 + r_1 i_1/r_2} = \frac{r_1 r_2}{r_1 + r_2};$$

or, in words, when resistances are connected in parallel the reciprocal of the combined resistance is equal to the sum of the reciprocals of the separate resistances. (This may be shown to hold for any number of resistances in parallel.) Using this formula, we find that $r_2 = 1$ ohm or 4 ohms.

3. (*a*) ·00503 ohm; (*b*) ·0005003 ohm.

4. Put (*a*) 99 ohms; (*b*) 999 ohms in series with the voltmeter.

5. The current E/r_v flows through the voltmeter, so that the current flowing through the conductor is $i - E/r_v$; the resistance of the conductor is therefore $\dfrac{E}{i - E/r_v}$.

6. (*a*) The number of watts required $= 500 \times 32 \times 1{\cdot}2$; hence the current in the circuit $= 500 \times 32 \times 1{\cdot}2/250$ amperes $= 64 \times 1{\cdot}2$ amperes, and the total resistance of the lamps $= 250/(64 \times 1{\cdot}2)$ ohms. The internal resistance of x cells in series $= 5\,x/1000$ ohms, and the pressure furnished by them is $2\,x$ volts. Hence

$$\frac{250}{64 \times 1{\cdot}2} + \frac{5\,x}{1000} = \frac{2\,x}{64 \times 1{\cdot}2} : x = 155\,.$$

(*b*) By a similar calculation, the number of cells required = 165.

7. The amounts of electricity on the four condenser plates are all the same. Hence we have $E_1 C_1 = E_2 C_2 = EC$, and the total pressure $E = E_1 + E_2$. Combining these equations, we have $1/C = 1/C_1 + 1/C_2$; or, in words, when condensers are connected in series the reciprocal of the combined capacity is equal to the sum of the reciprocals of the separate capacities. (This may be shown to hold for any number of capacities in series.) Using this formula, $C = {\cdot}99 \times 10^{-3}$ microfarad in the numerical case.

8. Including the condenser makes no difference to the total quantity of electricity. Hence $Q = C_V E = 0{\cdot}9\,E(C_V + 10^{-10})$, whence $C_V = 9\,.\,10^{-10}$ farad.

9. The area of a plate $= 25\,\pi/2 = 39{\cdot}3$ sq.cm. Let there be n plates. Equal numbers of lines of force proceed from either side of each inside plate, so that each of the latter must be counted twice. By equation (6), p. 69, $C = KA/d$, so that $\dfrac{8{\cdot}84\,.\,10^{-14}\,.\,2\,n\,.\,39{\cdot}3}{0{\cdot}11} = 5\,.\,10^{-10}$, whence $n = 8$. Thus in all there are $(2\,n + 1) = 17$ plates.

10. The capacity is multiplied by K' when the liquid is poured in. Now the capacity of a variable condenser with semi-circular plates is approximately

proportional to the reading on its scale, as the effective area is proportional to the reading. If the latter was 180° at first we have to set the condenser to $(180/K')°$ to obtain the same capacity as before.

11. The charge on a condenser is given by $Q = EC$. If the amount of electricity dQ flows away in time dt, the change in the pressure is $dE = -dQ/C$. Further, the current i is equal to dQ/dt, and by equation (1), p. 16, $E = ir = rdQ/dt$. Hence $-\frac{dt}{rC} = \frac{dE}{E}$: integrating, $-t/rC = \log_e E + C'$. Let the pressure at time $t = 0$ be E_0; then $C' = -\log_e E_0$ and we have

$$-t/rC = \log_e E/E_0\,, \quad \text{or } E = E_0\,e^{-t/rC}\,.$$

The product rC has the dimensions of time and gives the time taken by the charge to sink to $1/e$ of its original value. For the particular problem stated we put $E_1 = E_0/2$, and we have

$$\frac{1}{2} = e^{-t_1/rC}, \quad \text{or } t_1 = rC\log_e 2\,.$$

As a numerical example we may take $C = 10$ microfarads $= 10^{-5}$ farad and $r = 10^5$ ohms; since $\log_e 2 = 0\cdot69$, this gives $t_1 = 0\cdot69$ seconds.

12. As in last example, we have $dQ/dt = i = -CdE/dt$. Hence if E_g is the pressure at which the current vanishes,

$$E - E_g = ir = -CrdE/dt, \quad \text{or } Cr\frac{dE}{dt} + E - E_g = 0\,.$$

As may be verified by substitution, the general solution of this differential equation is of the form

$$E = Ae^{-t/rC} + E_g\,.$$

Let $E = E_0$ at time $t = 0$; then $A = E_0 - E_g$, and $E = (E_1 - E_g)\,e^{-t/rC} + E_g$. The lamp goes out when E falls to 160 volts; hence

$$160 = (220 - 157)e^{-t_1/rC} + 157$$

and $rC = 0\cdot38$, so that $t_1 = 0\cdot38\log_e 21 = 1\cdot15$ seconds.

13. We have $E = E_0e^{-t/rC}$ (see solution of ex. 11). Substituting $r = 10^{12}$, $t = 10$, $E = 0\cdot8\,E_0$, we have $C\log_e\frac{1}{0\cdot8} = 10^{-11}$, whence $C = 4\cdot5 \times 10^{-11}$ farad.

14. The quantity of electricity induced during one revolution of the plates is equal (p. 67) to $A\mathbf{D}$, where A is the total area of the movable plates, 21 sq.cm. in our case. By equation (3), p. 67, $\mathbf{D} = K\mathbf{R}$, and $\mathbf{R} = 220/2\,d$. The machine gives 10^{-8} coulombs per second, i. e. 10^{-9} coulombs per revolution. To determine d we thus have the equation $10^{-9} = 21.8\cdot84.10^{-14}.220/2\,d$, whence $d = 0\cdot21$ cm.

15. As l is large compared with R, we may assume that the lines of force are accurately radial in direction, i. e. we neglect the stray field at the ends of the condenser. By p. 67, $\mathbf{R}_0$, the intensity of the surface of the wire, is equal to $\mathbf{D}/K = Q/KA = Q/K.2\pi r_0 l$. We now imagine a very thin cylindrical surface of metal placed at a distance r from the wire. This does not alter the field in any way, as provided the surface is sufficiently thin the lines of force are not shortened and remain radial in direction. As all the lines of force end on the metal surface, a charge Q is induced on its inner surface. We can now write down the intensity. As in the case above, $\mathbf{R} = Q/K.2\pi rl$ for any point in the cylindrical condenser at a distance r from its axis. By equation (4a), p. 68,

$$E = \int_{r_0}^{R}\mathbf{R}\,dr = \frac{Q}{2lK\pi}\int_{r_0}^{R}\frac{1}{r}\,dr = \frac{Q}{2lK\pi}\log_e\frac{R}{r_0} \text{ and } C = \frac{Q}{E} = \frac{2lK\pi}{\log_e R/r_0}\,.$$

16. Using the result of ex. 15 and substituting $l = 10^5$ cm., $K = 3.8\cdot84.10^{-14}$, $x_0 = 0\cdot1$ cm., $R = 0\cdot6$ cm., we have $C = 9\cdot3.10^{-8}$ farad.

17. $5\cdot5.10^{10}$ electrons.

18. $2\cdot65.10^{-9}$ coulombs.

19. 0·17 ampere.
20. 6300 volts.
21. The capacity, and hence also the energy, is multiplied by n.
22. $1{\cdot}06.10^{-5}$ cm.
23. 10^{-5} volt-second.

24. Condenser, resistance, and galvanometer must be connected in parallel. Then half the electricity flows away through the resistance, so that the galvanometer deflection is half as great as before, by Ohm's law: the condenser must therefore be charged to 20 volts.

25. As the primary coil has 20 turns per cm. the intensity of the magnetic field inside it is 200 ampere-turns per cm. By equation (1 b), p. 106, the induced pressure is given by

$$E = \frac{d}{dt}(\mu n A \mathbf{H}) = \mu n \mathbf{H}\frac{dA}{dt}.$$

Let the required velocity be v; then $dA/dt = 5\,v$, so that

$$0{\cdot}01 = 1{\cdot}256.10^{-8}.10^3.200.5v, \text{ or } v = 0{\cdot}8 \text{ cm. per sec.}$$

26. In the successive positions of the coil the lines of force enter it first from the one side and then from the other; thus two integral pressures occur in immediate succession, each of magnitude $\mu n A.0{\cdot}2 = 1{\cdot}256.10^{-8}.n.10^3.0{\cdot}2$, and this must be equal to $0{\cdot}5.10^{-3}$ volt-second, so that $n = 200$.

In the first experiment it is the horizontal component of the earth's field that we measure, in the second it is the vertical component. These are combined by the parallelogram law to give the resultant magnetic force: we have $\tan\alpha = 2{\cdot}25$, so that $\alpha = 66°$.

27. We have $Q_1 = C_0E$ and $Q_2 = CE$. If we neglect the resistance of the conductor, the pressure E is the same in both condensers. Further, the total quantity of electricity is constant.

$$Q_1 = Q = 2\,C_0E_0 = C_0E + CE; \ \therefore 2\,E_0 = E\,(1 + C/C_0)\,. \qquad (1)$$

Calculating C_0 from equation (6), p. 69, we have $C_0 = K\,A/d = 8{\cdot}84.10^{-14}.10^2/0{\cdot}1 = 8{\cdot}84.10^{-11}$ farad.

Let the motion of the plate of the variable condenser be simple harmonic. Then

$$C = K\frac{A}{0{\cdot}1 + 0{\cdot}05\sin\omega t} = C_0\frac{1}{1 + 0{\cdot}5\sin\omega t},$$

and, by equation (1), above,

$$2\,E_0 = E\left(1 + \frac{1}{1 + 0{\cdot}5\sin\omega t}\right)$$

$$\text{or } E = \frac{2\,E_0}{1 + \dfrac{1}{1 + 0{\cdot}5\sin\omega t}}.$$

Differentiating with respect to t,

$$\frac{dE}{dt} = \frac{\omega.2\,E_0.0{\cdot}5\cos\omega t}{\left(1 + \dfrac{1}{1 + 0{\cdot}5\sin\omega t}\right)^2(1 + 0{\cdot}5\sin\omega t)^2} = \frac{E_0\,\omega\cos\omega t}{(2 + 0{\cdot}5\sin\omega t)^2}.$$

For the fixed condenser $dQ_1/dt = i = C_0dE/dt$,

$$\therefore i = \frac{E_0\,\omega\cos\omega t}{(2 + 0{\cdot}5\sin\omega t)^2} = \frac{C_0E_0\,\omega\cos\omega t}{(2 + 0{\cdot}5\sin\omega t)^2}.$$

The maxiumm value of this expression, as may be verified graphically, is

$$i_0 = \frac{C_0E_0\omega}{\sqrt{2}}.1{\cdot}12 = \frac{8{\cdot}84.10^{-14}.100}{0{\cdot}1}.10^3.2\pi.10^2\frac{1{\cdot}12}{1{\cdot}4} = 4{\cdot}4.10^{-5} \text{ ampere.}$$

28. The energy of the charged condenser is $\frac{1}{2}CE^2$, and the condenser is charged and discharged 40 times per second. Hence energy flows through the lamp at the rate of $2.40.0{\cdot}5\,C.100^2$ watts. This must be equal to 5; hence $C = 1{\cdot}25.10^{-5}$ farad.

29. $2{\cdot}78$ volts.

30. $\mathbf{H} = 1000.1{\cdot}2/50 = 24$ ampere-turns per cm. Let U be the velocity of the strip, n the number of revolutions per second, and A its area. Then $dA/dt = 10u = 10.2\,\pi rn$, and $E = \mu\mathbf{H}\,dA/dt = 1{\cdot}256.10^{-8}.24.10.2.50\,\pi n$, which must be equal to 10^{-3} volt; hence $n = 1{\cdot}06$.

31. $0{\cdot}0738$ ft.-lb.

32. The energy of the magnetic field in the second experiment must be equal to that of the charged condenser in the first, i. e. $\frac{1}{2}Li^2 = \frac{1}{2}CE^2$: hence $L = 4{\cdot}85.10^{-2}$ henry.

33. The strength of the magnetic field is $20\,i/10 = 2\,i$ ampere-turns per cm., and the cross-section of the coil is $\pi\,(2{\cdot}5)^2 = 20$ sq.cm. By equation (8), p. 142, the force exerted (in kg.) is $10{\cdot}2.\frac{1}{2}\mu\mathbf{H}^2A = 10{\cdot}2.\frac{1{\cdot}256}{2}.10^{-8}.4\,i^2.20$: this is to be equal to 1 gm., hence $i = 14$ amperes.

34. $6{\cdot}6.10^{-3}$ kilograms weight.

35. $1{\cdot}05.10^{-6}\,\mathbf{H}$ kg./cm^2.

36. The induced pressure E_i is equal to $L\,di/dt$ (p. 173) and is in the direction opposite to that of the external pressure E. Thus Ohm's law becomes

$$E - L\frac{di}{dt} = ir\,.$$

The general solution of this differential equation (see solution to ex. 12) is of the form

$$i = Ae^{-rt/L} + E/r\,.$$

At time $t = 0$, $i = 0$, so that $A = -E/r$ and

$$i = \frac{E}{r}\left(1 - e^{-rt/L}\right).$$

The current attains half its final value when

$$\frac{1}{2} = 1 - e^{-rt/L} = 1 - e^{-10r/50}\,.$$

Hence $r = 5\log_e 2 = 3.47$ ohms.

37. $0{\cdot}795$ ampere.

38. 50 cycles per second.

39. $0{\cdot}123$ henries.

40. $97{\cdot}5$ cycles per second and over.

41. $0{\cdot}16$ henry.

42. By pp. 174 –5 the pressures at the capacity and inductance differ in phase by 180°, and thus cancel each other if of equal magnitude. The pressure between the ends of the inductance is

$$E_i = 2\,\pi nLi\,.$$

We now write down the relationship between the current i and E_c, the pressure between the ends of the condenser. The condenser equation which we have often used already gives

$$i = -C\,dE_c/dt\,.$$

Let
$$E_c = E_0 \sin 2\,\pi nt:$$

then $i = -C.2\,\pi n\cos 2\,\pi nt$, and (apart from the difference of phase) the equation connecting the current and pressure is

$$E_c = \frac{i}{2\,\pi n C}\,.$$

Thus the effect of the inductance is destroyed if

$$2\,\pi n L = \frac{1}{2\,\pi n C}$$

$$\text{i.e.}\quad C = \frac{1}{4\,\pi^2 n^2 L}\,,$$

43. To avoid affecting the simple harmonic nature of the pressure in the secondary we have to confine ourselves to the straight part of the hysteresis loop i. e. to intensities below about 3 ampere-turns per cm. Alternating currents used in practice may usually be expressed by $i = i_0 \sin \omega t$, where i_0 is the maximum current. Mean values are often used as well, namely

(1) the mean current

$$i_m = \frac{\omega}{\pi}\int_0^{\pi/\omega} i_0 \sin \omega t\, dt = \frac{2}{\pi} i_0 = 0{\cdot}64\, i_0\,,$$

and (2) the effective or root mean square current

$$i_{\text{eff.}} = \left[\frac{\omega}{\pi}\int_0^{\pi/\omega} i_0^2 \sin^2 \omega t\, dt\right]^{\frac{1}{2}} = \frac{i_0}{\sqrt{2}} = 0{\cdot}71\, i_0\,.$$

The mean current is used where i is involved, e. g. in electrolytic processes, the effective current where i_2 is involved, e. g. in processes where heat is developed.

In our case the effective current is 2 amperes, so that the maximum value of the current is $2/0{\cdot}71 = 2{\cdot}8$ amperes. Then

$$2{\cdot}8 n = 3 \text{ ampere-turns per cm.}$$

$$\therefore\ n = 1{\cdot}1 \text{ turns per cm.}$$

44. Let $\mathbf{R}$ be the intensity of the field, a the ratio of the force of friction to the velocity, s the distance traversed by the electricity carrier, e its charge and m its mass. The force on the carrier is then given by

$$e\mathbf{R} = a\frac{ds}{dt} + m\frac{d^2 s}{dt^2}\,. \tag{1}$$

The solution of this differential equation is formed by adding a particular integral to the solution of the equation

$$\frac{ds}{dt} + \frac{m}{a}\frac{d^2 s}{dt^2} = 0\,,$$

which is
$$s = A e^{\lambda t} + C\,,$$

where
$$1 + m\lambda/a = 0\,,$$

. e.
$$s = A e^{-at/m} + C\,.$$

iA particular integral of equation (1) is easily found by substitution, e.g. $e\mathbf{R}t/a$ and the complete solution of equation (1) is then

$$s = A e^{-at/m} + C + e\mathbf{R}t/a\,.$$

Differentiating,

$$v = \frac{ds}{dt} = -A\frac{a}{m}e^{-at/m} + \frac{e\mathbf{R}}{a}\,.$$

At time $t = 0$, $v = 0$: hence $A = me\mathbf{R}/a^2$, and

$$v = \frac{e\mathbf{R}}{a}\left(1 - e^{-at/m}\right).$$

The final velocity $e\mathbf{R}/a$ is only reached after an infinite length of time.

45. 5.10^{16} electrons.

46. 28·3 ampere-turns per cm.

47. Use equation (4), p. 210: $4{\cdot}43.10^{-8}$ cm./sec.: 55 volts.

48. 215 hours.

49. 9·95 per cent.

50. 395 microfarads.

51. 296,000 coulombs.

52. The mention of the time is deliberately misleading, the quantity of copper deposited in one half-second being redissolved in the next. The essential point is at what instant the current is switched on or off. In the most favourable case the amount of copper will be that corresponding to the quantity of electricity which flows during a half-period of 10^{-2} second. To calculate the number of coulombs passing in a half-period we have to multiply the mean current (see solution of ex. 43) by the length of the half-period: $i_m = 0{\cdot}90\, i_{eff}$. Taking a gram-equivalent of copper (cupric) as 32, the mass of copper deposited is

$$\frac{32}{9{\cdot}65.10^4}.10.0{\cdot}9.10^{-2} = 3.10^{-5} \text{ gm.}$$

53. $3{\cdot}75.10^5$ ohms.

54. Taking the mean cross-section of the path of the current as $18\,\pi = 57$ sq.cm. we find 8·2 amperes for the current.

55. 2·08 kg.

56. 1·25 amperes: 5 amperes, 5/3 amperes respectively: 1/3.

57. If r_x, r_0 are the new and original resistances respectively, t the rise of temperature, and α the temperature coefficient of resistance, $r_x = r_0\,(1 + \alpha t)$: 63° C.

58. 160 metres.

59. The mean length of path of the current in the constantan is 3 cm. and in the copper may be taken to be 41 cm. Using data from Table VII, p. 248, the resistance of the constantan block is $4{\cdot}9.10^{-5}$ ohm and that of the copper rod $7{\cdot}0.10^{-5}$ ohm. By Table VIII, p. 269, the thermo-electric pressure is $26{\cdot}3.10^{-3}$ volt: hence the required current is 220 amperes.

60. Each α-ray gives rise to 20,000 ions per centimetre of its path (p. 292), and all the ions reach the electrodes. Let n be the number of α-particles emitted per second, then the quantity of electricity supplied per second is $n.5.20{,}000$ $.2.1{\cdot}6.10^{-19}.2$ coulombs (the reason for the last factor 2 is explained by the note on p. 296). This must be equal to 10^{-9} coulombs: hence $n = 1{\cdot}6.10^4$.

INDEX

Accumulator, 275.
α-rays, 287, 292, 296.
—, range of, 287, 293.
—, variations in emission with time, 297.
Alternating current, 116, 304.
—, commutator motor for, 164.
—, effective or root mean square value of, 117.
—, frequency of, 117.
—, instantaneous and maximum values of, 117.
—, production of high frequency, 304, 307.
—, production of very low frequency, 116, 183.
—, properties of high frequency, 186—193.
—, reason why used in preference to direct current, 177.
—, simple harmonic, 116, 149.
Alternating current generator with external poles, 149.
— with internal poles, 152.
— with no coils in the armature, 153.
Alternating current synchronous motor, 160.
Ammeter, 9: see also under Galvanometer.
—, hot-wire, 11, 12.
—, inertia of moving-coil, 117.
—, model of soft-iron, 5, 9.
—, moving-coil, with mechanical pointer, 9, 11, 18, 19.
—, moving-coil, with optical lever, 9, 11.
—, re-graduation as voltmeter, 16.
Ampere, definition of, 12.
Ampere-second (coulomb), 34, 36.
Ampere-turns, 92, 107.
— per centimetre, 92.
—, use of iron to increase, 94.
Amplifier, 214, 292.
Amplitude, 165, 304, 313.
Angular momentum, 95.
Angular velocity, 140.
Anode, 195.
Antenna, 25, 232.
Anticathode, 212, 221.
Arc, electric, 227.
Armature, 149.
—, drum, 150, 161.
—, ring, 150.
Armature (keeper), 128.
Aston, 216.
Atomic weight, 211, 224.
— of the electron, 210.
Atomic ions, 60, 235.
Atoms of electricity, in equilibrium on the surface of conductors, 42.
—, in neutral bodies, 47.
—, in or upon insulators, 56.
—, occurrence in pairs, 48.
Attraction between turns of a coil, 139.
— between two condenser plates, 75, 76.
— between two conductors, 5, 8.
— between two electric charges, 79.
— between two magnetic poles, 141.
Avogadro's number, 234, 293.

Ballistic deflection (fling), 30.
Barlow's wheel, 247.
Becquerel, 285.
β-rays, 288.
Biot and Savart's law, 112.
Blow-out for current interrupters, 180.
Bolometer, 251.
Brownian movement, 20.
Brush discharge, 226.

Canal rays, see Positive rays.
Capacity, 37.
— of an aluminium cell, 243.
— of a plane condenser, 69.
— of a sphere, 70.
Capillary electrometer, 279.
Cataphoresis, 198.
Cathode, 195, 225.
—, incandescent, 208.
Cathode fall of potential, 217.
Cathode glow, 216.
Cathode rays, 209, 216.
—, deviation in an electric field, 221.
—, finding the charge on, 218.
—, kinetic energy of, 209.
—, path in a uniform magnetic field, 210, 221.

Cathode ray oscillograph, 220.
Cell, concentration, 272.
—, Daniell, 272, 275.
—, dry, 276.
—, Meidinger, 272.
—, reversible, 275.
—, standard, 15, 277.
—, unpolarizable, 276.
Characteristic, falling, 230, 245.
— of an arc, 229, 230.
— of a dynamo, 152.
— of a valve, see Valve.
Choking coil, 174, 324.
Circuit of audible frequency, 307.
Circularly-polarized vibration, 165.
"Classical" theory of electricity, 123.
Coehn's law of charging, 62.
Commutator, 149.
— brushes, 152.
— motor for alternating current, 164.
Compass-needle, 3, 91.
Concentration cell, 272.
Condensation tracks, 289.
Condenser, 24.
—, adjustable, 38, 306.
—, change of energy when work is done, 79.
—, cylindrical, 199.
—, plane, 20, 37, 69.
—, paper, 31.
—, spherical, 69.
Condenser telephone, 77.
Conductance, specific, see Conductivity.
Conduction, dependent and independent, 196.
—, dependent, in a high vacuum, 208.
—, dependent, in insulating crystals, 258.
—, dependent, in liquids, 246.
—, dependent, through air, 198, 202.
—, electrolytic, see Electrolytic Conduction.
—, electronic, see Conduction in metals.
—, independent, in gases and vapours, 215, 225.
—, independent, in solid non-metals, 256.
— in liquids, 230.
— in metals, 246.
— in metals, effect of temperature on, 250.
—, mechanism of, 194, 259.
—, spontaneous, in air, 207.
Conduction current, 115, 122, 321.
— circulating without loss of energy, 140, 253.
— due to the decay of an electric field, 30, 35.
— model experiments on, 195.
Conductivity, electrical and thermal, 254.
— of electrolytes, 237.
— of highly concentrated electrolytes, 239.
— of metals, 248.
Conductor (in the old-fashioned sense), 25, 28.
Conductor, 29.
— of the first class, 268.
— of the second class, 271.
Conductor traversed by current, deviation by a magnetic field of, 143.
—, magnetic field of, 3, 5, 7.
Constriction of conductors by their own magnetic field, 249.
— of lines of electric force, 330.
Contact pressure between metal and electrolyte, 271.
— between two metals, 63, 267.
Continuous current, same as Direct current, 116.
Convection current, 197, 210, 260, 265.
Coolidge tube, see X-ray tube.
Cosmic radiation, ions due to, 207.
Coulomb (ampere-second), 37.
Coulomb's law, 79.
Couple on coil traversed by a current, 145, 161.
Crookes, 209, 285, 289.
Crookes dark space, 216, 217, 218.
Crystal detector, 154, 189, 284, 317.
Curie, 285.
Current, alternating, see Alternating current.
—, beginning of, 169.
—, conduction, see Conduction current.
—, convection, see Convection current.
—, direct, 116.
—, direction of, 100, 115.
—, displacement, see Displacement current.
—, dying-away of, 169.
—, eddy, see Eddy currents.
—, effective, 117.
—, heating effect of, 5, 240.
—, invariably closed, 119, 315.
—, physiological effect of, 18, 241.
—, properties of, 3.
—, root mean square, 117.
—, time-integral of, 34.
—, wattless, 161, 172.

Daniell cell, 272, 275.
D'Arsonval galvanometer, same as Moving-coil galvanometer.
Dead point of a motor, 161.
Decay of an electric field, 29, 30, 194, 303.
—, caused by radioactivity, 283.

Diamagnetic substances, 106, 146, 159.
Diamond, photo-electric conduction of, 258.
Dielectric, 38, 119.
Dielectric constant, 38, 57.
—, absolute, 67, 69.
Dipole, electric, 311, 313.
Direct current, 116.
Direct current generator, 149.
— with drum armature, 150.
Direct current motor, 161.
Discharge from points, 71, 225.
Disintegration of atoms, artificial, 299.
—, radioactive, 294.
Displacement, 67, 119.
—, rate of change of, 120.
Displacement current, 118, 119, 186, 188, 197, 307, 313, 316, 322, 329, 335.
—, magnetic field of, 121, 323.
—, measurement of, 120.
—, numerical examples of, 121, 188.
— of the dipole, 323.
Displacement-density, 63, 67, 119.
Dry cell, 276.
Dynamo, 151.
—, characteristic of, 152.
—, compound-wound, 152.
—, series-wound, 152.
—, shunt-wound, 152.

Eddy currents, 158.
—, damping of vibrations by, 158.
— within molecules, 159.
Electrical counter, Geiger's, 292.
Electrical equivalent of heat, 75.
Electric charge, 28.
— of a condenser, 37.
Electric current, see Current.
Electric endosmose, 264.
Electric field, 21.
—, forces in, 71.
— in the boundary layer between gases and liquids, 279, 280.
— in the boundary layer between metals and electrolytes, 271.
— in the boundary layer between two metals, 266.
—, intensity of, see Electric Intensity.
— of a moving charge, 300.
— rate of change of, 121, 122.
— used to remove dust, 202.
Electric intensity, 63, 66, 227, 319.
—, calculation for various fields, 69.
— in metals, 246, 250.
— in sunlight, 335.
—, line-integral of, 68, 115.
Electric motor, see Motor.
Electric osmosis, 264.
Electric screening, 49.
Electric substances, 28.
—, measurement of, 32.
Electric waves, 318.
—, eye as a receiver for, 335.
—, free, 323.
—, half-free, 331.
— in wires, 315.
—, measurement of velocity of, 321.
—, refraction of, 334.
Electric whirl, 71.
Electric wind, 225.
Electricity carriers, 25, 28, 36, 72, 73, 83, 195.
—, "fall" of, 209, 211.
—, measurement of charge on, 36, 83.
—, visible, 197, 199.
Electrodes, 195.
—, unpolarizable, 276.
Electrodynamometer (galvanometer with a fixed coil and a moving coil), model of, 8 (fig. 9, c, d).
Electrolysis of dilute sulphuric acid, 6, 10.
Electrolyte, 230.
—, conductivity of, 237, 239.
Electrolytic break, 178.
Electrolytic conduction, effect of temperature on, 237, 244.
— in fused salts and glasses, 243.
— in liquids, 231.
Electrolytic dissociation, 232, 243.
— of pure water, 245.
Electromagnet, iron-clad, 142.
Electromagnetic field, 114, 331.
Electromagnetic induction, experiments on, 77, 149.
—, law of, 104.
—, more thorough-going interpretation of, 114.
—, relativity of phenomena of, 113, 138.
— with high frequency alternating current, 186.
Electrometer, 14.
—, capillary, 279.
—, quadrant, 82.
—, unifilar, 81, 82.
Electromotive force, 1.
Electrons, 29, 61, 208.
—, atomic weight of, 210.
—, observation of single, 289.
—, thermal emission of, 282.
Electronic conduction, see Conduction in metals.
Electroplating, 199, 242.
Electroscope (ungraduated electrometer), 15.
Electrostatic induction, 46.

Electrostatic induction, distortion of field in, 46.
— in insulators, 59.
Electrostatic voltmeter, see Electrometer.
Elementary quantity of electricity, 85, 224.
Elliptically-polarized vibration, 165.
Energy, "electric", 141.
— of an electron in "volts", 282.
— of the electric field, 75.
— of the magnetic field, 139, 140, 300.
Equivalents, Faraday's law of, 233, 234, 237.
Exciter, 309, 319.
Extra current, 135.

Farad (ampere-second/volt), 38.
Faraday, 101, 148, 233.
Faraday box or cage, 49, 51.
Faraday dark space, 216, 225.
Feddersen sparks, 185.
Ferromagnetic substances, 131, 133.
Field strength, see Electric intensity, Magnetic intensity.
Flame collector, 65.
Fleming, J. A., 213.
Fling (ballistic deflection) of a galvanometer, 30.
Fluor spar, pleochroic haloes (caused by α-rays) in, 298.
Forces in the electric field, 71.
— in the magnetic field, 141.
— on uncharged bodies, 80.
Forest, Lee de, 213.
Frequency of an alternating current, 117.
Friction between electrons and metal, 247, 260.
—, internal, 239, 266.
Frictional electricity, 61, 84.
Foucault currents, see Eddy currents.

Galvanometer (sensitive ammeter), 9; see also under Ammeter.
—, graduation in ampere-seconds, 34.
—, graduation in volt-seconds, 102.
—, limit of sensitivity of, 20.
—, loop, 9, 12.
—, mirror, 9, 19, 30, 36.
—, moving-coil, see under Ammeter.
—, string, 7, 8.
γ-rays, 287.
1 gauss = 0·795 ampere-turns/cm.
Geiger's counting apparatus, 292.
Geissler tubes, 218.
Generators, 148—157.
—, external excitation of, 151, 161.
Goldstein, E., 216.
Gram-atom, 233.
Gram-equivalent, 234.
Gram-molecule, 233.
Graphite, conductivity of, 248.
Grid of a valve, 213.
Gypsum crystals, arrangement to form lines of force diagrams, 22, 80.

Half-period of a radioactive element, 296.
Hammer break, 135, 177.
Heaviside layer, 333.
Helium, given off by radium, 293.
Helium ions, divalent, 288.
Henry (volt-second/ampere), 136.
Hertzian transmitter, 325.
Heusler's alloys, 131.
High frequency alternating current, see Alternating current.
High vacuum, definition of, 208.
—, dependent conduction in, 208.
Hittorf, 209.
Hooke's law, 63.
Hot-wire ammeter, see Ammeter.
Hydrogen nucleus, see Proton.
Hysteresis, 132.

Impedance, 171, 174.
Incandescent lamp, energy required by, 156.
Inductance (coefficient of self-induction), 136.
Induction, see Electromagnetic induction, Electrostatic induction.
Induction coil, 177.
—, current and pressure graphs of, 179, 180.
—, interrupters for, 177, 178.
—, of type used by physiologists, 179.
Induction furnace, 176.
Induction machine, 19, 50, 54.
— acting as a motor, 53, 55.
Induction motor, 161, 166, 167.
—, one-phase, 167.
—, two-phase, 166, 167.
—, three-phase, 167.
Inertia, mechanical, 169, 306.
— of energy, 302.
— of the magnetic field, 168, 169, 181.
Infra-red, 335.
Instantaneous value of alternating current, 117.
Insulating liquids, 245.
Insulators, 2, 29.
Integral pressure, 102.
—, due to self-induction, 135.
Intensity, see Electric Intensity, Magnetic Intensity.

Interrupters for induction coils, 177, 178.
Ionic conduction, see Conduction.
Ionium (Io), 298.
Ionization by collision, 217.
Ionization potential, 284.
Ionizers, 202.
Ions, 60.
— as electricity carriers, 202.
—, atomic, 60, 235.
—, complex, 202.
—, conduction through lattice of, 256.
—, direct measurement of velocity of, 238.
—, due to cosmic radiation, 207.
—, friction between water and, 239.
—, from bunsen flame, 203.
—, from match, 202.
— in aqueous solution, 231.
— in metallic conduction, 246.
—, length of life of, 203.
—, mobility of, 205.
—, molecular, 60, 235.
—, recombination of, 203.
—, valency of, 61.
Iron circuit, 127, 148.
Iron core in a magnetic field, 94.
Isotopes, 224.

Jet break, 178.
Joule (watt-second), 74.

Kamerlingh Onnes, 253.
Kaufmann, W., 209, 301.
Keeper, 128.
Kilowatt, 156.
Kilowatt-hour, 75.

Lead tree, 6, 256.
Leads, 7.
Lenard, 209.
Lenard's window, 222.
Length of sparks in ordinary air, 227.
Lenz's law, 103, 156.
Leyden jar, 39.
Lichtenberg's figures, 56, 57.
Light, velocity of, 300, 320, 321.
Light waves same as electric waves, 333.
Lightning, 17, 215, 280.
Lines of electric force, closed, 74, 114, 192, 327.
—, connection with pressure, 25, 27, 75.
—, representation by gypsum crystals, 22, 24.
—, shortening by conductors and insulators, 58.
Lines of magnetic force, 3.
— of permanent magnets, 2, 3, 4.
— of a straight wire carrying a current, 4, 5.
Line-integral of electric intensity, 68, 115.
— of magnetic intensity, 107, 122.
Linking-together of electric and magnetic lines of force, 123.
Loop galvanometer, 9, 12.
Loschmidt's number, see Avogadro's number.
Loud-speaker, 292.
Luminous discharge, 215.
—, distribution of pressure in, 217.

Magnet, permanent, 95.
Magnetic circuit, 129.
Magnetic field, deviation of conductor traversed by a current in, 143.
—, direction of, 88.
—, energy of, 138, 142.
—, forces in, 141.
— in the interior of a conductor, 191, 249.
—, intensity of, see Magnetic intensity.
— of a coil, 88.
— of a dipole, 330.
— of a gas traversed by a current, 7, 10.
— of a liquid traversed by a current, 5, 9.
— of a solid traversed by a current, 3, 5.
— of the earth, 93.
—, rate of change of, 106, 122.
—, uniform, 90, 111.
— with no poles, 90, 104.
Magnetic flux, 125, 130.
— increased by iron, 127, 128.
—, measurement of, 127.
Magnetic induction, 125.
— and intensity of the magnetic field, 131.
Magnetic intensity, 92.
—, line-integral of, 107, 122.
Magnetic moment, 146.
— potentiometer, Rogowski's, 107.
— resistance, see Reluctance.
— screen, 98.
— substances, non-existence of, 89, 96.
Magnetism, apparently induced in magnetic field, 97.
—, residual, 95.
Magnetometer, 91.
—, graduation of, 92.
Magnetomotive force, 106, 114, 122, 130.
— in the field of a permanent magnet, 112.
— of a current, 109.
— of a uniform magnetic field, 111.
Magnetoscope, 91.
Magnus effect, 143.
Mass, dependence on velocity, 300.
—, electrical origin of, 299.
Mass spectrograph, Aston's, 224.

Match as a source of ions, 202.
— as a resistance, 292.
Matter, ultimate constituents of, 298.
Maximum value of an alternating current, 117.
Maxwell's equations, 101, 113, 115, 122, 123.
Mean free path, 198.
Meidinger cell, 272.
Meissner, Alexander, 308.
Mercury arc lamp, 228.
Mercury vapour rectifier, 229.
Micro-ampere (10^{-6} ampere), 20.
Micro-farad (10^{-6} farad), 38.
Micron, 228.
Microphone, 154, 213, 282.
Mirror galvanometer (sensitive ammeter with optical lever), 19, 30, 36.
Mobility of electricity carriers, 40, 201, 238.
Mol, see Gram-molecule.
Molecular currents, 94, 96, 126, 129, 159, 254.
Molecular forces, 281.
Molecular ions, 60, 235.
Molecules, average distance between them in air, 198.
—, number in a cubic centimetre, 198.
Moment of inertia, 140.
Motor, commutator, 164.
—, direct current, 161.
—, induction, 161, 166, 167.
—, series-wound, 163.
—, shunt-wound, 163.
—, synchronous, 160.
Multiplier process, 51.

Negative glow, 216.
Neon, 193, 219, 317.
Neon lamp, 219, 259.
Nernst, 281.
Normal cells, see Standard cells.
Nodes of a stationary wave, 314, 319, 329.

Ohm (volt/ampere), 16.
Ohmic pressure, 170.
— resistance, 172, 175.
Ohm's law for electrolytes, 235, 239.
— for metals, 16, 246.
—, mechanism of, 200.
Optical refraction, law of, 333.
Oscillations, automatic regulation of electrical, 305.
—, circularly polarized, 164.
—, damped, 183, 220, 305.
—, electrical and mechanical, 181, 182.
—, elliptically polarized, 165.
—, forced, 309.
Oscillations, linearly polarized, 164.
—, period of electrical and mechanical, 183.
—, undamped, 305.
—, very slow electrical, 183.
Oscillatory circuit, 181.
—, spark gaps in, 185.
Oscillograph, 9, 219.
—, cathode ray, 220.
—, glow-light, 219.

Paramagnetic substances, 106.
—, motion in a magnetic field, 146.
Peat, drying by electricity, 265.
Peltier effect, 270.
Pendulum, 181, 305, 309, 312.
—, automatic regulation of mechanical, 304, 305.
Permeability, 105.
—, absolute and relative values of, 105, 106.
— of diamagnetic substances, 159.
— of iron, 130—134.
Period of an oscillatory circuit, Kelvin's formula for, 184 (equation 3).
Permanent magnet, 95.
Perrin, 209.
Phase-difference, 165.
— between current and pressure, 174.
Photo-electric effect, 291.
Photographic plate, paths of α-rays in, 291, 292.
Pinch phenomenon, 249.
Pistol bullet, measurement of velocity of, 40.
Pitchblende, radioactivity of, 283.
Plücker, 209.
Polarization in crystals, 258.
— in electrolysis, 236, 273.
— of a dielectric, 59.
— of electric waves, 334.
Poles of a cell, definition of positive and negative, 6.
Poles of a magnetic field, 89.
Pole pieces of large electromagnets, 128.
Polonium (Po), 298.
Positive column, 216, 219, 220, 225, 244.
Positive rays, 216, 223, 224.
Potential difference (electric pressure), 1.
Potential fall, 70.
Potentiometer, 17, 34.
—, magnetic, 107.
Pressure between metals and electrolytes, 271.
— between two metals, 267.
—, connection with intensity, 67.
—, induced, 105, 171, 186.
—, integral, see Integral pressure.

Pressure, numerical values in volts, 17.
—, properties of electric, 2, 13.
—, time-integral of, 102.
Primary coil, 103.
Principle of energy, 103, 274.
Proof-plane, 28.
Proof-wire, 64, 217, 236.
Proton, 224, 299.
— as an electricity carrier in metals, 246.

Quadrant electrometer, 82.
Quantity of electricity, 28.
Quantum theory, Planck's, 123.

Radiation field of a dipole, 327, 328, 330.
Radioactive collector, 65.
Radioactive substances, rays from, 286, 287.
Radioactive transmutation of the elements, 284, 299.
Radium (Ra), 293, 297.
Radium A, B, C, 297.
Radium bromide, 286.
Radium emanation (radon), decay of, 297.
Radium series, 297.
Ram, hydraulic, 169.
Reactance, 174.
Receiver for detecting displacement currents, 189, 316, 324, 325, 332.
Rectifier, mercury vapour, 229.
— with aluminium anode, 243.
— with hot cathode (two-electrode valve), 215.
Refractive index and photo-electric conduction, 259.
Reluctance, 130.
Residual magnetism, 95.
Resistance, 15.
—, magnetic, see Reluctance.
—, specific, 236, 245, 248, 258.
Resistances, practical, 15, 18, 247, 258.
—, standard, 251.
Resistance thermometer, 251.
Resonance, 187, 310, 311.
Resonator, 309, 313.
Restoring couple, 11.
Ring armature, 150.
Ring current without electrodes, 192.
Röntgen, 285.
Röntgen rays, see X-rays.
Rotary field, 157, 166, 174.
Rotary field motor, see Induction motor.
Rotational induction, 138.
Rotor, 157, 158, 160, 166.
—, squirrel-cage, 158.
Rutherford, 293, 294.

Saturation current, 206.
Scintillation method, 289.
Secondary coil, 101.
—, explanation of action of, 114.
Secondary radiation, 287.
Secondary reactions in electrolysis, 232.
Selenium cell, 259.
Self-induction, 134, 300.
—, coefficient of, 136.
— in straight conductors, 188, 190.
Short circuit, 2.
Short-circuit current, 161.
Short-wave transmission, 333.
Skin effect, 186, 190.
Slip, 158.
Soddy, 285, 294.
Solenoid, 88.
Solution pressure of metals, 281.
Source of current, 42, 194; see also Cell.
Spark, 226.
—, lag of, 227.
—, length in ordinary air, 227.
—, photographs of, 178.
Spark gaps in oscillatory circuits, 185.
Specific conductance, see Conductivity.
Specific inductive capacity, see Dielectric constant.
Specific resistance, see Resistance.
Spectrum tube, 219.
Spinthariscope, 289.
Squirrel-cage rotor, 158.
St. Elmo's fire, 215.
Standard cells, 15, 277.
Starter of an electric motor, 162, 167.
Static electricity, same as current electricity, 20.
Stationary waves, 317, 327.
Stator, 160.
Stop-watch with switch, 34, 35.
Streaming currents, 265.
Strength of current, numerical values of, 18.
String, vibrating, 313.
String galvanometer, model of, 7, 9.
String voltmeter, 81.
Superconduction, 253.
Surface density, 67.
Surface tension, 278.
Suspended particles in cataphoresis, 263.
— in electric fields, 83, 294.
—, lines of force diagram of, 73.
Synchronous motor, 160.

Telephone, 154.
— as generator of alternating current, 154.
—, old-fashioned, 154, 155.

Temperature, absolute, 251.
—, effect on conductivity of electrolytes, 237, 244.
—, effect on metallic conduction, 250.
Tension (electric pressure), 1.
Terminals, 2.
Tesla transformer, 187.
Thermal conductivity of metals, 254.
Thermo-couple, 270.
Thermo-electric pressure, 269.
Thermo-element, see Thermo-couple.
Thermo-junction, crossed-wire, 270, 317.
Thomson, J. J., 209, 216.
Thorium A, B, 296.
Thorium emanation, decay of, 296.
Time-integral of current, 34, 35.
— of pressure, 102.
Torque, 91, 160.
Transformation of energy in generators, 155.
Transformer, 175.
—, Tesla, 187.
Transformer ratio, 176.
Transmitter, 324.

Ultra-violet, 228, 335.
Unifilar electrometer or voltmeter, 81, 82.
Unipolar induction, 138.
Unit coil, 125.
Units, absolute, 13, 79.
—, international practical, 12, 15.
Uranium I and II (U I, U II), 298.

Vacuum, electric field in a, 26.
Valency, 234.
Valve, three-electrode, 213, 306.
Valve, grid-plate characteristic of, 214.
—, two-electrode, 215.
Velocity of cathode rays, 212, 300.
— of electrons in metals, 249.
Vibrations, see Oscillations.
Volt, definition of, 15.
Voltage (electric pressure), 1.
Voltameter (galvanometer on electrolytic principle), 11.
Voltmeter, electrostatic, 14, 25, 27.
— using a current, 15.
— with an auxiliary field, 82.
Volt-second, 102.

Water, decomposition of, 232.
Water-dropper, 65.
Watt-second (volt-coulomb), 74.
Waves, electric, see Electric Waves.
— in a string, 318.
—, stationary, 317, 327.
Wave-length of free electric waves, 329.
Wehnelt cathode, 220, 221.
Wiedemann-Franz rule, 255.
Wien, 216.
Wilson, C. T. R., 289.
Wimshurst machine, see Induction machine.
Wireless telegraphy, 332.
Work done by an electric field, 74.
Work of extraction of an electron, 281, 282.

X-rays, 285, 335.
—, formation of ions by, 291.
X-ray tube, gas-filled, 215, 221.
—, high vacuum (Coolidge), 212.

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